

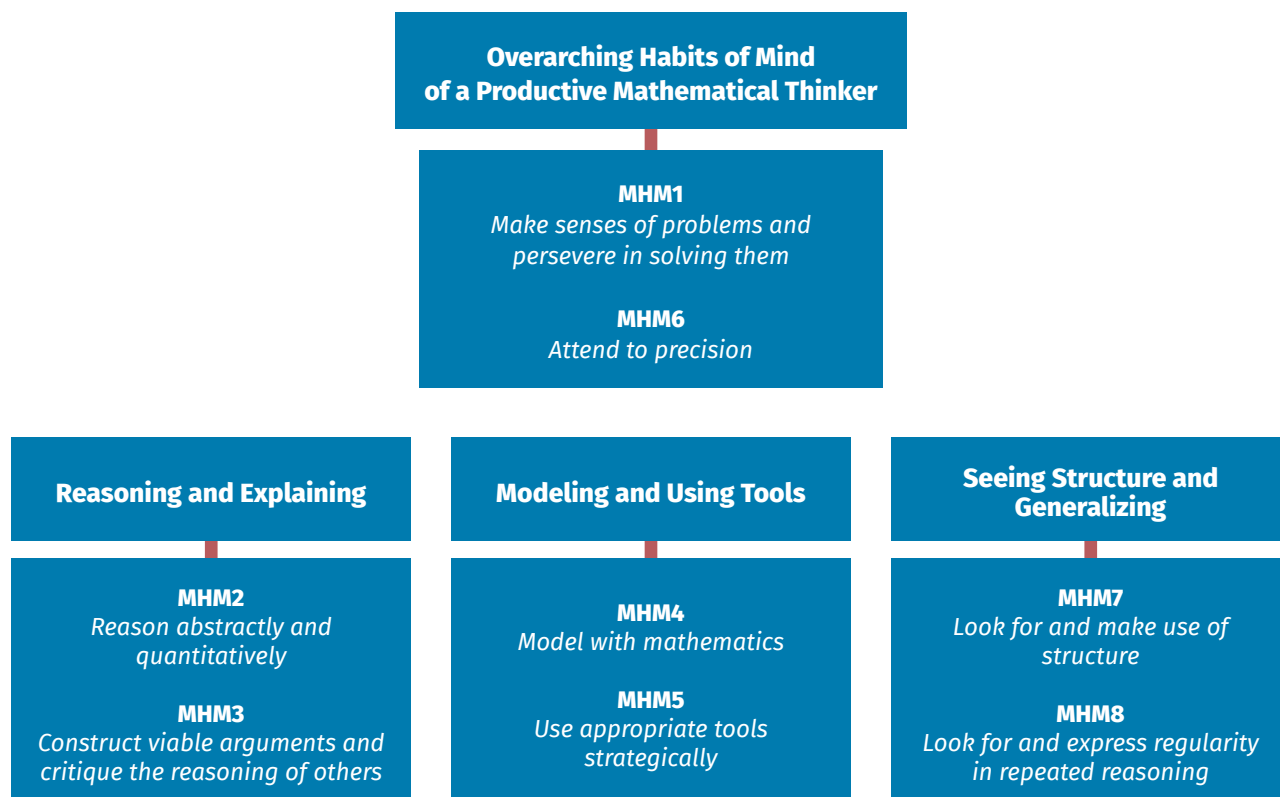
Mathematics – High School Mathematics I



Overview of the West Virginia College- and Career-Readiness Standards for Mathematics

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards. These standards address the skills, knowledge, and dispositions that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge – what students need to understand, know, and be able to do. The standards also require that the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards be connected. These connections are essential to support the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level or course-specific Mathematics Content Standards. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Mathematical Habits of Mind



The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important “processes and proficiencies “ with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics’ process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report Adding it Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010).

Ideally, several Mathematical Habits of Mind will be embedded in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MHM1), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck’s research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender a “growth mindset.” In Dweck’s estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind are able to use these skills not only in mathematics, but across disciplines and into their lives beyond school, college, and career.

Policy 2520.2B

West Virginia College- and Career-Readiness Standards for Mathematics

Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a

flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

MHM8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics – High School Mathematics I

