The Mathematical Habits of Mind Overview

The Mathematical Habits of Mind and the Mathematics Content Standards are integral components of the West Virginia College- and Career-Readiness Standards for Mathematics. The Mathematical Habits of Mind address the attributes and characteristics that students develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge—what students need to understand, know, and be able to do.

The Mathematical Habits of Mind are:

Connected: Ideally, several Mathematical Habits of Mind are evident in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. The content standards and the Mathematical Habits of Mind cannot be isolated from one another. Mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Equitable: All students must have access to the Mathematical Habits of Mind. The skills developed through the Habits of Mind are metacognition skills. Much like the content standards, students may need support, scaffolds, and increased opportunities to master the Habits of Mind.

Intentional: The Mathematical Habits of Mind must be taught as purposefully and practiced with the same intention as the Mathematics Content Standards. The Mathematical Habits of Mind represent a picture of what it looks like for students to understand and do mathematics both in and out of the classroom. Every math lesson should coherently and robustly integrate at least one of the Mathematical Habits of Mind.

Ongoing: The Mathematical Habits of Mind are developed throughout each year and across all grade levels and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject.

Authentic: The intent of the West Virginia College- and Career-Readiness Standards for Mathematics is to prepare all West Virginia students for college, careers, and civic life. The Mathematical Habits of Mind develop mathematically competent individuals who can use mathematics as a tool for making wise decisions in their personal lives, a foundation for rewarding work, and a means for comprehending and influencing the world in which they live.

Mathematical Habit of Mind 7 – Look for and make use of structure.

This document combines information from several sources into one in-depth look at Mathematical Habit of Mind 7.

Mathematical Habits of Mind in Policy

The following exert is from WV Policy 2510:

- The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

Overview of MHM7 – What it is, What it does, and What it looks like

MHM7. Look for and make use of structure.			
What it is	**What it does**	**What it looks like**	
Looking for patterns and recognizing the significant aspects of mathematical problems using clear definitions.	Allows students to identify multiple strategies, select the best one, and see complicated situations as being made of multiple parts. Students will use what they know is true to accurately solve a new problem. • See repeated calculations and look for generalizations and shortcuts. • See the overall process of the problem and still attend to the details in the problem-solving steps. • Understand the broader application of patterns and see the structure in similar situations. • Continually evaluate the reasonableness of their intermediate results.	***Students:*** • Look closely to discern patterns or structure. • Associate patterns with properties of operations and their relationships. • Step back for an overview and can shift perspective. • See complicated things, such as algebraic expressions, as single objects or as composed of several objects. (Younger children decompose and compose numbers.)	***Teacher:*** • Provides opportunities and time for students to explore patterns and relationships to solve problems. • Provides rich tasks and facilitates pattern seeking and understanding of relationships in numbers rather than following a set of steps and/or procedures.

Developing Mathematical Habits of Mind through Questions and Expressing in Student-Friendly Language

The following chart includes both the MHM in student-friendly language and examples of questions teachers might use to support mathematical thinking and student engagement.

Mathematical Habit of Mind	MHM Expressed in Student-Friendly Language	Questions to Develop Mathematical Thinking
MHM7. Look for and make use of structure.	I can use what I already know about math to solve the problem. I can see complicated problems as being made of multiple, often simpler, parts. When solving a problem, I can identify multiple ways to approach it and then, choose the best way. I can understand patterns in mathematics and see the structure in similar problems.	• How would you describe the problem in your own words? • How would you describe what you are trying to find? • What do you notice about _____? • What information is given in the problem? • Describe the relationship between the quantities. • Describe what you have already tried. What might you change? • Talk me through the steps you have used to this point. • What steps in the process are you most confident about? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin? • How else might you [organize, represent, show, etc.] _____?

Rubric – Implementing Mathematical Habits of Mind

Use the Task descriptors in developing lessons to ensure that classroom tasks help cultivate the MHMs. The teacher descriptors can be used during or after the lesson to evaluate how the task was carried out. The column titled "Proficient" describes the expected norm for task and teacher action, while the column titled "Exemplary" includes all features of the proficient column and more. A task is exemplary when meeting criteria in both the proficient and exemplary columns.

MHM7	DESCRIPTOR	NEEDS IMPROVEMENT	EMERGING (teacher does the thinking)	PROFICIENT (teacher mostly models)	EXEMPLARY (students take ownership)
Look for and make use of structure.	**Task**	• Requires students to automatically apply an algorithm to a task without evaluating its appropriateness.	• Requires students to analyze a task before automatically applying an algorithm.	• Requires students to analyze a task and identify more than one approach to the problem.	• Requires students to identify the most efficient solution to the task.
	Teacher	• Does not recognize students for developing efficient approaches to the task. • Requires students to apply the same algorithm to a task although there may be other approaches.	• Identifies individual students' efficient approaches, but does not expand understanding to the rest of the class. • Demonstrates the same algorithm to all related tasks although there may be other more effective approaches.	• Facilitates all students in developing reasonable and efficient ways to accurately perform basic operations. • Continuously questions students about the reasonableness of their intermediate results.	• Prompts students to identify mathematical structure of the task in order to identify the most effective solution path. • Encourages students to justify their choice of algorithm or solution path.

