The Mathematical Habits of Mind Overview

The Mathematical Habits of Mind and the Mathematics Content Standards are integral components of the West Virginia College- and Career-Readiness Standards for Mathematics. The Mathematical Habits of Mind address the attributes and characteristics that students develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge—what students need to understand, know, and be able to do.

The Mathematical Habits of Mind are:

Connected: Ideally, several Mathematical Habits of Mind are evident in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. The content standards and the Mathematical Habits of Mind cannot be isolated from one another. Mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Equitable: All students must have access to the Mathematical Habits of Mind. The skills developed through the Habits of Mind are metacognition skills. Much like the content standards, students may need support, scaffolds, and increased opportunities to master the Habits of Mind.

Intentional: The Mathematical Habits of Mind must be taught as purposefully and practiced with the same intention as the Mathematics Content Standards. The Mathematical Habits of Mind represent a picture of what it looks like for students to understand and do mathematics both in and out of the classroom. Every math lesson should coherently and robustly integrate at least one of the Mathematical Habits of Mind.

Ongoing: The Mathematical Habits of Mind are developed throughout each year and across all grade levels and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject.

Authentic: The intent of the West Virginia College- and Career-Readiness Standards for Mathematics is to prepare all West Virginia students for college, careers, and civic life. The Mathematical Habits of Mind develop mathematically competent individuals who can use mathematics as a tool for making wise decisions in their personal lives, a foundation for rewarding work, and a means for comprehending and influencing the world in which they live.

Mathematical Habit of Mind 4 – Model with mathematics.

This document combines information from several sources into one in-depth look at Mathematical Habit of Mind 4.

Mathematical Habits of Mind in Policy

The following exert is from WV Policy 2510:

- The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

Overview of MHM4 – What it is, What it does, and What it looks like

MHM4. Model with mathematics.			
What it is	**What it does**	**What it looks like**	
Using real graphics, articles, and data from newspaper or other sources to make math relevant. Students use math in science, art, music, and even reading.	Helps students use math to solve real-world problems, simplify complicated situations, organize data, and understand the world around them. • What math drawing or diagram could you make and label to represent the problem? • What are some ways to represent the quantities? • What is an equation or expression that matches the [diagram, number line, chart, table, etc.]? • Where did you see one of the quantities in the task in your equation or expression? • How would it help to create a [diagram, graph, table, etc.]? • What are some ways to visually represent _____? • What formula might apply in this situation?	***Students:*** • Apply the mathematics they know to everyday life, society, and the workplace. • Write equations to describe situations. • Are comfortable in making assumptions and approximations to simplify complicated situations. • Analyze relationships to draw conclusions. • Improve their model if it has not served its purpose.	***Teacher:*** • Provides problem situations that apply to everyday life. • Provides rich tasks that focus on conceptual understanding, relationships, etc.

Developing Mathematical Habits of Mind through Questions and Expressing in Student-Friendly Language

The following chart includes both the MHM in student-friendly language and examples of questions teachers might use to support mathematical thinking and student engagement.

Mathematical Habit of Mind	MHM Expressed in Student-Friendly Language	Questions to Develop Mathematical Thinking
MHM4. Model with mathematics.	I can use math to solve real-world problems and better understand the world around me. I can make drawings to represent the problem. I can use symbols and numbers to solve the problem.	• What math drawing or diagram could you make and label to represent the problem? • What are some ways to represent the quantities? • What is an equation or expression that matches the [diagram, number line, chart, table, etc.]? • Where did you see one of the quantities in the task in your equation or expression? • How would it help to create a [diagram, graph, table, etc.]? • What are some ways to visually represent _____? • What formula might apply in this situation?

Rubric – Implementing Mathematical Habits of Mind

Use the Task descriptors in developing lessons to ensure that classroom tasks help cultivate the MHMs. The teacher descriptors can be used during or after the lesson to evaluate how the task was carried out. The column titled "Proficient" describes the expected norm for task and teacher action, while the column titled "Exemplary" includes all features of the proficient column and more. A task is exemplary when meeting criteria in both the proficient and exemplary columns.

MHM4	DESCRIPTOR	NEEDS IMPROVEMENT	EMERGING (teacher does the thinking)	PROFICIENT (teacher mostly models)	EXEMPLARY (students take ownership)
Model with mathematics.	**Task**	• Requires students to identify variables and to perform necessary computations.	• Requires students to identify variables and to compute and interpret results.	• Requires students to identify variables, compute and interpret results, and report findings using a mixture of representations. • Illustrates the relevance of the mathematics involved. • Requires students to identify extraneous or missing information.	• Requires students to identify variables, compute and interpret results, report findings, and justify the reasonableness of their results and procedures within context of the task.
	Teacher	• Identifies appropriate variables and procedures for students. • Does not discuss appropriateness of model.	• Verifies that students have identified appropriate variables and procedures. • Explains the appropriateness of model.	• Asks questions to help students identify appropriate variables and procedures. • Facilitates discussions in evaluating the appropriateness of model.	• Expects students to justify their choice of variables and procedures. • Gives students opportunity to evaluate the appropriateness of model.

