Why Study Math?

Thesis: Mathematics is Useful, Important, and Interesting

Mathematics classes are required of all students in American public schools, from entry through graduation. Why do we, as a society, require students to learn mathematics? Are these reasons good ones, and how should they inform the way mathematics is taught? Put another way -- as many frustrated students have -- "Who cares about math and when am I ever going to need it?"

Mathematician John Allen Paulos writes,

> "As a mathematician, I'm often challenged to come up with compelling reasons to study mathematics. If the questioner is serious, I reply that there are three reasons or, more accurately, three broad classes of reasons to study mathematics. Only the first and most basic class is practical. It pertains to job skills and the needs of science and technology. The second concerns the understandings that are essential to an informed and effective citizenry. The last class of reasons involves considerations of curiosity, beauty, playfulness, perhaps even transcendence and wisdom." [Paulos, *A Mathematician Reads the Newspaper*]

Broadly speaking, we should study math (or anything) because it is

- Useful:
  - Mathematical problems abound in daily life
  - Mathematical proficiency is required for many jobs
  - Mathematics is essential for science, engineering, and research
- Important:
  - A mathematically informed citizenry will make better economic and political decisions about risk, policy, and resource allocation
- Interesting:
  - "Mathematics, rightly viewed, possesses not only truth, but supreme beauty" (Bertrand Russell) and should be studied in its own right
  - The landmark accomplishments of mathematics stand alongside the masterworks of art and music as cultural triumphs that all educated persons should be able to appreciate
  - Doing mathematics teaches patterns of problem-solving and insight that transfer to other knowledge domains
  - Mathematical proof teaches skills in rigor, argumentation and persuasion that transfer to other knowledge domains

Good reasons to study math do not include

- My Counselor/Parents/School Board is making me take it
- I need it for college/10th grade/chemistry class
- It will be on the SAT/ACT/state exam
- Mathematics builds character

These simply beg the question: ability can be just as well judged or character built by memorizing the dictionary or walking uphill barefoot in the snow. Instead let's expand on the worthy reasons to study math.

Mathematics is Interesting: A Historical Perspective

Mathematics has been a part of a traditional western education since classical times -- the door to Plato's Academy in ancient Greece reportedly bore the inscription "Let No One Ignorant of Geometry Enter Here." In The Republic, Plato lays out a course of study for citizens of an ideal society. His higher education begins with Arithmetic, of which "all arts and sciences necessarily partake," and which "leads to the apprehension of truth:"

> "Arithmetic is a kind of knowledge which legislation may fitly prescribe; and we must endeavour to persuade those who are to be the principal men of our State to go and learn arithmetic, not as amateurs, but they must carry on the study until they see the nature of numbers with the mind only; nor again, like merchants or retail-traders, with a view to buying or selling, but for the sake of their military use, and of the soul herself; and because this will be the easiest way for her to pass from becoming to truth and being." [Republic VII.525b]

It continues with the "kindred science" of Geometry:

> "Geometry is the knowledge of the eternally existent ... it would tend to draw the soul to truth, and would be productive of a philosophic attitude of mind, directing upward the faculties that now wrongly are turned earthward. ... We must require that the men of your Fair City shall never neglect geometry, for even the by-products of such study are not slight: ... its uses in war, and also we are aware that for the better reception of all studies there will be an immeasurable difference between the student who has been imbued with geometry and the one who has not." [Republic VII.527b-c]

In his view, mathematics should be taught both because it "compels the soul to turn her gaze toward that place, where is the full perfection of being, which she ought, by all means, to behold" [VII], and because it prepares people for the responsibilities of a citizen and soldier. "Plato maintained that only those individuals who survive this program are really fit for the highest offices of the state and capable of being entrusted with the noblest of all tasks, those of maintaining and dispensing justice." [EB] For Plato, the 'interesting' and 'important' arguments are bound together: citizens imbued with an appreciation of what is true and eternal are the best defense against tyrants and demagogues.

Furthermore, Plato considers mathematics to be an important subject of study not only in its own right but also for the salutary benefits in abstraction, insight, and intelligence that carry over to all studies: "Even the dull, if they have had an arithmetical training, although they may derive no other advantage from it, always become much quicker than they would otherwise have been."

Medieval and Renaissance Education

From Aristotle into Medieval times the educational curriculum was standardized to include the preparatory "Trivium" (grammar, rhetoric and logic) and the "Quadrivium" (arithmetic, geometry, music and astronomy). In Aristotle's systemization of education (widely adopted by medieval culture) all students learned basic arithmetic in the Trivium (age 7 to puberty) and continued studying mathematics through the Quadrivium (puberty to age 17). This program set the template for what is now understood to comprise a 'Classical Education.' Although there have been movements towards and away from this ideal ever since, its basic structure endures.

It's important to recognize that this tradition, even through the humanist revolution, considers the 'math is interesting and important' argument to vastly outweigh the 'math is useful' viewpoint. In the 15th century, the early humanist Vergerio wrote

> "Arithmetic, which treats of the properties of numbers, Geometry, which treats of the properties of dimensions, lines, surfaces, and solid bodies, are weighty studies because they possess a peculiar element of certainty. ... The knowledge of Nature -- animate and inanimate -- the laws and the properties of things in heaven and in earth [...] -- this is a most delightful, and at the same time most profitable, study for youth. [Vergerio p109]

The renaissance scholar Montesquieu echoed a similar sentiment: "The first motive which ought to impel us to study is the desire to augment the excellence of our nature, and to render an intelligent being yet more intelligent." [Montesquieu quoted in Arnold].

