West Virginia math4life

PLC Guide

Mathematical Habits of Mind

for Middle and Secondary Mathematics

West Virginia DEPARTMENT OF EDUCATION

#math4lifeWV



West Virginia Board of Education
2021-2022

Miller L. Hall, President
Thomas W. Campbell, CPA, Vice President
F. Scott Rotruck, Financial Officer

Robert W. Dunlevy, Member
A. Stanley Maynard, Ph.D., Member
Daniel D. Snavely, M.D., Member
Debra K. Sullivan, Member
Nancy J. White, Member
James S. Wilson, D.D.S., Member

Sarah Armstrong Tucker, Ph.D., Ex Officio
Chancellor
West Virginia Higher Education Policy Commission
West Virginia Council for Community and Technical College Education

W. Clayton Burch, Ex Officio
State Superintendent of Schools
West Virginia Department of Education

The Mathematical Habits of Mind: Embedding Them into Daily Practice

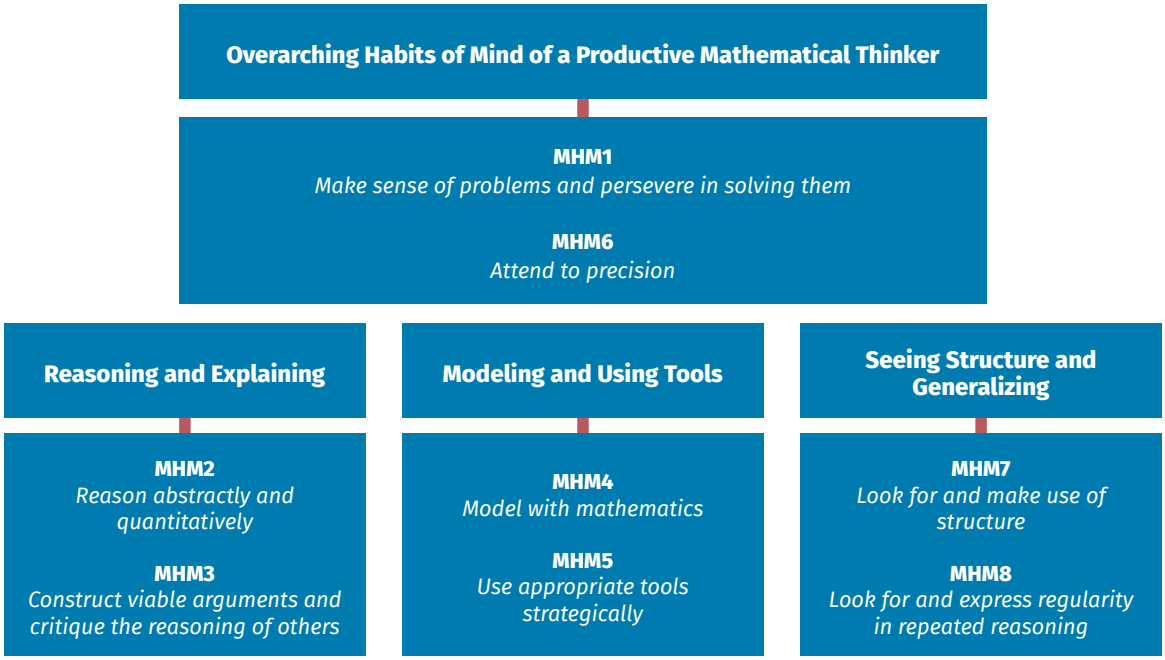
Session 1
Introduction

Foreword

Please read the following excerpt. Highlight, underline, and take notes of what you feel is of particular interest.

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards. These standards address the skills, knowledge, and dispositions students should develop to foster mathematical understanding and expertise for what students need to understand, know, and be able to do. The standards also require integration of the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards. These connections are essential in supporting the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level or course-specific Mathematics Content Standards. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Mathematical Habits of Mind



The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated with the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important "processes and proficiencies" with longstanding importance in mathematics education. The first of these is the National Council of Teachers of Mathematics' (NCTM) process standards of problem-solving, reasoning and proof, communication, representation, and connections. The second is the strands of mathematical proficiency specified in the National Research Council's report Adding It Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO, 2010).

