The Mathematical Habits of Mind Overview

The Mathematical Habits of Mind and the Mathematics Content Standards are integral components of the West Virginia College- and Career-Readiness Standards for Mathematics. The Mathematical Habits of Mind address the attributes and characteristics that students develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge—what students need to understand, know, and be able to do.

The Mathematical Habits of Mind are:

Connected: Ideally, several Mathematical Habits of Mind are evident in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. The content standards and the Mathematical Habits of Mind cannot be isolated from one another. Mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Equitable: All students must have access to the Mathematical Habits of Mind. The skills developed through the Habits of Mind are metacognition skills. Much like the content standards, students may need support, scaffolds, and increased opportunities to master the Habits of Mind.

Intentional: The Mathematical Habits of Mind must be taught as purposefully and practiced with the same intention as the Mathematics Content Standards. The Mathematical Habits of Mind represent a picture of what it looks like for students to understand and do mathematics both in and out of the classroom. Every math lesson should coherently and robustly integrate at least one of the Mathematical Habits of Mind.

Ongoing: The Mathematical Habits of Mind are developed throughout each year and across all grade levels and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject.

Authentic: The intent of the West Virginia College- and Career-Readiness Standards for Mathematics is to prepare all West Virginia students for college, careers, and civic life. The Mathematical Habits of Mind develop mathematically competent individuals who can use mathematics as a tool for making wise decisions in their personal lives, a foundation for rewarding work, and a means for comprehending and influencing the world in which they live.

Mathematical Habit of Mind 5 – Use appropriate tools strategically.

This document combines information from several sources into one in-depth look at Mathematical Habit of Mind 5.

Mathematical Habits of Mind in Policy

The following exert is from WV Policy 2510:

- The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

Overview of MHM5 – What it is, What it does, and What it looks like

MHM5. Use appropriate tools strategically.			
What it is	**What it does**	**What it looks like**	
Deciding what tool is appropriate to use, i.e. spreadsheet, graph, or computer software.	Gives students the opportunity to select the appropriate math tool to use to correctly solve problems. • Use available tools including visual models, recognizing the strengths and limitations of each. • Use estimation and other mathematical knowledge to detect possible errors. • Identify relevant external mathematical resources to pose and solve problems. Use technological tools to deepen understanding of mathematics.	***Students:*** • Consider available tools when solving a mathematical problem. • Are familiar with a variety of mathematics tools and use them appropriately to explore and deepen their understanding of concepts.	***Teacher:*** • Provides a variety of tools and technology for students to explore to deepen their understanding of math concepts. • Provides problem solving tasks that require students to consider a variety of tools for solving. (Tools might include pencil/paper, concrete models, ruler, protractor, calculator, spreadsheet, computer algebra system, statistical package, or dynamic geometry software, etc.)

Developing Mathematical Habits of Mind through Questions and Expressing in Student-Friendly Language

The following chart includes both the MHM in student-friendly language and examples of questions teachers might use to support mathematical thinking and student engagement.

Mathematical Habit of Mind	MHM Expressed in Student-Friendly Language	Questions to Develop Mathematical Thinking
MHM5. Use appropriate tools strategically.	I can choose the correct math tool to use to solve problems. I can estimate to determine if my answer is reasonable. I can use math tools, pictures, drawings, and objects to solve problems. I know technology tools help me to understand mathematics.	• What mathematical tools could we use to visualize and represent the situation? • What information do you have? • What do you know that is not stated in the problem? • What approach would you consider trying first? • What estimate did you make for the solution? • In this situation, would it be helpful to use a [graph, number line, ruler, diagram, calculator, manipulatives, etc.]? • Why was it helpful to use _____? • What can using a _____ show us that _____ may not? • In what situations might it be more informative or helpful to use _____?

Rubric – Implementing Mathematical Habits of Mind

Use the Task descriptors in developing lessons to ensure that classroom tasks help cultivate the MHMs. The teacher descriptors can be used during or after the lesson to evaluate how the task was carried out. The column titled "Proficient" describes the expected norm for task and teacher action, while the column titled "Exemplary" includes all features of the proficient column and more. A task is exemplary when meeting criteria in both the proficient and exemplary columns.

MHM5	DESCRIPTOR	NEEDS IMPROVEMENT	EMERGING (teacher does the thinking)	PROFICIENT (teacher mostly models)	EXEMPLARY (students take ownership)
Use appropriate tools strategically.	**Task**	• Does not incorporate additional learning tools.	• Lends itself to one learning tool. • Does not involve mental computations or estimation.	• Lends itself to multiple learning tools. • Gives students opportunity to develop fluency in mental computations.	• Requires multiple learning tools (i.e., graph paper, calculator, manipulatives). • Requires students to demonstrate fluency in mental computations.
	Teacher	• Does not incorporate additional learning tools.	• Demonstrates use of appropriate learning tool.	• Chooses appropriate learning tools for student use. • Models error checking by estimation.	• Allows students to choose appropriate learning tools. • Creatively finds appropriate alternatives where tools are not available.