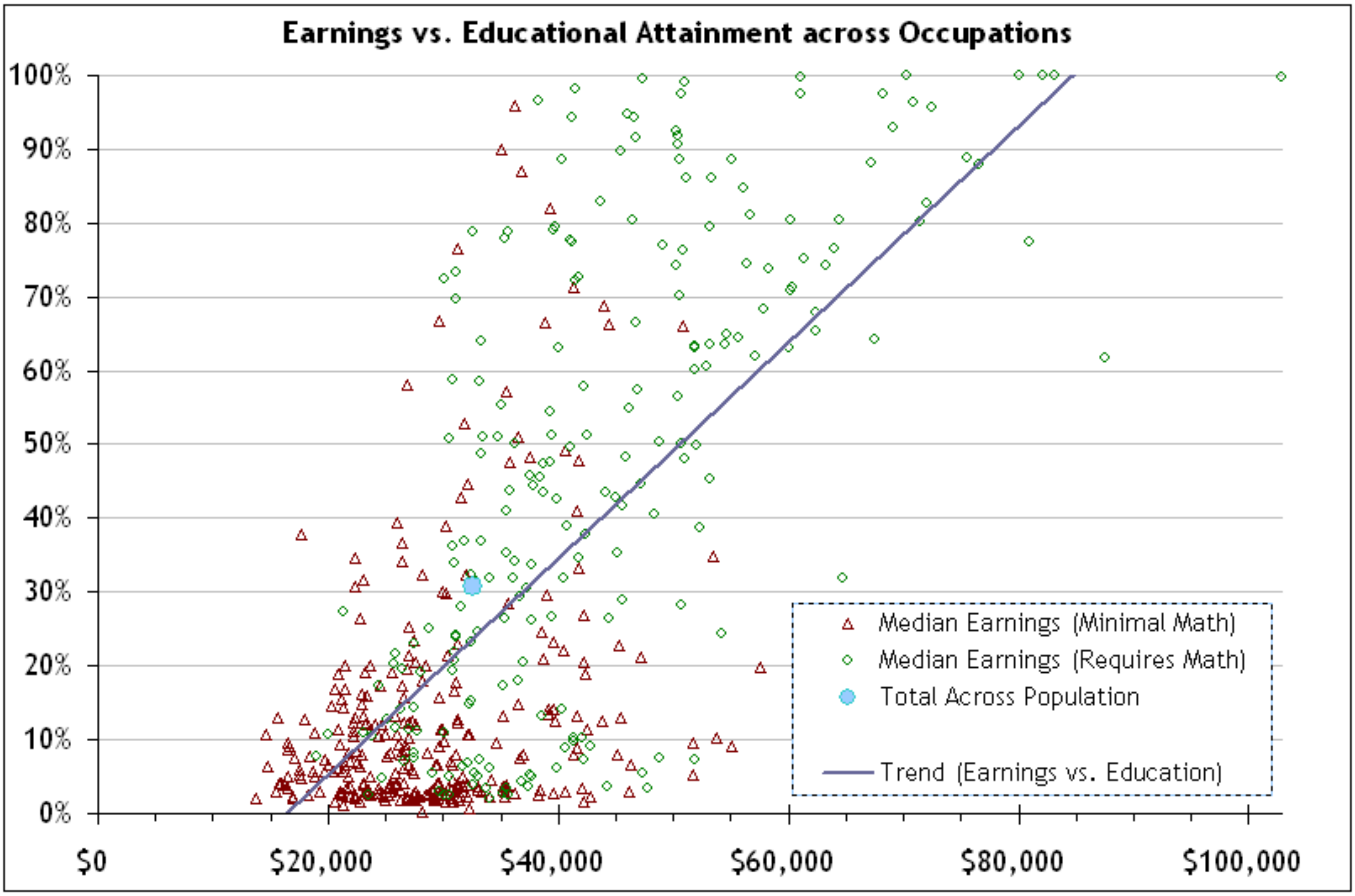
The classical scholars actually hold any appeal to the utility of mathematics in disdain. According to Plato, mathematics should be studied "in the spirit of a philosopher, and not of a shopkeeper!" [Republic VII.525d], while Aristotle felt that students "should not be taught all the useful arts: we entitle vulgar ... the industries that earn wages; for they make the mind preoccupied and degraded." [Aristotle, Politics VIII.1337b]

However, we today consider mathematics to be worthy of study not only because it is beautiful and eternal, but also because it is useful. (Perhaps we are preoccupied and degraded in any case.)

Mathematics is Useful

Mathematics is Essential in the Workplace

It should be no surprise that education is essential for finding and enjoying a satisfying and remunerative career, but it's instructive to examine just how strong the correlation is. Here is 1999 data from the US Census Department's "Earnings By Occupation and Education" survey:



Each point on the graph represents a unique occupation (see table): the vertical axis shows the percentage of workers in that job who have a college degree and the horizontal axis shows its median earnings. (The Bureau of Labor Statistics provides descriptions for each profession in the survey). For example, 88% of Petroleum Engineers went to college to prepare for their profession, and they earned a median salary of $76,529, putting them right on trend.

The general trend (shown as a blue line) is clear: jobs that generally require a college degree pay better than jobs that don't. In fact, among occupations in which more than 40% of workers had college degrees, only two ('Religious Workers, All Other' and 'Directors, Religious Activities and Education') earned less than $30,000. Only one job (Air Traffic Controller) paying above $60,000 had lower than a 60% degree status. (The outlying point on the right side of the graph at 62%/$87,500 is 'Chief Executive,' a nice racket if you can get it.)

Moreover, jobs that require math pay better than jobs that do not. I went through the census data (see table) and indicated, for each job, whether it required significant mathematical training or skills. Engineers, Sociologists, Drafters and Bank Tellers, to name a few, use mathematics daily in their jobs. Public Relations Managers, Bus Drivers and Roofers for the most part do not. (There are of course mathematical aspects to any job, and I'm sure that in some cases my classification was simplistic or wrong. However, I tried to indicate what jobs do not demand *significant* mathematical skill. For example, a waiter or waitress does deal with money and therefore with calculation; but the processes are simple and repetitive, and don't extend beyond elementary arithmetic). In the graph above, occupations that demand mathematics are shown as green circles, while occupations that do not are shown as red triangles.

