The Mathematical Habits of Mind Overview

The Mathematical Habits of Mind and the Mathematics Content Standards are integral components of the West Virginia College- and Career-Readiness Standards for Mathematics. The Mathematical Habits of Mind address the attributes and characteristics that students develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge—what students need to understand, know, and be able to do.

The Mathematical Habits of Mind are:

Connected: Ideally, several Mathematical Habits of Mind are evident in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. The content standards and the Mathematical Habits of Mind cannot be isolated from one another. Mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Equitable: All students must have access to the Mathematical Habits of Mind. The skills developed through the Habits of Mind are metacognition skills. Much like the content standards, students may need support, scaffolds, and increased opportunities to master the Habits of Mind.

Intentional: The Mathematical Habits of Mind must be taught as purposefully and practiced with the same intention as the Mathematics Content Standards. The Mathematical Habits of Mind represent a picture of what it looks like for students to understand and do mathematics both in and out of the classroom. Every math lesson should coherently and robustly integrate at least one of the Mathematical Habits of Mind.

Ongoing: The Mathematical Habits of Mind are developed throughout each year and across all grade levels and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject.

Authentic: The intent of the West Virginia College- and Career-Readiness Standards for Mathematics is to prepare all West Virginia students for college, careers, and civic life. The Mathematical Habits of Mind develop mathematically competent individuals who can use mathematics as a tool for making wise decisions in their personal lives, a foundation for rewarding work, and a means for comprehending and influencing the world in which they live.

Mathematical Habit of Mind 8 – Look for and express regularity in repeated reasoning.

This document combines information from several sources into one in-depth look at Mathematical Habit of Mind 8.

Mathematical Habits of Mind in Policy

The following exert is from WV Policy 2510:

- The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$ and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Overview of MHM8 – What it is, What it does, and What it looks like

MHM8. Look for and express regularity in repeated reasoning.			
What it is	**What it does**	**What it looks like**	
Showing students how a problem works, looking at shortcuts, repeated calculations, and attending to details.	Allows students to take their mathematical reasoning, apply it to other situations, and generalize to other problem types. • See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details in the problem-solving steps. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results.	***Students:*** • Notice if calculations are repeated and look for both general methods and shortcuts. • Pay attention to regularity and use it to solve problems. • Use regularity to develop a general formula and generalizations. • Maintain oversight of the process of solving a problem while attending to details and continually evaluate the reasonableness of immediate results.	***Teacher:*** • Provides problem situations that allow students to explore regularity and repeated reasoning. • Provides rich tasks that encourage students to use repeated reasoning to form generalizations and provides opportunities for students to communicate these generalizations.

Developing Mathematical Habits of Mind through Questions and Expressing in Student-Friendly Language

The following chart includes both the MHM in student-friendly language and examples of questions teachers might use to support mathematical thinking and student engagement.

Mathematical Habit of Mind	MHM Expressed in Student-Friendly Language	Questions to Develop Mathematical Thinking
MHM8. Look for and express regularity in repeated reasoning.	I can use a strategy I used to solve another math problem. I can notice repeated calculations and use them to create shortcuts.	• Explain how this strategy works in other situations. • Is this always true, sometimes true, or never true? • How would we prove that _____? • What do you notice about _____? • What is happening in this situation? What would happen if_____ ? • Is there a mathematical rule for _____? • What predictions or generalizations can this pattern support? • What mathematical consistencies do you notice? • How is this situation like and different from other situations using this operation?

Rubric – Implementing Mathematical Habits of Mind

Use the Task descriptors in developing lessons to ensure that classroom tasks help cultivate the MHMs. The teacher descriptors can be used during or after the lesson to evaluate how the task was carried out. The column titled "Proficient" describes the expected norm for task and teacher action, while the column titled "Exemplary" includes all features of the proficient column and more. A task is exemplary when meeting criteria in both the proficient and exemplary columns.

MHM8	DESCRIPTOR	NEEDS IMPROVEMENT	EMERGING (teacher does the thinking)	PROFICIENT (teacher mostly models)	EXEMPLARY (students take ownership)
Look for and express regularity in repeated reasoning.	**Task**	• Is disconnected from prior and future concepts. • Has no logical progression that leads to pattern recognition.	• Is overly repetitive or has gaps that do not allow for development of a pattern.	• Reviews prior knowledge and requires cumulative understanding. • Lends itself to developing a pattern or structure.	• Addresses and connects to prior knowledge in a non-routine way. • Requires recognition of pattern or structure to be completed.
	Teacher	• Does not show evidence of understanding the hierarchy within concepts. • Presents or examines task in isolation.	• Hides or does not draw connections to prior or future concepts.	• Connects concept to prior and future concepts to help students develop an understanding of procedural shortcuts. • Demonstrates connections between tasks.	• Encourages students to connect task to prior concepts and tasks. • Prompts students to generate exploratory questions based on current task. • Encourages students to monitor each other's intermediate results.

The Vertical Progression of the Mathematical Habit of Mind 8

The Mathematical Habits of Mind are an integral part of the West Virginia College- and Career-Readiness Standards for Mathematics. This Vertical Progression document has taken grade specific information about the Mathematical Habits of Mind from the West Virginia Educators' Guides for Mathematics to display how the Habits of Mind develop and grow from Kindergarten to High School. The document also showcases the similarities of the Habits of Mind at each grade level.