The Vertical Progression of the Mathematical Habit of Mind 5

The Mathematical Habits of Mind are an integral part of the West Virginia College- and Career-Readiness Standards for Mathematics. This Vertical Progression document has taken grade specific information about the Mathematical Habits of Mind from the West Virginia Educators' Guides for Mathematics to display how the Habits of Mind develop and grow from Kindergarten to High School. The document also showcases the similarities of the Habits of Mind at each grade level.

MHM5 – Use appropriate tools strategically.	
Kindergarten Students:	• begin to consider available tools when solving a mathematical problem. • decide when particular tools might be helpful. For example: » use linking cubes to represent two quantities and then compare the two representations side by side. » make math drawings of the quantities. • explain why particular mathematical tools were used.
Grade 1 Students:	• begin to consider available tools (including estimation) when solving a mathematical problem. • decide when particular tools might be helpful. For example: » decide it might be best to use colored chips to model an addition problem. • use tools such as counters, place-value (base-ten) blocks, hundreds number boards, concrete geometric shapes (e.g., pattern blocks or three-dimensional solids), and virtual representations to support conceptual understanding and mathematical thinking. • determine which tools are appropriate to use. For example: » when solving 12 + 8 = __, explain why place-value blocks are appropriate to use to solve the problem.
Grade 2 Students:	• consider available tools (including estimation) when solving a mathematical problem. • decide when certain tools might be better suited than others. For example: » solve a problem by making a math drawing rather than writing an equation. • use tools such as snap cubes, place-value (base-ten) blocks, hundreds number boards, number lines, rulers, virtual manipulatives, diagrams, and concrete geometric shapes (e.g., pattern blocks, three-dimensional solids). • understand which tools are the most appropriate to use. For example: » while measuring the length of the hallway, students are able to explain why a yardstick is more appropriate to use than a ruler.
Grade 3 Students:	• consider available tools (including drawings, concrete objects, or estimation) when solving a mathematical problem. • decide when particular tools might be helpful. For example: » use graph paper to find all the possible rectangles that have a given perimeter. » compile the possibilities into an organized list or a table. » determine whether they have all the possible rectangles.
Grade 4 Students:	• consider available tools (including estimation) when solving a mathematical problem. • decide when certain tools might be helpful. For example: » use graph paper, a number line, or drawings of dimes and pennies to represent and compare decimals. » use protractors to measure angles. • use other measurement tools to understand the relative size of units within a system. • express measurements given in larger units in terms of smaller units.

MHM5 – Use appropriate tools strategically.	
Grade 5 Students:	• consider available tools, including estimation. • decide which tools might help them solve mathematical problems. For example: » use unit cubes to pack a rectangular prism. » use a ruler to measure the dimensions to find a pattern for volume using the lengths of the sides. • use graph paper to accurately create graphs, solve problems, or make predictions from real-world data.
Grade 6 Students:	• consider available tools, including estimation and technology. • decide when particular tools might be helpful. For example: » represent figures on the coordinate plane to calculate area. » use number lines to create dot plots, histograms, and box plots to visually compare the center and variability of the data. » use visual fraction models to represent situations involving division of fractions. » use physical objects or applets to construct nets and calculate the surface area of three-dimensional figures.
Grade 7 Students:	• consider available tools (including estimation and technology) when solving a mathematical problem. • decide if particular tools might be helpful. For example: » represent data sets using dot plots with the same scale to create a visual comparison of the center and variability of the data. » use physical objects, spreadsheets, or applets to generate probability data. » use graphing calculators or spreadsheets to manage and represent data in different forms.
Grade 8 Students:	• consider available tools (including estimation and technology) when solving a mathematical problem. • decide when particular tools might be helpful. For example: » translate a set of data given in tabular form into a graphical representation to compare it with another data set. » draw pictures, use applets, or write equations to show the relationships between the angles created by a transversal that intersects parallel lines.
Algebra I and Math I Students:	• develop a general understanding of the graph of an equation or function as a representation of that object. • use tools such as graphing calculators or graphing software to create graphs in more complex examples to interpret results. • construct diagrams to solve problems.
Geometry and Math II Students:	• use visual tools for representing geometry, such as simple patty paper, transparencies, or dynamic geometry software.
Algebra II and Math III Students:	• use graphing technology to deepen understanding of the behavior of polynomial, rational, square root, and trigonometric functions.