The clustering of non-mathematical jobs towards the bottom end of the graph is striking. In the latest census, 74 occupations had median yearly earnings above $50,000; only seven, comprising less than 2% of the persons employed in such high-earnings jobs, can be said to require insignificant mathematical training.

It's also important to recognize the extent to which jobs that require mathematical understanding extend into those that *don't require* a college education. For example, among jobs that are found to have less then a 10% college-degree rate, Machinists, Carpenters, (and other craftsmen), Electricians, Surveyors, and Bank Tellers all involve non-trivial mathematics including geometry, number sense, application of complex formulas, large data sets, and more.

There is, of course, more to choosing a career besides its salary (which was used above because it's easy to measure). Yet the jobs in the top right of the graph tend to be more demanding and enjoyable than the jobs on the bottom left. Driven by technological improvements in productivity, jobs that require mathematical training are also the ones that are being created the fastest. The Bureau of Labor Statistics' Occupational Outlook states that

> "Among all occupations in the economy, computer and healthcare occupations are expected to grow the fastest. ... Three-fourths of the job growth in professional and related occupations is expected among computer and mathematical occupations; healthcare practitioners and technical occupations; and education, training, and library occupations. ... Professional and related occupations are the only major group projected to generate more openings from job growth than from replacement needs."

It is also easier, given adequate training in math and other subjects, to find or change jobs (and therefore to find a career that you enjoy). In a famous essay ("What You'll Wish You'd Known"), Paul Graham describes this as "staying upwind:"

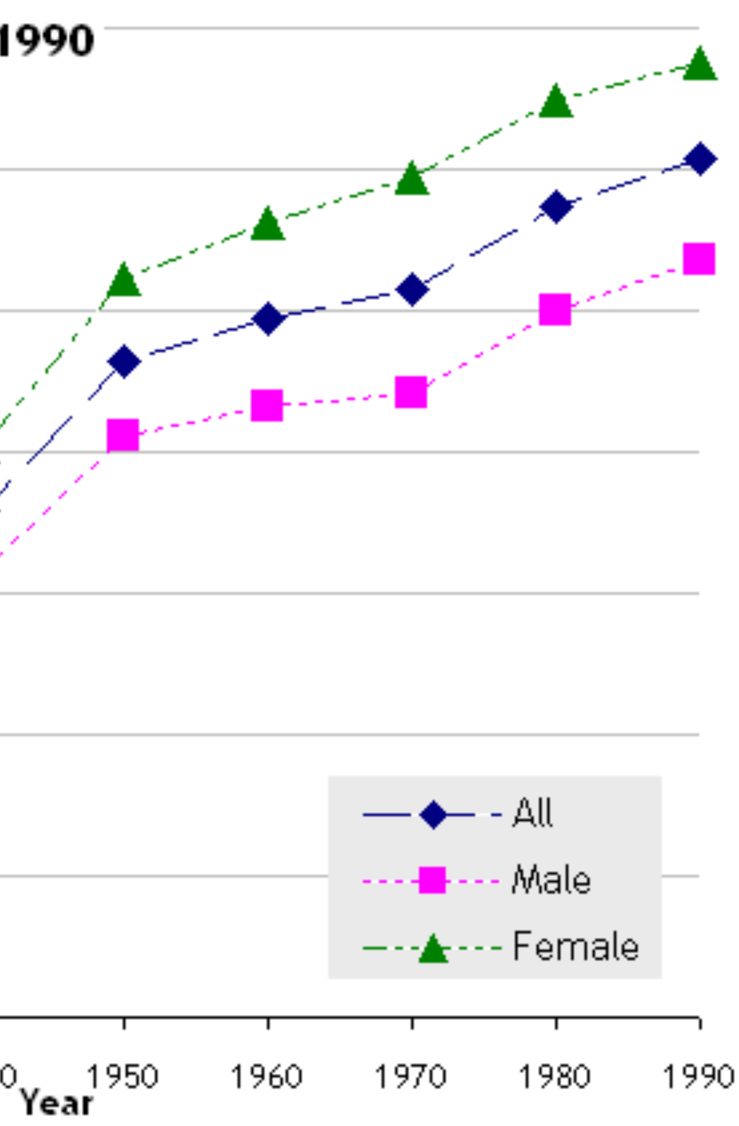
> Look at the options available now, and choose those that will give you the most promising range of options afterward. ... Suppose you're a college freshman deciding whether to major in math or economics. Well, math will give you more options: you can go into almost any field from math. ... Flying a glider is a good metaphor here. Because a glider doesn't have an engine, you can't fly into the wind without losing a lot of altitude. If you let yourself get far downwind of good places to land, your options narrow uncomfortably. As a rule you want to stay upwind. So I propose that as a replacement for "don't give up on your dreams." Stay upwind.