The Vertical Progression of the Mathematical Habit of Mind 7

The Mathematical Habits of Mind are an integral part of the West Virginia College- and Career-Readiness Standards for Mathematics. This Vertical Progression document has taken grade specific information about the Mathematical Habits of Mind from the West Virginia Educators' Guides for Mathematics to display how the Habits of Mind develop and grow from Kindergarten to High School. The document also showcases the similarities of the Habits of Mind at each grade level.

MHM7 – Look for and make use of structure.	
Kindergarten Students:	• begin to discern a pattern or structure in the number system. For instance, students recognize that 3 + 2 = 5 and 2 + 3 = 5. • use counting strategies, such as counting on, counting all, or taking away, to build fluency with facts to 5. • notice the written pattern in the "teen" numbers—that the numbers start with 1 (representing 1 ten) and end with the number of additional ones. • should be asked questions such as, "What do you notice when _____?"
Grade 1 Students:	• look for patterns and structures in the number system and other areas of mathematics. • begin to recognize the commutative property—for example, 7 + 4 = 11, and 4 + 7 = 11. • realize any two-digit number can be broken up into tens and ones (e.g., 35 = 30 + 5, 76 = 70 + 6). • make use of structure when working with subtraction as an unknown addend problem. For example: 13 – 7 = __ can be written as 7 + __ = 13 and can be thought of as "How much more do I need to add to 7 to get to 13?"
Grade 2 Students:	• look for patterns and structures in the number system. • notice number patterns within the tens place when connecting counting by tens to corresponding numbers on a hundreds chart. • see structure in the base-ten number system by understanding 10 ones equal a ten, and 10 tens equal a hundred. • adopt mental math strategies based on patterns (making ten, fact families, doubles). • use structure to understand subtraction as an unknown addend problem. For example: 50 – 33 = __ can be written as 33 + __ = 50 and can be thought of as "How much more do I need to add to 33 to get to 50?"
Grade 3 Students:	• look closely to discover a pattern or structure. • use properties of operations (e.g., commutative and distributive properties) as strategies to multiply and divide.
Grade 4 Students:	• look closely to discover a pattern or structure. • use properties of operations to explain calculations (partial products model). • generate number or shape patterns that follow a given rule.
Grade 5 Students:	• look closely to discover a pattern or structure. • use properties of operations as strategies to add, subtract, multiply, and divide with whole numbers, fractions, and decimals. • examine numerical patterns and relate them to a rule or a graphical representation.

MHM7 – Look for and make use of structure.	
Grade 6 Students:	• seek patterns or structures to model and solve problems. • notice patterns existing in ratio tables, recognizing both the additive and multiplicative properties. • apply properties to generate equivalent expressions. For example: 6 + 3x = 3(2 + x) by the distributive property. • solve equations. For example: 2c + 3 = 15, 2c = 12 by the subtraction property of equality, c = 6 by the division property of equality. • compose and decompose two- and three-dimensional figures to solve real-world problems involving area and volume.
Grade 7 Students:	• seek patterns or structures to model and solve problems. • recognize patterns existing in ratio tables, making connections between the constant of proportionality in a table with the slope of a graph. • apply properties to generate equivalent expressions and solve equations. • compose and decompose two- and three-dimensional figures to solve real-world problems involving scale drawings, surface area, and volume. • examine tree diagrams or systematic lists to determine the sample space for compound events and verify all possibilities are listed. • recognize solving an equation such as $8 = 4(x - \frac{1}{2})$ is easier when making use of structure, temporarily viewing $(x - \frac{1}{2})$ as a single entity.
Grade 8 Students:	• seek patterns or structures to model and solve problems. • apply properties to generate equivalent expressions and solve equations. • examine patterns in tables and graphs to generate equations and describe relationships. • experimentally verify the effects of transformations and describe them in terms of congruence and similarity.
Algebra I and Math I Students:	• develop formulas such as $(a \pm b)^2 = a^2 \pm 2ab + b^2$ by applying the distributive property. • observe the expression $5 + (x - 2)^2$ as 5 plus "something squared," and because "something squared" must be positive or zero, the expression can be no smaller than 5.
Geometry and Math II Students:	• construct triangles in quadrilaterals or other shapes and use congruence criteria of triangles to justify results about those shapes.
Algebra II and Math III Students:	• view the operations of complex numbers as extensions of the operations for real numbers. • understand the periodicity of sine and cosine and use these functions to model periodic phenomena.