The Vertical Progression of the Mathematical Habit of Mind 4

The Mathematical Habits of Mind are an integral part of the West Virginia College- and Career-Readiness Standards for Mathematics. This Vertical Progression document has taken grade specific information about the Mathematical Habits of Mind from the West Virginia Educators' Guides for Mathematics to display how the Habits of Mind develop and grow from Kindergarten to High School. The document also showcases the similarities of the Habits of Mind at each grade level.

MHM4 – Model with mathematics.	
Kindergarten Students:	• begin to represent problem situations in multiple ways—by using numbers, objects, words or mathematical language, acting out the situation, making a chart or list, drawing pictures, creating equations, and so forth. For example: » may use cubes or tiles to show the different number partners for 5. » place three objects on a 10-frame and then determine how many more are needed to "make a ten." • rely on manipulatives while solving tasks. • record an answer with a drawing or equation.
Grade 1 Students:	• experiment with representing problem situations in multiple ways, including writing numbers, using words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, or creating equations. • should be provided opportunities to connect different representations and explain connections. • are able to use representations as needed. • model real-life mathematical situations with an equation. • make sure equations accurately match the problem context. • use concrete models and pictorial representations while solving tasks. • write equations to model problem situations. For example: » "There are 11 bananas on the counter. If you eat 4 bananas, how many are left?" write the equation 11 – 4 = 7.
Grade 2 Students:	• experiment with representing problem situations in multiple ways, including writing numbers, using words (mathematical language), drawing pictures, using objects, acting out, making a chart or list, or creating equations. • should be provided opportunities to connect different representations and explain connections. • model real-life mathematical situations with an equation. • make sure equations accurately match the problem context. • use concrete manipulatives or math drawings (or both) to explain an equation. • create an appropriate problem situation from an equation. For example: » create a story problem for the equation 43 + ____ = 82. » "There were 43 mini-balls in the machine. Tom poured in some more mini-balls. There are 82 mini-balls in the machine now." » "How many balls did Tom pour in?"

MHM4 – Model with mathematics.	
Grade 3 Students:	• represent problem situations in multiple ways using numbers, words (mathematical language), objects, and math drawings. • represent a problem by acting it out or by creating charts, lists, graphs, or equations. • use various contexts and a variety of models (e.g., circles, squares, rectangles, fraction bars, and number lines) to represent and develop an understanding of fractions. • use models to represent both equations and story problems and explain thinking. • evaluate results in the context of the situation and reflect on whether the results make sense.
Grade 4 Students:	• experiment with representing problem situations in multiple ways, including writing numbers; using words (mathematical language); creating math drawings; using objects; making a chart, list, or graph; and creating equations. • should be provided opportunities to connect different representations and explain connections. • evaluate results in the context of the situation and reflect on whether the results make sense.
Grade 5 Students:	• experiment with representing problem situations in multiple ways—for example, by using numbers, mathematical language, drawings, pictures, objects, charts, lists, graphs, and equations. • should be provided opportunities to represent problems in various ways and explain connections. • evaluate results in the context of the situation and explain whether answers to problems make sense. • evaluate the utility of models and can determine which models are the most useful and efficient for solving particular problems.
Grade 6 Students:	• model problem situations symbolically, graphically, in tables, contextually, and with drawings of quantities as needed. • form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. • begin to explore covariance and represent two quantities simultaneously. • use number lines to compare numbers and represent inequalities. • use measures of center and variability and data displays (e.g., box plots and histograms) to draw inferences about and make comparisons between data sets. • should be provided many opportunities to make sense of and explain the connections between the different representations. • use representations, as appropriate, and apply the representations to a problem context.
Grade 7 Students:	• model real-world situations symbolically, graphically, tabularly, and contextually. • form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. • use experiments or simulations to generate data sets and create probability models.

MHM4 – Model with mathematics.	
Grade 8 Students:	• model real-world problem situations symbolically, graphically, in tables, and contextually. • working with the new concept of a function, learn relationships between variable quantities in the real world often satisfy a dependent relationship, in that one quantity determines the value of another. • form expressions, equations, or inequalities from real-world contexts and connect symbolic and graphical representations. • use scatter plots to represent data and describe associations between variables. • use representations as appropriate to problem context.
Algebra I and Math I Students:	• discover mathematics through experimentation and examination of data patterns from real-world contexts. • apply new mathematical understanding of exponential, linear, and quadratic functions to real-world problems.
Geometry and Math II Students:	• apply new mathematical understanding to real-world problems. • learn how transformational geometry and trigonometry can be used to model the physical world.
Algebra II and Math III Students:	• apply new mathematical understanding to real-world problems. • make use of expanding repertoire of functions in modeling. • discover mathematics through experimentation and by examining patterns in data from real-world contexts.