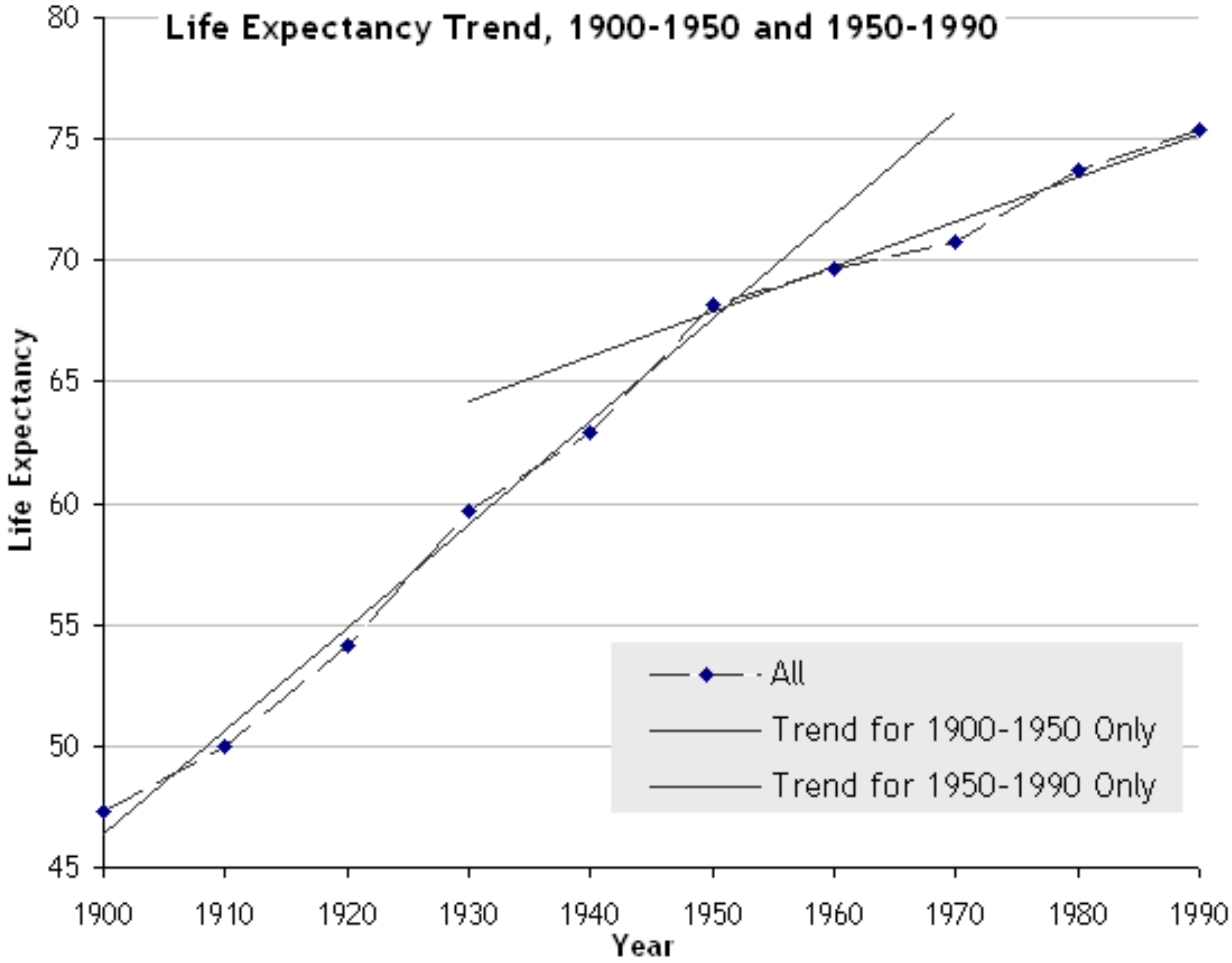
Mathematics is Useful in Other Ways

Mathematical situations abound in daily life:

- Balancing a checkbook
- Calculating a tip
- Choosing a cell-phone plan
- Using a recipe
- Playing pool
- Building a deck
- Betting on poker
- Investing for retirement

These tasks demand proficiency with arithmetic, geometry, combinatorics, and estimation. Furthermore, mathematics connects directly and indirectly with many other domains of knowledge. Its applications to science and engineering are clear, but mathematical strategies apply in many fields. For example, a history class might consider this data for life expectancy vs. birth year:





In a history ***or*** a mathematics class these graphs should raise the questions

- Why is life expectancy increasing? Are these benefits equally distributed across class, race, and gender?
- Why do women outlive men?
- Did world events cause some decades to be above trend and others to be below trend?
- Why is the rate of increase for life expectancy slowing down -- Maximum age? Risk Calculations? Disease? War?

Mathematics, History, Biology and Sociology go into answering these questions. Students of even the most humanitarian bent should be able to construct these graphs from a table of data, interpret their meaning, find the best-fit lines, and explain the ramifications of both the overall slope and the changing slope for the trend lines.

There is mathematics in the fine arts as well -- not only the obvious connections (perspective, symmetry, proportion) but many subtle ones as well. The artists Escher and Calder were strongly influenced by mathematics and corresponded with practicing mathematicians for insight and inspiration. A productive collaboration between a math and art teacher would be to have their students visit an art museum and describe the paintings in terms of their art and their mathematics (see this paper for an example).

How much mathematics can you find within these works of art?

Mathematics is Important

Our society as a whole benefits from having a mathematically fluent populace. Aristotle believed education should be a responsibility of the state, because it benefits a society to have an educated citizenry: "And inasmuch as the end for the whole state is one, it is manifest that education also must necessarily be one and the same for all, and that the superintendence of this must be public, and not on private lines. ... It is clear then that there should be legislation about education and that it should be conducted on a public system." [Politics VIII.1337a]

Many of the reasons above (for an individual to study mathematics) extend here -- our society benefits from a productive labor base, especially as much of our economic growth stems from technological jobs that demand high mathematical competence.

> "The economic cost of mathematical ignorance is gauged, in part, by people who, though they can perform the basic arithmetical operations, don't know when to do one and when to do another: clerks who are perplexed by discounts and sales taxes, medical personnel who have difficulty reckoning correct dosages, quality control managers who don't understand simple statistical concepts. The supply of mathematically capable individuals is also a factor in the U.S.'s position in many new scientific technologies, among them fuel-efficient engines, precision bearings, optical glasses, industrial instrumentation, laser devices, and electronic consumer products. As Labor Secretary Robert Reich and others have written, those jobs and job classifications requiring higher mathematics, language, and reasoning abilities are growing much more rapidly than those that do not." [Paulos, aMRtN]

Beyond this, a mathematically informed citizenry will make better economic and political decisions about risk, policy, and resource allocation.