Ideally, several Mathematical Habits of Mind will be evident in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students make sense and persevere in solving problems (MHM1), teachers need to allow students to struggle productively, and teachers must be attentive to the type of feedback they provide to students. Dr. Carol Dweck's research (Dweck, 2006) revealed that feedback offering praise of effort and perseverance seems to engender a "growth mindset." In Dweck's estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind can use these skills not only in mathematics, but also across other disciplines and into their lives beyond school, college, and career.

Discuss the importance and goals of a PLC. Do what is best for students!

Facilitate a discussion exploring Rick DuFour's four critical questions of a PLC (Keep these in mind as one progresses). These questions should be asked sequentially to define a focus on the first question. Use a whiteboard, sticky notes, etc. to share responses.

1. What do we want students to learn?
2. How do we know students learned it?
3. What do we do when students don't learn it?
4. What do we do when students authentically learn it?

Facilitate a discussion on the PTSA Cycle and the focus on the Mathematical Habits of Mind:

PTSA (Plan, Teach, Study, Act) Cycle

- Plan (Lesson Planning)
- Teach (Teaching and Monitoring)
- Study (Analyze Evidence of Student Learning)
- Act (Next Steps/Interventions)

Introduction to the Mathematical Habits of Mind PLC. Present the MHMs but do not discuss them.

The Mathematical Habits of Mind

- MHM1. Make sense of problems and persevere in solving them
- MHM2. Reason abstractly and quantitatively
- MHM3. Construct viable arguments and critique the reasoning of others
- MHM4. Model with mathematics
- MHM5. Use appropriate tools strategically
- MHM6. Attend to precision
- MHM7. Look for and make use of structure
- MHM8. Look for and express regularity in repeated reasoning

Before exploring the eight Mathematical Habits of Mind, teachers should reflect on their practice and identify specific ways they have implemented the habits of mind in their practice.

As a PLC/mathematics department/team, teachers should reflect privately and then record the answers to key questions such as:

- In your opinion, what are the purposes of the eight Mathematical Habits of Mind?
- What am I doing in my classroom to implement the Mathematical Habits of Mind? How often do I implement them into my lesson?
- Which Mathematical Habit(s) of Mind do I incorporate into my classroom most frequently? Least frequently?
- Do I fully understand the eight Mathematical Habits of Mind and how to utilize them in my classroom?
- How am I effectively and intentionally implementing the Mathematical Habits of Mind?
- How am I using Mathematical Habits of Mind to ensure ALL students are learning deeply?
- How is my use of the Mathematical Habits of Mind improving student learning and mastery of the West Virginia College- and Career-Readiness Standards?
- How have I educated my students and their parents about Mathematical Habits of Mind?

Action Step

Reflection responses should be documented to revisit throughout the Mathematical Habits of Mind PLC.

During the time between PLCs, teachers should compile and analyze their classroom lessons to identify specific Mathematical Habits of Mind being utilized in their lessons. Teachers are encouraged to focus on one specific class during this PLC. Additionally, a lesson that is rich in mathematics may enact several Mathematical Habits of Mind; however, teachers must identify at most one or two primary Mathematical Habits of Mind. Teachers must track daily lessons for this class until the next PLC meeting and be ready to provide specific examples of the Mathematical Habits of Mind in their lessons.

Session 2

Reflection on Teaching the Mathematical Habits of Mind and MHM1/MHM6

As a PLC/department/team, address the following questions:

