The Mathematical Habits of Mind Overview

The Mathematical Habits of Mind and the Mathematics Content Standards are integral components of the West Virginia College- and Career-Readiness Standards for Mathematics. The Mathematical Habits of Mind address the attributes and characteristics that students develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge—what students need to understand, know, and be able to do.

The Mathematical Habits of Mind are:

Connected: Ideally, several Mathematical Habits of Mind are evident in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. The content standards and the Mathematical Habits of Mind cannot be isolated from one another. Mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Equitable: All students must have access to the Mathematical Habits of Mind. The skills developed through the Habits of Mind are metacognition skills. Much like the content standards, students may need support, scaffolds, and increased opportunities to master the Habits of Mind.

Intentional: The Mathematical Habits of Mind must be taught as purposefully and practiced with the same intention as the Mathematics Content Standards. The Mathematical Habits of Mind represent a picture of what it looks like for students to understand and do mathematics both in and out of the classroom. Every math lesson should coherently and robustly integrate at least one of the Mathematical Habits of Mind.

Ongoing: The Mathematical Habits of Mind are developed throughout each year and across all grade levels and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject.

Authentic: The intent of the West Virginia College- and Career-Readiness Standards for Mathematics is to prepare all West Virginia students for college, careers, and civic life. The Mathematical Habits of Mind develop mathematically competent individuals who can use mathematics as a tool for making wise decisions in their personal lives, a foundation for rewarding work, and a means for comprehending and influencing the world in which they live.

Mathematical Habit of Mind 1 – Make sense of problems and persevere in solving them.

This document combines information from several sources into one in-depth look at Mathematical Habit of Mind 1.

Mathematical Habits of Mind in Policy

The following exert is from WV Policy 2510:

- The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

Overview of MHM1 – What it is, What it does, and What it looks like

MHM1. Make sense of problems and persevere in solving them.			
What it is	**What it does**	**What it looks like**	
Working to understand the problem, finding a way to attack it, and working until it is done by planning a solution pathway, comparing, and checking to see if answers make sense.	Allows students to work through a tough task using reasoning skills; the math becomes about the process and not about one right answer. • Interpret and make meaning of the problem to find a starting point. • Analyze what is given in order to explain the meaning of the problem. • Plan a solution pathway instead of jumping to a solution. • Monitor progress and change the approach if necessary. • See relationships between various representations. • Relate current situations to concepts or skills previously learned and connect mathematical ideas to one another. • Continually ask, "Does this make sense?" • Understand various approaches to solutions.	***Students:*** • Are actively engaged in solving problems and thinking is visible (i.e., DOING MATHEMATICS vs. FOLLOWING STEPS OR PROCEDURES). • Are analyzing givens, constraints, relationships, and goals. • Are discussing with one another, making conjectures, planning a solution pathway, not jumping into a solution attempt or guessing at the direction to take. • Relate current "situation" to concept or skill previously learned and check answers using different methods. • Continually ask self, "Does this make sense?"	***Teacher:*** • Provides time and facilitates discussion in problem solutions. • Facilitates discourse in the classroom so students UNDERSTAND the approaches of others. • Provides opportunities for students to explain themselves, the meaning of a problem, etc. • Provides opportunities for students to connect concepts to "their" world. • Provides students TIME to think and become "patient" problem solvers. • Facilitates and encourages students to check their answers using different methods (not calculators). • Provides problems that focus on relationships and are "generalizable".

Developing Mathematical Habits of Mind through Questions and Expressing in Student-Friendly Language

The following chart includes both the MHM in student-friendly language and examples of questions teachers might use to support mathematical thinking and student engagement.

Mathematical Habit of Mind	MHM Expressed in Student-Friendly Language	Questions to Develop Mathematical Thinking
MHM1. Make sense of problems and persevere in solving them.	I can try many strategies to understand and solve a math problem. I can discuss my strategies with fellow students. I can relate this math problem to what I have previously learned. I can check my answers to determine they makes sense.	• How would you describe the problem in your own words? • How would you describe what you are trying to find? • What do you notice about _____? • What information is given in the problem? • Describe the relationship between the quantities. • Describe what you have already tried. What might you change? • Talk me through the steps you have used to this point. • What steps in the process are you most confident about? • What are some other strategies you might try? • What are some other problems that are similar to this one? • How might you use one of your previous problems to help you begin? • How else might you [organize, represent, show, etc.] _____?

Rubric – Implementing Mathematical Habits of Mind

Use the Task descriptors in developing lessons to ensure that classroom tasks help cultivate the MHMs. The teacher descriptors can be used during or after the lesson to evaluate how the task was carried out. The column titled "Proficient" describes the expected norm for task and teacher action, while the column titled "Exemplary" includes all features of the proficient column and more. A task is exemplary when meeting criteria in both the proficient and exemplary columns.