> "Gullible citizens are a demagogue's dream. Charlatans yearn for people who can't recognize trade-offs between contrary desiderata; who lack a visceral grasp of the difference between millions of dollars for the National Endowment for the Arts and hundreds of billions of dollars for the savings-and-loan bailout; or who insist on paralyzing regulation of rare and minuscule health risks, whose cumulative expense helps to ensure the incomparably greater health hazard of poverty. Almost every political issue -- health care, welfare reform, NAFTA, crime -- has a quantitative aspect." [Paulos, aMRtN]

They will be less likely to be seduced by pseudoscience, hoodwinked by quacks and charlatans, or misled by demagogues.

> "Innumeracy also perpetuates welfare, harms health, and weakens families. Without requisite quantitative skills, individuals will find it very difficult to make a transition from welfare to work. Without critical skills to assess medical claims, individuals will often fall victim to false claims and questionable treatments. Without the skills to manage a household budget, many become victims of easy credit or consumer fraud. In short an innumerate citizen today is as vulnerable as the illiterate peasant of Gutenberg's time." [Steen 1997]

Objection: Mathematics, as currently taught, is Uninteresting, Useless, and Obscure

So mathematics is Interesting, Useful, and Important -- what of it? The mathematics that is taught in school is profoundly disconnected from the mathematics that people enjoy, use, or require. Beyond the obvious dangers of 'teaching to the test,' there is an overemphasis on form over process. In "When Good Teaching Leads to Bad Results: The Disasters of 'Well-Taught' Mathematics Courses," Alan Schoenfeld describes how typical mathematics courses bring about the following beliefs:

- *Belief 1:* The processes of formal mathematics (e.g. 'proof') have little or nothing to do with discovery or invention. Corollary: Students fail to use information from formal mathematics when they are in 'problem-solving mode.'
- *Belief 2:* Students who understand the subject matter can solve assigned mathematics problems in five minutes or less. Corollary: Students stop working on a problem after just a few minutes because, if they haven't solve it, they didn't understand the material (and therefore will not solve it).
- *Belief 3:* Only geniuses are capable of discovering, creating, or really understanding mathematics. Corollary: Mathematics is studied passively, with students accepting what is passed down 'from above' without the expectation that they can make sense of it for themselves.
- *Belief 4:* One succeeds in school by performing the tasks, to the letter, as described by the teacher. Corollary: Learning is an incidental by-product to 'getting the work done'" [Schoenfeld 1988 p151]

If proofs are memorized, answers judged to the decimal point, and problems assigned in a piece-meal bite-sized manner, students will inevitably arrive at the above 'disasters.' Mathematics classes are traditionally taught topic-by-topic (among other things, it's easier to ensure you're meeting state standards that way): first you teach the quadratic formula, then you teach square roots, then you teach rational expressions. Within each unit, problems concern the topic at hand ("Write '$(7+x)^2=4x$' in normal form. Okay now write '$(y+3)(y-4)=2y$' in normal form. Okay now ..." and so on) in a fully decontextualized manner.

Mathematics may be worthwhile, but how is a student supposed to guess it from such a curriculum? What is being taught is not mathematics but the procedures that mathematicians use. Mathematical problems do not come with a note attached reading "please complete the square;" in fact, a primary challenge in doing mathematics is deciding when to apply a particular tool. Real problems -- in life and in higher math -- demand strategies drawn from anywhere within the full spectrum of mathematics, and often more than one of them in series. Real problems come with a motivating context and require creativity, persistence, and often more than five minutes of time. Real problems are solved by ordinary people, who care about the problem's resolution, and that resolution is usually then shared with other people in a way that requires clarity and persuasion (whether it is a solution to Fermat's theorem or the 2Q04 Sales Figures). These features are rarely true of 'school mathematics' -- so students' perception that mathematics is done to satisfy their teacher or the school board or the Educational Testing Service is quite justified.

Is Algebra is the new Latin?

There are also many ways in which the case for mathematics is overstated. Many of the arguments for studying mathematics resemble the old arguments for studying Latin: it's said that mathematics 'promotes logical thinking,' 'has always been part of a classical education,' and 'provides the underpinning for all the things you'll want to learn.'

These arguments, as they apply to Latin, have for the most part been rejected by education theorists and education customers alike. If you want to understand the beautiful language of Virgil and the Medieval Philosophers, or have a fundamental philological understanding of the Romance languages, then by all means, study Latin. If, however, you want to learn French -- study French, not Latin. If you want to learn logical thinking -- study logic, not Latin. If you want to understand medical terminology -- study medicine, not Latin. Latin is a useful adjunct to those other subjects, and a worthwhile subject in its own right. However, Latin does not provide a royal road to learning.

Educational theorists call this wider applicability 'transfer', defined as the "ability to extend what has been learned in one context to new contexts." Thorndike and other researches examined the assumption that "learning Latin and other difficult subject had broad-based effects, such as developing general skills of learning and attention. But these studies raised serious questions about the fruitfulness of designing educational experiences based on [that assumption]. Rather than developing some kind of 'general skill' or 'mental muscle' that affected a wide range of performances, people seemed to learn things that were more specific." [HPL Chapter 3]

Is Knowledge for Knowledge's Sake a Leisure Pursuit?