All West Virginia teachers are responsible for classroom instruction that integrates content standards and the Mathematical Habits of Mind. Students in this course will focus on five critical domains that deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Students in Mathematics 1 will use properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades and develop connections between the algebraic and geometric ideas studied. The Mathematical Habits of Mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Expressions and Equations	Functions
Interpret algebraic expressions and transforming them purposefully to solve problems (e.g., in solving a problem about a loan with interest rate r and principal P, seeing the expression $P(1+r)^n$ as a product of P with a factor not depending on P).	Understand contextual relationships of variables and constants (e.g., Annie is picking apples with her sister; The number of apples in her basket is described by $n = 22t + 12$, where t is the number of minutes Annie spends picking apples. What do the numbers 22 and 12 tell you about Annie's apple picking?).
Geometry: Basics of Geometry	Geometry: Transformations and Congruence
Prove theorems about triangles and other figures (e.g., that the sum of the measures of the angles in a triangle is 180°).	- Given a transformation, work backwards to discover the sequence that led to the transformation. - Given two quadrilaterals that are reflections of each other, find the line of that reflection.
Statistics and Probability	
Use linear regression techniques to describe the relationship between quantities and assess the fit of the model (e.g., use the high school and university grades for 250 students to create a model that can be used to predict a student's university GPA based on his high school GPA).	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within High School Mathematics I:

Expressions and Equations	
Interpret the structure of expressions and equations in terms of the context they model.	Standards A1HS.1-2
Extend the properties of exponents to rational exponents.	Standards A1HS.3-4
Write expressions in equivalent forms to solve problems.	Standard A1HS.5
Create equations that describe numbers or relationships.	Standards A1HS.7-9
Solve equations and inequalities in one variable.	Standard A1HS.10
Solve systems of equations.	Standards A1HS.12-14
Represent and solve equations and inequalities graphically.	Standards A1HS.16-18
Functions	
Understand the concept of a function and use function notation.	Standards A1HS.19-21

Interpret functions that arise in applications in terms of a context.	Standard A1HS.22
Analyze functions using different representations.	Standards A1HS.23-25
Build a function that models a relationship between two quantities.	Standards A1HS.26-27
Build new functions from existing functions.	Standard A1HS.28
Construct and compare linear and exponential models and solve problems.	Standard A1HS.29
Geometry: Basics of Geometry	
Experiment with transformations in the plane.	Standard GHS.1
Identify and utilize inductive and deductive reasoning.	Standard GHS.2
Prove geometric theorems.	Standard GHS.3
Use coordinates to prove simple geometric theorems algebraically.	Standard GHS.4
Make geometric constructions.	Standard GHS.5
Geometry: Transformations and Congruence	
Experiment with transformations in the plane.	Standards GHS.6-9
Understand congruence in terms of rigid motions.	Standards GHS.10-13
Use coordinates to prove simple geometric theorems algebraically.	Standards A1HS.30-31, GHS.16
Statistics and Probability	
Summarize, represent, and interpret data on a single count or measurement variable.	Standards A1HS.32-34
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standard A1HS.35
Interpret linear models.	Standards A1HS.36-37

Cluster	Interpret the structure of expressions and equations in terms of the context they model.
M.A1HS.1	Interpret linear and exponential expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. c. Interpret the parameters in a linear function or exponential function of the form $f(x) = a \cdot b^x$ in terms of a context.
M.A1HS.2	Use the structure of exponential expressions to identify ways to rewrite them.
Cluster	Extend the properties of exponents to rational exponents.
M.A1HS.3	Explain the connections between expressions with rational exponents and expressions with radicals using properties of exponents. Extend from application of properties of exponents for expressions with integer exponents.
M.A1HS.4	Rewrite expressions involving radicals, including simplifying, and rational exponents using the properties of exponents.
Cluster	Write expressions in equivalent forms to solve problems.
M.A1HS.5	Choose and produce an equivalent form of linear and exponential expressions to reveal and explain properties of the quantity represented by the expression through connections to a graphical representation of the function. • Use the properties of exponents to transform expressions in exponential functions. For example, the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. Instructional Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.
Cluster	Create equations that describe numbers or relationships.
M.A1HS.7	Create equations and inequalities in one variable, representing linear and exponential relationships, and use them to solve problems. In the case of exponential equations, limit to situations with integer inputs.
M.A1HS.8	Create equations in two or more variables, representing linear and exponential relationships between quantities. In the case of exponential equations, limit to situations with integer inputs.

M.A1HS.9	Represent constraints by linear equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.
Cluster	Solve equations and inequalities in one variable.
M.A1HS.10	Solve linear equations, including equations with coefficients represented by letters, simple exponential equations that rely only on application of the laws of exponents, and compound linear inequalities in one variable.
Cluster	Solve systems of equations.
M.A1HS.12	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve simple cases by inspection (e.g., $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6). c. Solve real-world and mathematical problems leading to two linear equations in two variables (e.g., given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair).
M.A1HS.13	Understand and demonstrate ways to manipulate a system of two equations and two variables while preserving its solution set.
M.A1HS.14	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Include examples of solution sets with no solutions, an infinite number of solutions, and one solution.
Cluster	Represent and solve equations and inequalities graphically.
M.A1HS.16	Recognize that the graph of a linear or exponential equation in two variables is the set of all its solutions plotted in the coordinate plane.
M.A1HS.17	Explain why the x-coordinates of the points where the graphs of the linear and/or exponential equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations).

