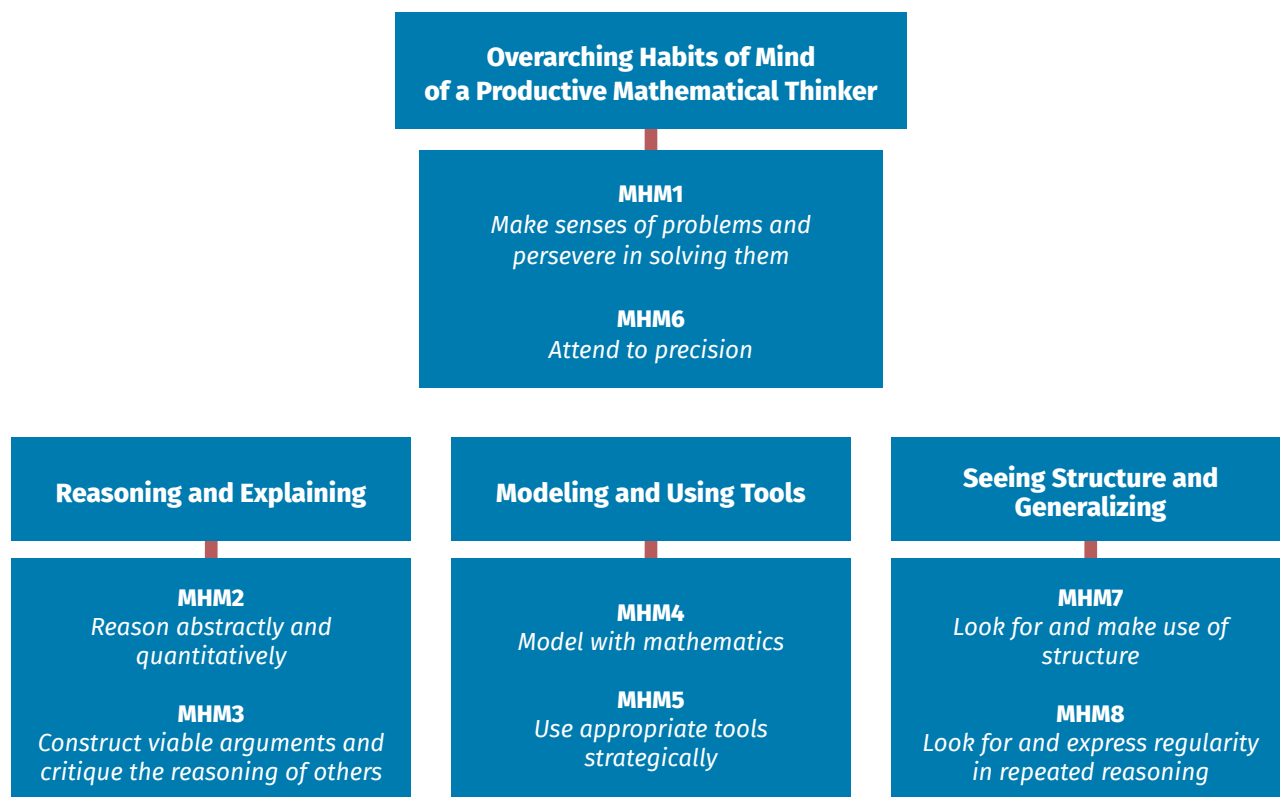


Overview of the West Virginia College- and Career-Readiness Standards for Mathematics

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards. These standards address the skills, knowledge, and dispositions that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge – what students need to understand, know, and be able to do. The standards also require that the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards be connected. These connections are essential to support the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level or course-specific Mathematics Content Standards. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Mathematical Habits of Mind



The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important “processes and proficiencies “ with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics’ process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council’s report Adding it Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010).

Ideally, several Mathematical Habits of Mind will be embedded in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MHM1), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck’s research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender a “growth mindset.” In Dweck’s estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind are able to use these skills not only in mathematics, but across disciplines and into their lives beyond school, college, and career.

Policy 2520.2B

West Virginia College- and Career-Readiness Standards for Mathematics

Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a

flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y .

MHM8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics – Grade 7

West Virginia teachers who provide mathematics instruction must integrate content standards with the MHM. Students in the seventh grade will focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions and working with two- and three-dimensional shapes to solve problems involving area, surface area and volume; and (4) drawing inferences about populations based on samples. The MHM, which should be integrated in these content areas, include: making sense of problems and persevering in solving them; reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision; looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students in seventh grade will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from sixth grade, the following chart represents the mathematical understandings that will be developed in seventh grade:

Ratios and Proportional Reasoning	The Number System
Analyze proportional relationships (e.g., by graphing in the coordinate plane), and distinguish proportional relationships from other kinds of mathematical relationships (e.g., buying 10 times as many items will cost you 10 times as much, but taking 10 times as many aspirin will not lower your fever 10 times as much).	- Solve percent problems (e.g., tax, tips, and markups and markdowns). - Solve word problems that have a combination of whole numbers, fractions, and decimals (e.g., a woman making \$25 per hour receives a 10% raise; she will make an additional $\frac{1}{10}$ of his or her salary an hour, or \$2.50, for a new salary of \$27.50).
Expressions and Equations	Geometry
Solve equations such as $\frac{1}{2}(x - 3) = \frac{3}{4}$ quickly and accurately and write equations of this kind to solve word problems.	Solve problems involving scale drawings, circle measurements, triangle attributes, and angle relationships.
Statistics and Probability	
Use statistics and probability to draw inferences and make comparisons (e.g., deciding which candidate is likely to win an election based on a survey).	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Ratios and Proportional Relationships	
Analyze proportional relationships and use them to solve real-world and mathematical problems.	Standards 1-3
The Number System	
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	Standards 4-6
Expressions and Equations	
Use properties of operations to generate equivalent expressions.	Standards 7-8
Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	Standards 9-10
Geometry	
Draw, construct and describe geometrical figures and describe the relationships between them.	Standards 11-13
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	Standards 14-16