One point that fans of a 'classical education' neglect is that higher education has historically been a privilege of the upper classes, of those who could afford to pursue knowledge for knowledge's sake. Students' reluctance to study math may be a quite reasonable economic decision: mathematics may be beautiful and eternal and all the rest, but it won't help a Roofer or a Rock Star put food on the table (or at least most students don't see how it will help). Ironically, mathematics tells us that people make these risk-reward calculations quite well. However, if the calculation draws on faulty perceptions of the input factors then the outcome will be faulty too. (See *Technological Risk* by H.W. Lewis for a good discussion of this phenomenon).

Response: Mathematics Education must emphasize what is Interesting, Important, and Useful

Even given these objections, the case for studying mathematics remains as strong today as it did in ancient Greece, though our cultural calculations may have shifted somewhat.

More people may be learning mathematics than have done so in the past, but this represents a triumph and a necessity of our technological age. All persons in our society deserve an equal chance to become full and educated citizens. No one should have to suffer the tyranny of ignorance due to the circumstances of their birth or poor choices made while young.

Although the case for transfer is overstated for mathematics and for Latin, the utility of mathematics as a general vehicle for teaching problem solving and rigorous thought is not ungrounded. After all, human reasoning is largely based in pattern recognition and analogy, clear examples of transfer ('Gee, this problem is just like this other problem I have already met'). However, transfer is not automatic and will not occur unless instruction is designed to foster it.

And if the design of our current mathematical education system is not addressing the real reasons why one should study math, we must redesign our mathematics curriculum with eyes firmly fixed on our ultimate goals: the reasons for mathematics education laid out above.

Teach Mathematics that is Useful and Relevant

If mathematics is worthy of study because it is useful, and we want students to come to see that mathematics is useful, then mathematics instruction should center around contexts in which mathematics is useful.

Teach Mathematics in Context

We ask that students learn mathematics because of the many ways in which it will be useful in their later lives.

It turns out that the right way to teach mathematics is to embed it within

meaningful contexts.

Meaningful does not ahve to be 'real world' -- it just has to have a future relevance

contexts should not be artificial

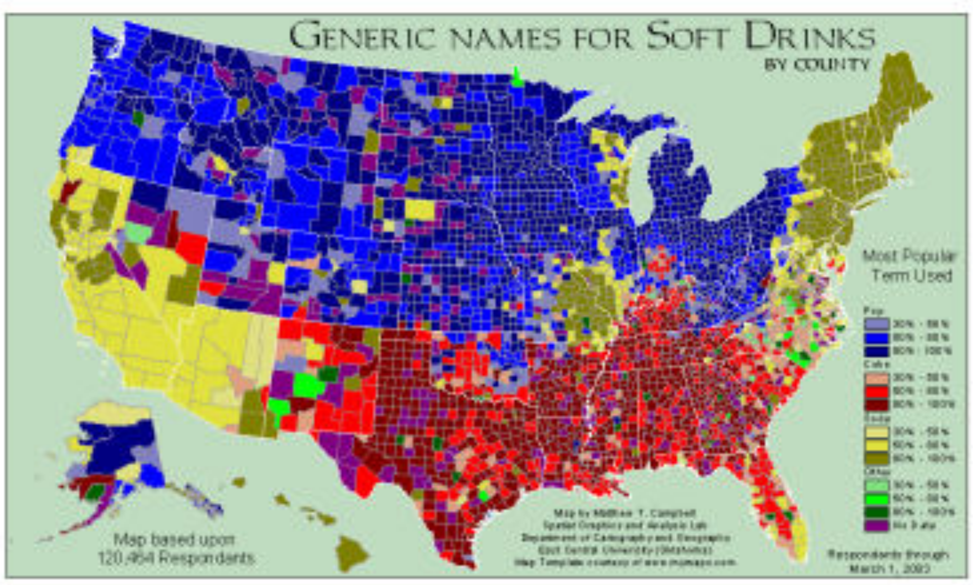
Introduce Large Data Sets Early and Often

One major shift in emphasis is to introduce students early and often to data-rich problems, since so much of the mathematics of research, politics and the workplace have to do with synthesizing and interpreting large data sets. It is easy to find examples that are useful, interesting and important; besides the two above (Earnings vs. Education and Mathematical Proficiency, and Life Expectancy by Decade), consider:

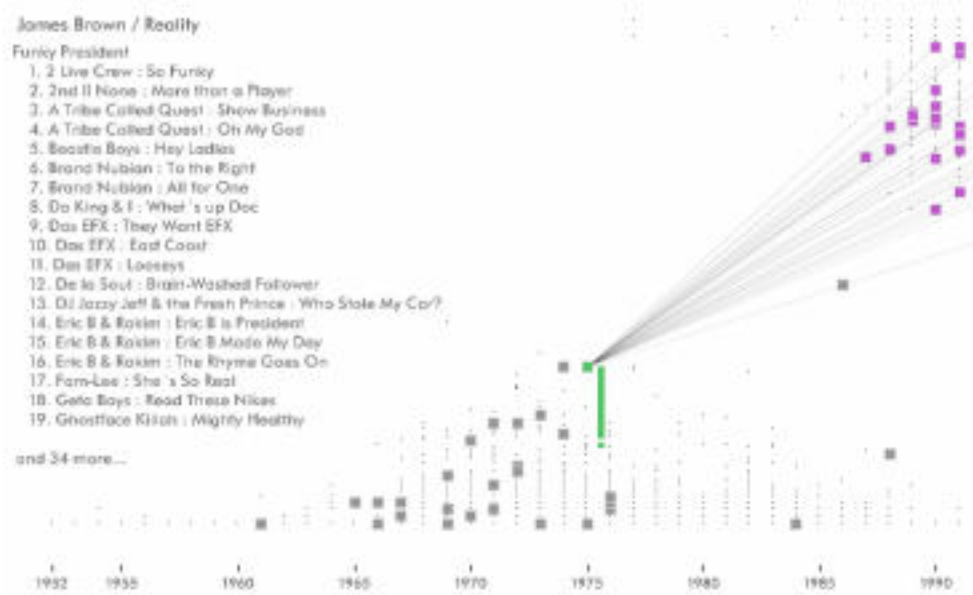
- The Sequence of DNA bases in a genome
- The schedule, milestones, and critical paths for a $100 million construction project
- The geographical distribution of generic soft drink names
- Price and sales figures for an industry or commercial operation
- Graphical History of Sampling in Hip-Hop
- In-Season standings or Historical Trends in Major League Baseball
- Word Frequency in a large corpus of written material