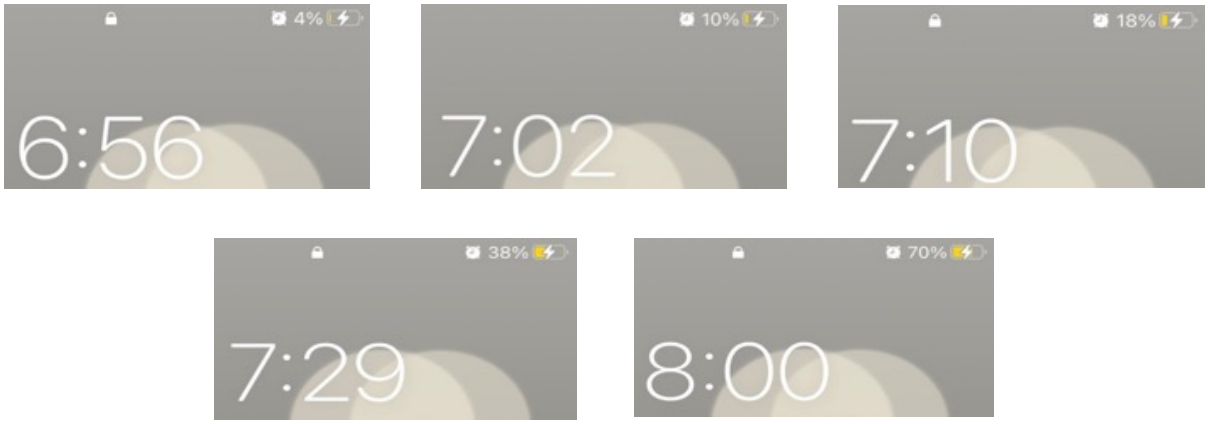
- What Mathematical Habits of Mind were used the most in your lessons?
- What Mathematical Habits of Mind were used the least in your lessons?
- How did this compare with your reflections practice prior to tracking the use of the Mathematical Habits of Mind in your lesson?
- Does the use of the Mathematical Habits of Mind correlate more with your level of comfort utilizing them in your classroom or to your view of mastering the West Virginia College- and Career-Readiness Standards in your class?

Find/create an activity to simulate MHM1/MHM6.

Throughout this activity emphasize the importance of MHM1 and MHM6.

Sample activity:

1. Provide four minutes of individual think time to produce notice and wonder statements/questions about the following images:



What do you notice? What do you wonder?

2. Discuss the notice and wonder statements/questions as a group.
3. Next, separate participants into small groups to produce a problem from the given images. The groups will need to generate an appropriate grade-level question and solve the problem. This activity should last 15 minutes.
4. Upon completion of the group discussion, there are two options for sharing:
   a. Other group participants can try to solve the created questions/problems from each small group. Then discuss their solutions.
   b. A brief conversation about each question posed, created, and discovered can be presented by each group to the whole group.

5. Ask all participants to work individually on the following prompts related to the images provided:
    a. Use multiple representations to display the given information. Attend to precision when representing the data.
    b. What type of function best represents the given data? Why?
    c. Find a line of best fit for the data.
    d. Predict the time at which the phone will be fully charged. Explain your reasoning.
    e. Would this data, information, and prediction be accurate for all phones and chargers? Why or why not?
6. Have participants return to small groups to discuss their results.
7. Finally, allow each small group to share their findings.

This activity is an adaptation of the "Charge!" DESMOS activity found here: *Charge! • Activity Builder by Desmos*

Facilitate a discussion with the following questions. Use whiteboards, sticky notes, etc. to gather responses from participants.

- How do you know if a task, activity, problem, etc. incorporates MHM1 and MHM6?
- What can mathematically proficient students do to demonstrate MHM1 and MHM6?

Use the following resources from the West Virginia Department of Education's website to continue the discussion about MHM1 and MHM6.

- *20941_MHM1-Resources-v2.pdf (wvde.us)*
- *20941_MHM6-Resources-v2.pdf (wvde.us)*

Action Step

- Create and implement a lesson that utilizes MHM1 and/or MHM6 in your classroom. Explain how the lesson uses these Mathematical Habits of Mind.
    - » After the lesson, reflect on your experience incorporating these Mathematical Habits of Mind in your classroom. Including this lesson, how many times were MHM1 and MHM6 present in your lessons since the last PLC session?
- Continue to track your use of the other Mathematical Habits of Mind with specifically focused evidence on MHM1 and MHM6.