Statistics and Probability

Use random sampling to draw inferences about a population.	Standards 17-18
Draw informal comparative inferences about two populations.	Standards 19-22
Investigate chance processes and develop, use, and evaluate probability models.	Standards 23-26

Ratios and Proportional Relationships

Cluster	Analyze proportional relationships and use them to solve real-world and mathematical problems.
M.7.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units (e.g., if a person walks $\frac{1}{2}$ mile in each $\frac{1}{4}$ hour, compute the unit rate as the complex fraction $\frac{1/2}{1/4}$ miles per hour, equivalently 2 miles per hour).
M.7.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin). b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations (e.g., if total cost t is proportional to the number n of items purchased at a constant price p , the relationship between the total cost and the number of items can be expressed as $t = pn$). d. Explain what a point (x,y) on the graph of a proportional relationship means in terms of the situation. Focus special attention on the points $(0,0)$ and $(1,r)$ where r is the unit rate.
M.7.3	Use proportional relationships to solve multistep ratio and percent problems (e.g., simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and/or percent error).

Cluster	Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
M.7.4	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. - Describe situations in which opposite quantities combine to make 0 (e.g., a hydrogen atom has 0 charge because its two constituents are oppositely charged). - Understand $p + q$ as the number located a distance q from p, in the positive or negative direction, depending on whether q is positive or negative. (e.g., to add “$p + q$” on the number line, start at “p” then move q in the positive or negative direction depending on whether “q” is positive or negative). Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. - Understand subtraction of rational numbers as adding the additive inverse, $p - q = p + (-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts. - Apply properties of operations as strategies to add and subtract rational numbers.
M.7.5	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. - Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1) = 1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. - Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then $-(p/q) = (-p)/q = p/(-q)$. Interpret quotients of rational numbers by describing real-world contexts. - Apply properties of operations as strategies to multiply and divide rational numbers. - Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
M.7.6	Solve real-world and mathematical problems involving the four operations with rational numbers. Instructional Note: Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Cluster	Use properties of operations to generate equivalent expressions.
M.7.7	Apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients.
M.7.8	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related (e.g., $a + 0.05a = 1.05a$ means that “increase by 5%” is the same as “multiply by 1.05”).
Cluster	Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
M.7.9	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies (e.g., if a woman making \$25 an hour gets a 10% raise, she will make an additional $\frac{1}{10}$ of her salary an hour, or \$2.50, for a new salary of \$27.50; if you want to place a towel bar $9\frac{3}{4}$ inches long in the center of a door that is $27\frac{1}{2}$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation).
M.7.10	Use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities. - Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach (e.g., the perimeter of a rectangle is 54 cm; its length is 6 cm; what is its width?; an arithmetic solution similar to “$54 - 6 - 6$ divided by 2” may be compared with the reasoning involved in solving the equation $2w + 12 = 54$; an arithmetic solution similar to “$54/2 - 6$” may be compared with the reasoning involved in solving the equation $2(w + 6) = 54$). - Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem (e.g., as a salesperson, you are paid \$50 per week plus \$3 per sale; this week you want your pay to be at least \$100; write an inequality for the number of sales you need to make and describe the solutions).

Geometry

Cluster	Draw, construct and describe geometrical figures and describe the relationships between them.
M.7.11	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
M.7.12	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine the following: - a unique triangle (e.g., three side measures satisfy the triangle inequality theorem), - more than one triangle (e.g., given three angles whose sum is 180 degrees), or - no triangle (e.g., angle sum is not 180 degrees or sum of the measures of two sides does not exceed the measure of the third side).
M.7.13	Describe the two-dimensional figures (face shapes) that result from slicing three-dimensional figures with cuts made parallel to, perpendicular to, or neither parallel nor perpendicular to the bases of right rectangular prisms and right rectangular pyramids.
Cluster	Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
M.7.14	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. Instructional Note: Include problems where students must determine area given circumference but not vice versa.
M.7.15	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
M.7.16	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics and Probability

Cluster	Use random sampling to draw inferences about a population.
M.7.17	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

M.7.18	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions (e.g., estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data; gauge how far off the estimate or prediction might be).
Cluster	Draw informal comparative inferences about two populations.
M.7.19	Given two data displays, distinguish measures of center and measures of variation.
M.7.20	Compare two numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and describing any overall pattern(s). d. Giving quantitative measures of variability (interquartile range (IQR), range, and/or mean absolute deviation (MAD)) and describing any striking deviations from the overall pattern(s). e. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
M.7.21	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability (e.g., the mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable).
M.7.22	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations (e.g., decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book).
Cluster	Investigate chance processes and develop, use, and evaluate probability models.
M.7.23	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely and a probability near 1 indicates a likely event.
M.7.24	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency and predict the approximate relative frequency given the probability (e.g., when rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times). Instructional Note: Provide adequate models for students to understand relative frequency is the experimental probability which when given enough trials will match the theoretical probability.

M.7.25	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. - Develop a uniform probability model by assigning equal probability to all outcomes and use the model to determine probabilities of events (e.g., if a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected). - Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process (e.g., find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down; do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?).
M.7.26	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. - Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. - Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. - Design and use a simulation to generate frequencies for compound events (e.g., use random digits as a simulation tool to approximate the answer to the question: if 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?).