- O-Ring damage in the Space Shuttle (compare: chart-redesign)

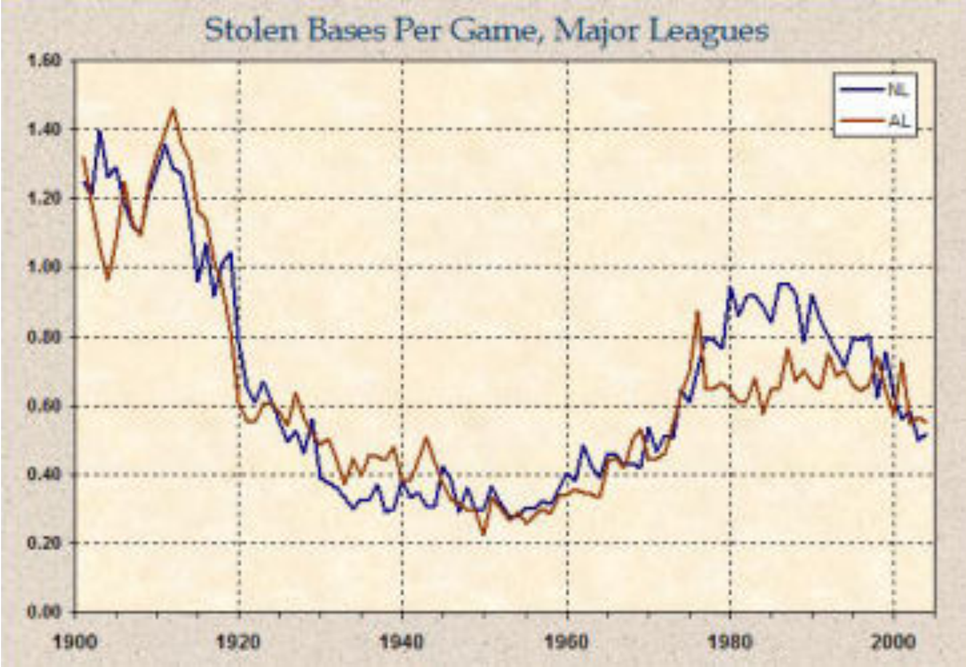
Interesting Large Data Sets



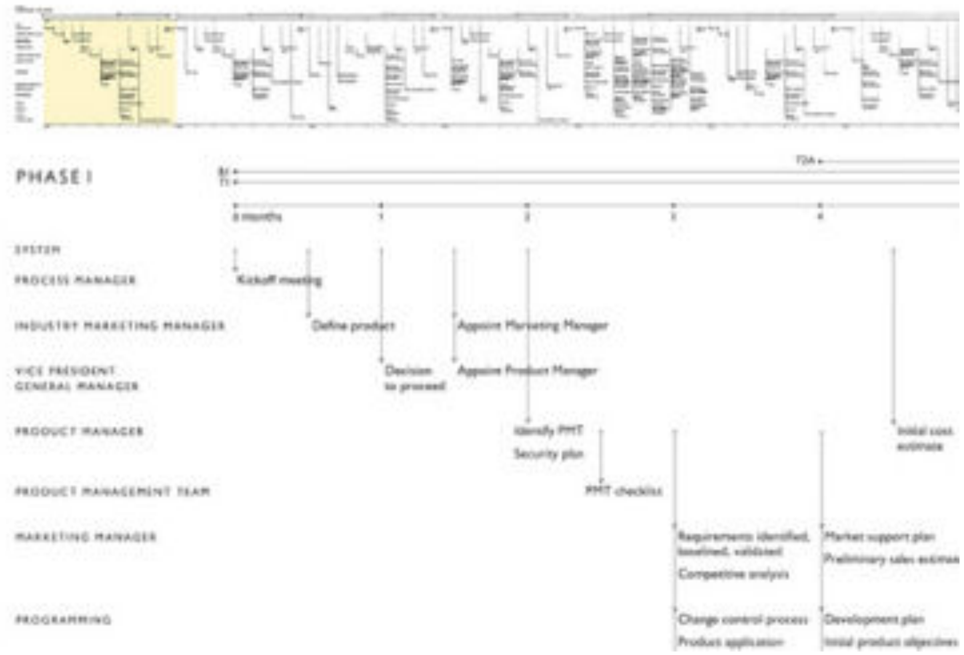
Soda vs. Pop



History of Sampling in Hip-Hop



Historical Trends in Baseball



Timeline for a large project

The natural context of such data sets makes them self-motivating: obvious questions arise that can be analyzed mathematically (Why did stolen bases increase in the 1980s? Why less so in the AL? Are there other trends that correlate or explain this trend?) Students will come to see mathematics as a useful tool that any intelligent person can call on to solve a problem of interest. Such problems are also open-ended -- they admit many questions to ask and many methods to answer those questions. When taught with an eye towards the way mathematics is actually used, it also becomes interesting and important.

Furthermore, analysis of data-rich problems leads directly to fundamental mathematical ideas and processes. Many properties of linear equations and graphs may be fruitfully explored in the context of a linear fit to summarize a trend. Students can learn the meaning and importance of the slope and intercept of a line by actually using them in their natural context. Later, after several examples have been met, the abstract general properties of a line become interesting and intelligible.