Session 3
Reasoning and Explaining - MHM2 and MHM3

Facilitate reflection/discussion for everyone using Rick DuFour's four critical questions of a PLC. Keep in mind the focus should be on how these questions were addressed using the lesson that emphasized MHM1 and MHM6.

1. What do we want students to learn?
2. How do we know students learned it?
3. What do we do when students don't learn it?
4. What do we do when students authentically learn it?

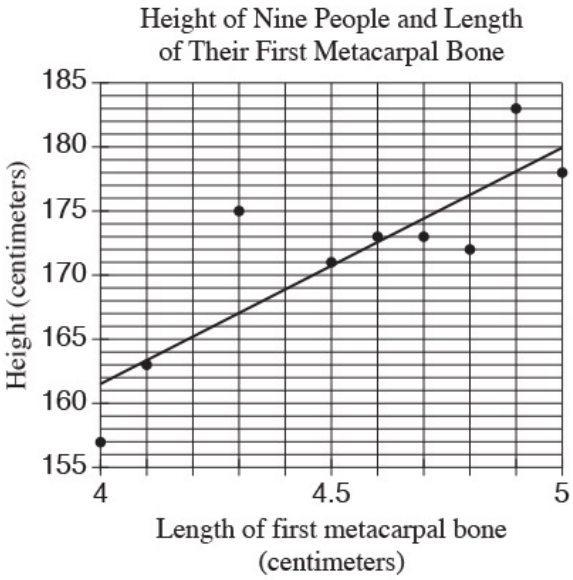
Find/create an activity to simulate MHM2/MHM3.

Throughout this activity emphasize the importance of MHM2 and MHM3.

Sample activity:

1. Read the prompt to the participants and look at the graph below:

   The first metacarpal bone is located in the wrist. The scatterplot below shows the relationship between the length of the first metacarpal bone and height for 9 people. The line of best fit is also shown.



   This prompt and graph were part of SAT practice problems found here: *M:C:MC:SET:2A | SAT Suite of Assessments (collegeboard.org)*

2. Provide two minutes of individual think time to produce notice statements and wonder questions:

   What do you notice? What do you wonder?

3. Share notice statements and wonder questions aloud, using technology, on whiteboards, or with sticky notes, etc. to facilitate a whole group discussion.

4. Next, in small groups, create a question for students using the prompt and graph. Then, answer the question as a group.
5. As a whole group, allow each group to share their questions, answers, and discussion that took place in their small groups.
6. After that, share the actual questions from the SAT practice exam below and have participants solve the problems:

   How many of the nine people have an actual height that differs by more than 3 centimeters from the height predicted by the line of best fit?

   Which of the following is the best interpretation of the slope of the line of best fit in the context of this problem?

   Based on the line of best fit, what is the predicted height for someone with a first metacarpal bone that has a length of 4.45 centimeters?

7. Assign participants to random groups to discuss one question and its solution.
8. Use the jigsaw method to have participants share solutions with their original notice/wonder small groups.

Facilitate a discussion with the following questions. Use whiteboards, sticky notes, etc. to gather responses from participants.

- How do you know if a task, activity, problem, etc. incorporates MHM2 and MHM3?
- What can mathematically proficient students do to demonstrate MHM2 and MHM3?

Use the following resources from the West Virginia Department of Education's website to continue the discussion about MHM2 and MHM3.

- *20941_MHM2-Resources-v2.pdf (wvde.us)*
- *20941_MHM3-Resources-v2.pdf (wvde.us)*

Action Step

- Create and implement a lesson that utilizes MHM2 and MHM3 in your classroom. Explain how the lesson uses these MHMs.
  - » After the lesson, reflect on your experience incorporating these MHMs in your classroom. Including this lesson, how many times have MHM2 and MHM3 been present in your lessons since the last PLC session?
- Continue to track your use of the other Mathematical Habits of Mind with specifically focused evidence on MHM2 and MHM3.

Session 4
Modeling and Using Tools - MHM4 and MHM5

Facilitate reflection/discussion for everyone using Rick DuFour's four critical questions of a PLC. Keep in mind the focus should be on how these questions were addressed using the lesson that emphasized MHM2 and MHM3.