MHM8 – Look for and express regularity in repeated reasoning.	
Kindergarten Students:	• notice repetitive actions in counting, computations, and mathematical tasks. For example: » the next number in a counting sequence is 1 more when counting by ones, » 10 more when counting by tens (or 1 more group of 10). • should be given problems like: » "There are 8 crayons in the box. Some are red and some are blue. How many of each could there be?" » realize 8 crayons could include 4 of each color (8 = 4 + 4), 5 of one color and 3 of another (8 = 5 + 3), and so on. » For each solution, students repeatedly engage in the process of finding two numbers to join together to equal 8.
Grade 1 Students:	• notice repetitive actions in counting and computation. • gain a better understanding of place value. • check work by asking, "Does this make sense?" • begin to look for regularity in problem structures when solving mathematical tasks. For example: » add three one-digit numbers by using strategies such as "make a ten" or doubles. • recognize when and how to use strategies to solve similar problems. For example: » when evaluating 8 + 7 + 2, a student may say, "I know 8 and 2 equals 10, then I add 7 to get to 17. It helps if I can make a ten out of two numbers when I start." • use repeated reasoning while solving a task with multiple correct answers. For example: » "There are 12 crayons in the box. Some are red and some are blue. How many of each color could there be?" » use repeated reasoning to find pairs of numbers that add up to 12 (e.g., the 12 crayons could include 6 of each color [6 + 6 = 12], 7 of one color and 5 of another [7 + 5 = 12], and so on).
Grade 2 Students:	• notice repetitive actions in counting and computation (e.g., number patterns to count by tens or hundreds). • check for the reasonableness of solutions during and after completion of a task by asking themselves, "Does this make sense?"
Grade 3 Students:	• notice repetitive actions in computations and look for "shortcut" methods. • use the distributive property as a strategy to work with products of numbers they know to solve products they do not know. For example: » to find the product of 7 × 8, decompose 7 into 5 and 2 and then multiply 5 × 8 and 2 × 8 to arrive at 40 + 16, or 56. • evaluate work by asking, "Does this make sense?"

Grade 4 Students:	• notice repetitive actions in computation to make generalizations. • use models to explain calculations and understand how algorithms work. • examine patterns and generate algorithms. For example: » use visual fraction models to write equivalent fractions.
Grade 5 Students:	• use repeated reasoning to understand algorithms and make generalizations about patterns. • connect place value and prior work with operations to understand and use algorithms to extend multi-digit division from one-digit to two-digit divisors and to fluently multiply multi-digit whole numbers. • use various strategies to perform all operations with decimals to hundredths. • explore operations with fractions with visual models and begin to formulate generalizations.
Grade 6 Students:	• use repeated reasoning to understand algorithms and make generalizations about patterns. • make use of opportunities to solve and model problems designed to support generalizing through noticing $\frac{a}{b} \div \frac{c}{d} = \frac{ad}{bc}$ and construct other examples and models that confirm generalizations. • connect place value and prior work with operations to understand algorithms to fluently divide multi-digit numbers. • perform all operations with multi-digit decimals. • begin to make connections between covariance, rates, and representations that show the relationships between quantities.
Grade 7 Students:	• use repeated reasoning to understand algorithms and make generalizations about patterns. • notice that $\frac{a}{b} = \frac{c}{d}$ if and only if ad = bc and construct other examples and models that confirm generalizations.
Grade 8 Students:	• use repeated reasoning to understand the slope formula and to make sense of rational and irrational numbers. • through modelling linear relationships, notice the slope of the graph of the linear relationship and the rate of change of the associated function are the same. For example: » after repeatedly checking whether points are on the line with a slope of 3 go through the point (1, 2), abstract the equation of the line in the form $\frac{y-2}{x-1} = 3$. • divide to find decimal equivalents of rational numbers (e.g., $\frac{2}{3} = 0.\overline{6}$) and generalize observations. • use iterative processes to determine more precise rational approximations for irrational numbers.
Algebra I and Math I Students:	• realize the key feature of a line in the plane is an equal difference in outputs over equal intervals of inputs, and the result of evaluating the expression $(y_2-y_1)/(x_2-x_1)$ for points on the line is always equal to a certain number m. Therefore, if (x, y) is a generic point on this line, the equation $m = \frac{y_2 - y_1}{x_2 - x_1}$ will give a general equation of the line.
Geometry and Math II Students:	• explore rotations, reflections, and translations, noticing some attributes of shapes (e.g., parallelism, congruency, orientation) remain the same. • develop properties of transformations by generalizing these observations.

Algebra II and Math III Students:	• observe patterns in geometric sums—for example, the first several sums of the form $\sum_{k=0}^{n}(2^k-1)$ can be written as follows: $n=0, \sum_{k=0}^{0}(2^k-1) = 2^0-1$ $= 1-1$ $= 0$ $n=1, \sum_{k=0}^{1}(2^k-1) = (2^0-1)+(2^1-1)$ $= 0+(2-1)$ $= 0+1$ $= 1$ $n=2, \sum_{k=0}^{2}(2^k-1) = (2^0-1)+(2^1-1)+(2^2-1)$ $= 0+1+(4-1)$ $= 0+1+3$ $= 4$ $n=3, \sum_{k=0}^{3}(2^k-1) = (2^0-1)+(2^1-1)+(2^2-1)+(2^3-1)$ $= 0+1+3+(8-1)$ $= 0+1+3+7$ $= 11$