M.A1HS.18	Graph the solutions of a linear inequality in two variables as a half-plane and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.
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Functions

Cluster	Understand the concept of a function and use function notation.
M.A1HS.19	Use multiple representations of linear and exponential functions to recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Develop function notation utilizing the definition of a function to represent situations both algebraically and graphically.
M.A1HS.20	Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
M.A1HS.21	Recognize arithmetic and geometric sequences are functions, sometimes defined recursively, whose domain is a subset of the integers (e.g., the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$).
Cluster	Interpret functions that arise in applications in terms of a context.
M.A1HS.22	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of quantities, and sketch graphs showing key features given a verbal description of the relationship. Relate the domain of a function to its linear and exponential graphs and, where applicable, to the quantitative relationship it describes. • Key features of linear and exponential graphs include: intercepts; and intervals where the function is increasing, decreasing, positive, or negative.
Cluster	Analyze functions using different representations.
M.A1HS.23	Graph linear and exponential functions expressed symbolically and show key features of the graph. - For linear functions, focus on intercepts. - For exponential functions, focus on intercepts and end behavior. Instructional Note: Provide opportunities for students to graph and show key features by hand and using technology.
M.A1HS.24	Compare properties of two linear or exponential functions each represented in a different way (algebraically, graphically, numerically in tables, or from verbal descriptions).
M.A1HS.25	Write a function defined by a linear or exponential expression in different but equivalent forms to reveal and explain different properties of the function. • Use the properties of exponents to interpret expressions in exponential functions.
Cluster	Build a function that models a relationship between two quantities.

M.A1HS.26	Write linear and exponential functions that describe a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations.
M.A1HS.27	Construct linear and exponential functions, including explicit forms of arithmetic and geometric sequences to model situations, given a graph, a description of a relationship, or given input-output pairs (include reading these from a table).
Cluster	Build new functions from existing functions.
M.A1HS.28	Identify the effect on the graphs of linear and exponential functions, $f(x)$, with $f(x) + k$, for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.
Cluster	Construct and compare linear, quadratic, and exponential models and solve problems.
M.A1HS.29	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

Geometry: Basics of Geometry

Cluster	Experiment with transformations in the plane.
M.GHS.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
Cluster	Identify and utilize inductive and deductive reasoning.
M.GHS.2	Construct and justify the validity of a logical argument. a. Identify the converse, inverse, and contrapositive of a conditional statement. b. Translate a short, verbal argument into symbolic form. c. Use Venn diagrams to represent set relationships. d. Use inductive and deductive reasoning.

Cluster	Prove geometric theorems.
M.GHS.3	Use appropriate methods of proof to prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent.
Cluster	Use coordinates to prove simple geometric theorems algebraically.
M.GHS.4	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
Cluster	Make geometric constructions.
M.GHS.5	Make formal geometric constructions with a variety of tools and methods, such as a compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.: a. copying a segment; b. copying an angle; c. bisecting a segment; d. bisecting an angle; e. constructing perpendicular lines, including the perpendicular bisector of a line segment; and f. constructing a line parallel to a given line through a point not on the line.

Geometry: Transformations and Congruence

Cluster	Experiment with transformations in the plane.
M.GHS.6	Build on prior knowledge from rigid motions to: a. Represent transformations using geometric concepts in the plane. b. Describe transformations as functions that take points in the plane as inputs and give other points as outputs. c. Compare transformations that preserve distance and angle to those that do not.
M.GHS.7	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
M.GHS.8	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
M.GHS.9	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, for example, graph paper, tracing paper, or geometry software. Describe a sequence of transformations that will carry a given figure onto another.
Cluster	Understand congruence in terms of rigid motions.
M.GHS.10	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.

M.GHS.11	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
M.GHS.12	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
M.GHS.13	Use congruence criteria for triangles to solve problems and to prove relationships in geometric figures.
Cluster	Use coordinates to prove simple geometric theorems algebraically.
M.A1HS.30	Prove the slope criteria for parallel and perpendicular lines and use the slope criteria to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).
M.A1HS.31	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles. Instructional Note: Using the distance formula provides practice with the distance formula and its connection with the Pythagorean theorem.
M.GHS.16	Use coordinates to prove simple geometric theorems about right triangles.

Statistics and Probability

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.
M.A1HS.32	Select applicable representations to display data on the real number line (e.g., dot plots, histograms, and box plots).
M.A1HS.33	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation only as a tool to describe spread and not to explicitly find standard deviation) of two or more different data sets.
M.A1HS.34	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
Cluster	Summarize, represent, and interpret data on two categorical and quantitative variables.
M.A1HS.32	Represent data on two quantitative variables on a scatter plot and describe how the variables are related. - Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. - Informally assess the fit of a function by plotting and analyzing residuals. Focus should be on situations for which linear models are appropriate. - Fit a linear function for scatter plots that suggest a linear association.
Cluster	Interpret linear models.