1. What do we want students to learn?
2. How do we know students learned it?
3. What do we do when students don't learn it?
4. What do we do when students authentically learn it?

Find/create an activity to simulate MHM4/MHM5.

Throughout this activity emphasize the importance of MHM4 and MHM5.

Sample activity:

Follow the directions on this modeling activity on DESMOS.

- *Card Sort: Modeling • Activity Builder by Desmos*

Instructional Note: DESMOS provides teachers opportunities to create activities through the Activity Builder feature. Explore this option to create your own activity (for example, another card sort) that addresses grade-level appropriate standards.

While this activity does include multiple representations (verbally, graphically, algebraically) of modeling, this does not incorporate these two Mathematical Habits of Mind with intent in which they are written.

- First, let us examine MHM4. There is modeling of relationships between quantities. However, the specific models are chosen by the DESMOS cards or the teacher.
- Second, DESMOS is the tool being utilized to complete this activity. However, this tool was uniquely identified by the teacher for this activity.

These two Mathematical Habits of Mind must be student-centered not teacher-centered as represented in this activity. To properly incorporate MHM4, students must choose the representation to model a real-world scenario provided to them. Students will connect the mathematics with the model that they feel works best for the situation. For MHM5, once again students must choose the appropriate tool. Students will determine what tool assists them in solving a problem. The key to these two MHMs is ensuring that students are not told what model or what tool to use by the teacher but instead are choosing the model and tool that will help them solve an authentic problem.
How can this DESMOS activity be modified to ensure MHM4 and MHM5 are being properly implemented?

Facilitate a discussion with the following questions. Use whiteboards, sticky notes, etc. to gather responses from participants.

- How do you know if a task, activity, problem, etc. incorporates MHM4 and MHM5?
- What can students do to proficiently demonstrate MHM4 and MHM5?

Use the following resources from the West Virginia Department of Education's website to continue the discussion about MHM4 and MHM5.

- *20941_MHM4-Resources-v2.pdf (wvde.us)*
- *20941_MHM5-Resources-v1.pdf (wvde.us)*

Action Step

- Create and implement a lesson that utilizes MHM4 and MHM5 in your classroom. Explain how the lesson uses these Mathematical Habits of Mind.
  - » After the lesson, reflect on your experience incorporating these Mathematical Habits of Mind in your classroom. Including this lesson, how many times have MHM4 and MHM5 been present in your lessons since the last PLC session?
- Continue to track your use of the other Mathematical Habits of Mind with specifically focused evidence on MHM4 and MHM5.

Session 5
Seeing Structure and Generalizing - MHM7 and MHM8

Facilitate reflection/discussion for everyone using Rick DuFour's four critical questions of a PLC. Keep in mind the focus should be on how these questions were addressed using the lesson that emphasized MHM4 and MHM5.

1. What do we want students to learn?
2. How do we know students learned it?
3. What do we do when students don't learn it?
4. What do we do when students authentically learn it?

Find/create an activity to simulate MHM7/MHM8.

Throughout this activity emphasize the importance of MHM7 and MHM8.

Sample Activity:

One of the most difficult aspects of the Mathematical Habits of Mind is recognizing the difference between MHM7 and MHM8. Here are two activities to help identify the difference in the standards.

- MHM7
  - » Go to this website and download SAT Practice Test 1
    - *Free Official SAT Practice Tests | College Board*
  - » Participants should have five to ten minutes to try to solve these problems and find multiple ways to find answers. Make sure work is shown for all problems.
    - Math Test – No Calculator #8
    - Math Test – No Calculator #14
    - Math Test – Calculator #4
  - » Break into small groups to discuss answers.
  - » As a whole group, discuss ways to answer the questions.
  - » Download and review the SAT Practice Test 1 Answer Explanations to see how they are explained. Note the emphasis on structure in the explanations. Discuss how this will change your approach to emphasizing structure in your classroom.

- MHM8
  - » Complete the activity on Exploring Exponents from Exploring Exponents
    - *YouCubed: Exploring-Exponents-Handout.pdf (netdna-ssl.com)*
  - » As a whole group, discuss the findings of the activity and specifically talk about the ways repeated reasoning was used.

