The Mathematical Habits of Mind Overview

The Mathematical Habits of Mind and the Mathematics Content Standards are integral components of the West Virginia College- and Career-Readiness Standards for Mathematics. The Mathematical Habits of Mind address the attributes and characteristics that students develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge—what students need to understand, know, and be able to do.

The Mathematical Habits of Mind are:

Connected: Ideally, several Mathematical Habits of Mind are evident in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. The content standards and the Mathematical Habits of Mind cannot be isolated from one another. Mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Equitable: All students must have access to the Mathematical Habits of Mind. The skills developed through the Habits of Mind are metacognition skills. Much like the content standards, students may need support, scaffolds, and increased opportunities to master the Habits of Mind.

Intentional: The Mathematical Habits of Mind must be taught as purposefully and practiced with the same intention as the Mathematics Content Standards. The Mathematical Habits of Mind represent a picture of what it looks like for students to understand and do mathematics both in and out of the classroom. Every math lesson should coherently and robustly integrate at least one of the Mathematical Habits of Mind.

Ongoing: The Mathematical Habits of Mind are developed throughout each year and across all grade levels and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject.

Authentic: The intent of the West Virginia College- and Career-Readiness Standards for Mathematics is to prepare all West Virginia students for college, careers, and civic life. The Mathematical Habits of Mind develop mathematically competent individuals who can use mathematics as a tool for making wise decisions in their personal lives, a foundation for rewarding work, and a means for comprehending and influencing the world in which they live.

Mathematical Habit of Mind 6 – Attend to precision.

This document combines information from several sources into one in-depth look at Mathematical Habit of Mind 6.

Mathematical Habits of Mind in Policy

The following exert is from WV Policy 2510:

- The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

Overview of MHM6 – What it is, What it does, and What it looks like

MHM6. Attend to precision.			
What it is	**What it does**	**What it looks like**	
Speaking and solving mathematical problems with exactness, using clear definitions.	Enables students to make use of precise and exact math language. Their measurements will be exact, numbers will be precise, and explanations will be detailed. • Use available tools including visual models, recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. • Use technological tools to deepen their understanding of mathematics.	***Students:*** • Use and clarify mathematical definitions in discussions and in their own reasoning (orally and in writing). • Use, understand and state the meanings of symbols. • Express numerical answers with a degree of precision.	***Teacher:*** • Facilitates, encourages and expects precision in communication. • Provides opportunities for students to explain and/or write their reasoning to others.

Developing Mathematical Habits of Mind through Questions and Expressing in Student-Friendly Language

The following chart includes both the MHM in student-friendly language and examples of questions teachers might use to support mathematical thinking and student engagement.

Mathematical Habit of Mind	MHM Expressed in Student-Friendly Language	Questions to Develop Mathematical Thinking
MHM6. Attend to precision.	I can use correct mathematical terms in my discussions and thinking. I can use and understand the meaning of mathematical symbols. I can check to see if my strategies are well thought out and my calculations are correct.	• What mathematical terms apply in this situation? • How did you know your solution was reasonable? • Explain how you might show that your solution answers the problem. • What would be a more efficient strategy? • How are you showing the meaning of the quantities? • What symbols or mathematical notations are important in this problem? • What mathematical language, definitions, properties (and so forth) can you use to explain _____? • Can you say it in a different way? • Can you say it in your own words? And, now say it in mathematical words. • How could you test your solution to see if it answers the problem?

Rubric – Implementing Mathematical Habits of Mind

Use the Task descriptors in developing lessons to ensure that classroom tasks help cultivate the MHMs. The teacher descriptors can be used during or after the lesson to evaluate how the task was carried out. The column titled "Proficient" describes the expected norm for task and teacher action, while the column titled "Exemplary" includes all features of the proficient column and more. A task is exemplary when meeting criteria in both the proficient and exemplary columns.