MHM1	DESCRIPTOR	NEEDS IMPROVEMENT	EMERGING (teacher does the thinking)	PROFICIENT (teacher mostly models)	EXEMPLARY (students take ownership)
Make sense of problems and persevere in solving them.	**Task**	• Is strictly procedural. • Does not require students to check solutions for errors.	• Is overly scaffolded or procedurally "obvious". • Requires students to check answers by plugging in numbers.	• Is cognitively demanding. • Has more than one entry point. • Requires a balance of procedural fluency and conceptual understanding. • Requires students to check solutions for errors using one other solution path.	• Allows for multiple entry points and solution paths. • Requires students to defend and justify their solution by comparing multiple solution paths.
	Teacher	• Does not allow for wait time; asks leading questions to rush through task. • Does not encourage students to individually process the tasks. • Is focused solely on answers rather than processes and reasoning.	• Allots too much or too little time to complete task. • Encourages students to individually complete tasks but does not ask them to evaluate the processes used. • Explains the reasons behind procedural steps. • Does not check errors publicly.	• Allows ample time for all students to struggle with task. • Expects students to evaluate processes implicitly. • Models making sense of the task (given situation) and the proposed solution.	• Differentiates to keep advanced students challenged during work time. • Integrates time for explicit meta-cognition. • Expects students to make sense of the task and the proposed solution.

The Vertical Progression of the Mathematical Habit of Mind 1

The Mathematical Habits of Mind are an integral part of the West Virginia College- and Career-Readiness Standards for Mathematics. This Vertical Progression document has taken grade specific information about the Mathematical Habits of Mind from the West Virginia Educators' Guides for Mathematics to display how the Habits of Mind develop and grow from Kindergarten to High School. The document also showcases the similarities of the Habits of Mind at each grade level.

MHM1 - Make sense of problems and persevere in solving them.	
Kindergarten Students:	• begin to understand mathematics involves solving problems and discussing how they were solved. • explain the meaning of a problem and look for ways to solve it. • use real-life experiences to connect mathematics to the world. • use concrete objects or pictures to help conceptualize and solve problems.
Grade 1 Students:	• realize doing mathematics involves solving problems and discussing how they were solved. • explain the meaning of a problem and look for ways to solve it. • use concrete objects or math drawings to help conceptualize and solve problems.
Grade 2 Students:	• realize doing mathematics involves reasoning about and solving problems. • explain the meaning of a problem and look for ways to solve it. • use concrete objects or pictures to help conceptualize and solve problems. • make conjectures about the solution and plan out a problem-solving approach.
Grade 3 Students:	• know doing mathematics involves solving problems and discussing how they were solved. • explain the meaning of a problem and look for ways to solve it. • use concrete objects, pictures, or drawings to help conceptualize and solve problems; such as: » "Jim purchased 5 packages of muffins. Each package contained 3 muffins. How many muffins did Jim purchase?" or "Describe another situation where there would be 5 groups of 3 or 5 × 3." • listen to other students' strategies and make connections between various methods of solving a given problem.
Grade 4 Students:	• know doing mathematics involves solving problems and discussing how they were solved. • explain the meaning of a problem and look for ways to solve it. • use an equation strategy to solve a word problem. For example: » "Chris bought clothes for school. She bought 3 shirts for $12 each and a skirt for $15. How much money did Chris spend on her new school clothes?" Students could solve this problem with the equation 3 × $12 + $15 = a. • use visual models to help conceptualize and solve problems. • listen to the strategies of others and try different approaches. • often use another method to check answers.

MHM1 - Make sense of problems and persevere in solving them.	
Grade 5 Students:	• solve problems by applying understanding of operations with whole numbers, decimals, and fractions that include mixed numbers. • solve problems related to volume and measurement conversions. • seek the meaning of a problem and look for efficient ways to represent and solve it. For example, » "Sonia had $2\frac{1}{3}$ sticks of gum. She promised her brother that she would give him $\frac{1}{2}$ of a stick of gum. How much will she have left after she gives her brother the amount she promised?"
Grade 6 Students:	• solve real-world problems through the application of algebraic and geometric concepts involving ratio, rate, area, and statistics. • seek the meaning of a problem and look for efficient ways to represent and solve it. • explain the relationships between equations, verbal descriptions, and tables and graphs. • check answers to problems using a different method.
Grade 7 Students:	• solve problems involving ratios and rates and discuss how they were solved. • solve real-world problems through the application of algebraic and geometric concepts. • seek the meaning of a problem and look for efficient ways to represent and solve it. • compare arithmetic and algebraic solutions to the same problem (M.7.10a), identifying correspondences between different approaches.
Grade 8 Students:	• solve real-world problems through the application of algebraic and geometric concepts. • seek the meaning of a problem and look for efficient ways to represent and solve it.
Algebra I and Math I Students:	• learn patience is often required to understand what a problem is asking. • discern between useful and extraneous information. • expand repertoire of expressions and functions that can be used to solve problems.
Geometry and Math II Students:	• construct accurate diagrams of geometry problems to help make sense of them. • organize work so others can follow the reasoning (e.g., in proofs).
Algebra II and Math III Students:	• apply understanding of various functions to real-world problems. • approach complex mathematics problems and break them down into smaller problems, synthesizing the results when presenting solutions.