Facilitate a discussion with the following questions. Use whiteboards, sticky notes, etc. to gather responses from participants.

- » How do you know if a task, activity, problem, etc. incorporates MHM7 and MHM8?
- » What can mathematically proficient students do to demonstrate MHM7 and MHM8?

Use the following resources from the West Virginia Department of Education's website to continue the discussion about MHM7 and MHM8.

- *20941_MHM7-Resources-v2.pdf (wvde.us)*
- *20941_MHM8-Resources-v2.pdf (wvde.us)*

Action Step

- Create and implement a lesson that utilizes MHM7 and MHM8 in your classroom. Explain how the lesson uses these Mathematical Habits of Mind.
    - » After the lesson, reflect on your experience incorporating these Mathematical Habits of Mind in your classroom. Including this lesson, how many times have MHM7 and MHM8 been present in your lessons since the last PLC session?
- Continue to track your use of the other Mathematical Habits of Mind with specifically focused evidence on MHM7 and MHM8.

Session 6
Reflection on MHM PLC

Facilitate a reflection/discussion for everyone using Rick DuFour's four critical questions of a PLC. Keep in mind the focus should be on how these questions were addressed using the lesson that emphasized MHM7 and MHM8.

1. What do we want students to learn?
2. How do we know students learned it?
3. What do we do when students don't learn it?
4. What do we do when students authentically learn it?

Each participant should respond to each statement/question. Facilitate a discussion around these statements/questions.

- Rank the MHMs in order of use throughout this PLC. Reflect on this list. Were you surprised by your results? Why or why not? Did your exploration of the MHMs and intentionality of use increase during this PLC? Why or why not?
- After completing this PLC, what are the purposes of the eight Mathematical Habits of Mind?
- What changes did you make in your classroom to implement the Mathematical Habits of Mind? How often were they implemented into your lessons?
- Which Mathematical Habit(s) of Mind did you feel was best incorporated in your classroom that addressed the intent, rigor, and depth of grade-level appropriate standards? Why?
- Which MHM did you feel was the most difficult for you to incorporate into your lessons to address the intent, rigor, and depth of grade-level appropriate standards? Why?
- Which Mathematical Habit(s) of Mind do you incorporate into my classroom most frequently? Least frequently? Does this correlate with your understanding of the Mathematical Habits of Mind? Why or why not?
- Do you have a better understanding of the eight Mathematical Habits of Mind and how to utilize them in your classroom? Why or why not?
- How are you using Mathematical Habits of Mind to ensure ALL students are learning deeply?
- How have you educated your students and their parents about the MHMs?

Reflection by each participant:

The National Council of Teachers of Mathematics (NCTM) discusses the importance of personal and collective professional growth towards effective teaching and learning of mathematics.

"As professional, mathematics teachers recognize that their own learning is never finished and continually seek to improve and enhance their mathematical knowledge for teaching, their knowledge of mathematical pedagogy, and their knowledge of students as learners of mathematics." (Principles to Action, NCTM, 2014)

1. Did the PLC review of the Mathematical Habits of Mind offer opportunities to enhance your mathematical knowledge for teaching? Identify and explain this growth in mathematical understanding.
2. Might these opportunities encourage you to continue to acquire mathematical knowledge related to the same concepts? How could you go about researching these concepts?
3. How did the focus on Mathematical Habits of Mind provide opportunities to enhance your knowledge of mathematical pedagogy? Explain how this new knowledge can serve to inform classroom instruction.
4. Did the Mathematical Habits of Mind PLC provide opportunities to enhance your knowledge of students as learners of mathematics? What new insights regarding student reasoning developed because of this exploration?
5. How has intentionally connecting the Mathematical Habits of Mind into your lesson plan changed your approach to teaching the West Virginia College- and Career-Readiness standards? Has the focused attention on Mathematical Habits of Mind increased the depth and rigor in your lessons? How?
6. How has the use of the PTSA Cycle impacted your teaching and learning?



W. Clayton Burch
West Virginia Superintendent of Schools