MHM6	DESCRIPTOR	NEEDS IMPROVEMENT	EMERGING (teacher does the thinking)	PROFICIENT (teacher mostly models)	EXEMPLARY (students take ownership)
Attend to precision.	**Task**	• Gives imprecise instructions.	• Has overly detailed or wordy instructions.	• Has precise instructions.	• Includes assessment criteria for communication of ideas.
	Teacher	• Does not intervene when students are being imprecise. • Does not point out instances when students fail to address the question completely or directly.	• Inconsistently intervenes when students are imprecise. • Identifies incomplete responses but does not require student to formulate further response.	• Consistently demands precision in communication and in mathematical solutions. • Identifies incomplete responses and asks student to revise their response.	• Demands and models precision in communication and in mathematical solutions. • Encourages students to identify when others are not addressing the question completely.

The Vertical Progression of the Mathematical Habit of Mind 6

The Mathematical Habits of Mind are an integral part of the West Virginia College- and Career-Readiness Standards for Mathematics. This Vertical Progression document has taken grade specific information about the Mathematical Habits of Mind from the West Virginia Educators' Guides for Mathematics to display how the Habits of Mind develop and grow from Kindergarten to High School. The document also showcases the similarities of the Habits of Mind at each grade level.

MHM6 - Attend to precision.	
Kindergarten Students:	• begin to develop precise communication skills, calculations, and measurements. • describe actions, strategies, and reasoning using grade-level appropriate vocabulary. • work with pictorial representations and concrete objects to help develop understanding and descriptive vocabulary. For example: » analyze and compare two- and three-dimensional shapes. » sort objects based on appearance. • make sure objects are directly beside each other when measuring. • check work to ensure the accuracy and reasonableness of solutions.
Grade 1 Students:	• use clear and precise language in discussions with others and when explaining reasoning. • use precise communication, calculation, and measurement skills. • describe solution strategies for mathematical tasks using grade-level appropriate vocabulary, precise explanations, and mathematical reasoning. • measure objects iteratively (repetitively), checking to make sure there are no gaps or overlaps. • check work to ensure the accuracy and reasonableness of solutions.
Grade 2 Students:	• use clear and precise language in discussions with others and when explaining reasoning. • communicate clearly, using grade-level appropriate vocabulary accurately and using precise explanations and reasoning to explain processes and solutions. For example: » carefully line up the measurement tools to get an accurate measurement. » consider if answers are reasonable and accurate.
Grade 3 Students:	• develop mathematical communication skills. • use clear and precise language in discussions with others and when explaining reasoning. • specify units of measure. • state the meaning of symbols.
Grade 4 Students:	• develop mathematical communication skills. • use clear and precise language in discussions with others and when explaining reasoning. • specify units of measure. • state the meaning of symbols.
Grade 5 Students:	• continue to refine mathematical communication skills through the use of clear and precise language in discussions with others and when explaining reasoning. • use appropriate terminology when referring to expressions, fractions, geometric figures, and coordinate grids. • specify units of measure. • state the meaning of symbols.

MHM6 - Attend to precision.	
Grade 6 Students:	• continue to refine mathematical communication skills through the use of clear and precise language in discussions with others and when explaining reasoning. • use appropriate terminology when referring to rates, ratios, geometric figures, data displays, and components of expressions, equations, or inequalities. • specify units of measure and label axes to clarify the correspondence with quantities in a problem when using ratio reasoning in solving problems. • express numerical answers with an appropriate degree of precision when working with rational numbers in a situational problem.
Grade 7 Students:	• continue to refine mathematical communication skills through the use of clear and precise language in discussions with others and when explaining reasoning. • define variables, specify units of measure, and label axes accurately. • use appropriate terminology when referring to rates, ratios, probability models, geometric figures, data displays, and components of expressions, equations, or inequalities.
Grade 8 Students:	• continue to refine mathematical communication skills through the use of clear and precise language in discussions with others and when explaining reasoning. • use appropriate terminology when referring to the number system, functions, geometric figures, and data displays.
Algebra I and Math I Students:	• understand a rational number has a specific definition and irrational numbers exist. • make use of the definition of function when deciding if an equation can describe a function by asking, “Does every input value have exactly one output value?”
Geometry and Math II Students:	• develop and use precise definitions of geometric terms. • verify a particular shape has specific properties and justify the categorization of the shape (e.g., a rhombus versus a quadrilateral).
Algebra II and Math III Students:	• make note of the precise definition of complex number, understanding real numbers are a subset of complex numbers. • utilize units in real-world problems. • use unit analysis as a method of verifying answers.