Emphasize Communication and Persuasion

Almost all real mathematics is done in a social environment. Mathematicians' proofs are probably best viewed as a process of developing community consensus about what is correct, important and beautiful. (The ideas that underlie a proof may be eternal and independent of discovery, but the process of sharing and convincing your peers that it is so is a social one.)

> I think that if students grew up in a mathematical culture where discourse, thinking things through, and *convincing* were important parts of their engagement with mathematics, then proofs would be seen as a natural part of their mathematics ("Why is this true? It's because . . .") rather than as an artificial imposition. [Schoenfeld 1994]

The social nature of this process is just as important in the workplace and the public forum:

> It has long been understood that getting the right answer is only the beginning rather than the end of being effective on the job. The ability to communicate our thinking convincingly is equally important. Where better than in mathematics classes to learn this skill? In short, the mathematical skills that will enhance the preparation of those who aspire to careers in mathematics are the very same skills that will help people become informed and flexible citizens, workers, and consumers. [Schoenfeld 2001]

Promote Knowledge Transfer

If mathematics is to equip students with general skills and strategies for reasoning, abstraction and problem-solving, we must teach in ways that promote knowledge transfer. *How People Learn* gives the following principles:

- Knowledge must be complete and connected within the original domain to transfer to other domains.
- Transfer is "enhanced by helping students see potential transfer implications of what they are learning."
- Transfer is "especially difficult when a subject is taught only in a single context rather than in multiple contexts."
- Abstract representations of knowledge transfer better than overly contextualized knowledge.
- Transfer is easier among similar problem domains.

- Transfer is improved by metacognition -- by training students to "actively evaluate their strategies and current levels of understanding"
  [HPL]

Note that we must teach not only abstract representations of knowledge but also multiple specific instantiations. This instruction may proceed from specific to general ("What did these examples have in common?," "Here is a method that applies to each of the cases we just explored, and many more besides") or from general to specific ("Can you use the properties of prime numbers to prove your observations?" "Can you see an analogy between this case and that general fact?") If there is only one context, students will identify the new fact strictly with that context. If there is only the abstract case, there may as well be no context at all -- students will never know when or why to apply the knowledge.

Teach Mathematics that is Important

Exercise skills in Estimation and Number Sense

For students to be informed citizens, producers, and consumers of information, they must become "numerate" -- the mathematical analogue to literacy. Fundamental to that is an ability to estimate and reckon with numbers no matter how small or large, a habit of comparing numbers with meaningful touch-points from past experience, and a rich gazette of such touch-points.

Too much of our current education in arithmetic focuses on exact calculation with paper or calculator. Students should receive frequent practice in estimation and reckoning. "How much time do all the students at this school spend doing homework?" "How big would a stack of one-dollar bills equivalent to the US Federal Deficit be?" "What is the total number of miles driven by all the cars in the US each year?" These so-called Fermi Problems, or back-of-the-envelope calculations, should emphasize order-of-magnitude correctness and creative analogies for estimation.

In these problems and in general, teachers and students should demand meaning in mathematics:

> "What is most important from the point of view of understanding arithmetic is the basic idea that [its] statements ... are *statements about the real world which we can use the real world to check if and when we want to*." [Holt p202]

For example, suppose the answer to an abstract area problem is a 20x1 rectangle. Follow up: what is something in the real world with that shape? (A ribbon, a sidewalk or a bowling lane -- not a swimming pool, sheet of paper or human hair). Teachers should instill the habit, when meeting an unfamiliar figure or measurement, of comparing it to a known and tangible quantity. "A moose weighs 1400 pounds and is 8 feet long? That's the size of 8-10 large men standing front-to-back!" A mathematics classroom should make numbers meaningful by "making available to them as many measuring instruments as possible: rulers, measuring tapes (in both feet and meters), scales, watches and stopwatches, thermometers, metronomes, barometers, light meters, decibel meters, and so on." [Holt p222] In these ways, students will build a catalog of meaningful numbers and measurements, by which they can index and interpret the world around them.

Explicitly Integrate Current Events, Politics, and Finances

Students should learn to skeptically and dispassionately evaluate the statements of politicians, salesmen and the media -- a skill that can and should be practiced. As a teacher I plan to have students bring in graphics, advertisements or news articles with mathematical content that they find interesting. (This is not hard -- the "median number of numbers on a full-length newspaper page is almost always well over 100; the mean is over 500." [Usiskin]) Students should learn to ask questions such as (adopted from the Annals of Improbable Research Teachers' Guide):

- Is this author right -- and what does "right" mean, anyway?
- Can you think of even one different explanation that works as well or better?
- Did the test really, really, truly, unquestionably, completely test what the author thought it did?
- Is the scientist ruthlessly honest with him-or-herself about how well this idea explains everything, or could the scientist be suffering from wishful thinking?
- Some people might say this is absolutely correct and important. Should you take their word for it?

Teach Mathematics that is Interesting

Expose Students to Beauty and Elegance in Mathematics

Show that mathematics is about pattern and insight, not calculation and form, by designing activities that emphasize pattern and insight, not calculation and form. For one example, see "Using Polynomials to Amaze" by C. Mulligan, which connects polynomials with Fibonacci-like sequences in an elegant manner. Proofs and constructions at the high school level should emphasize discovery and knowledge construction over rigor and certainly over memorization or recipe. Finally, there are many mathematical topics of great beauty that are accessible at the High School level, including

- Fractals
- Knot theory
- Cellular automata
- Patterns in geometric and combinatoric sequences
- Game Theory
- Graph Theory

Combat Negative Societal Messages

Our educational process obscures, for many students, the beauty and clarity of mathematics -- many facets of that process should be redesigned to make its elegance accessible to all. With that said, we live in a deeply anti-intellectual age, one that transmits foolish and destructive attitudes towards learning, especially towards mathematics.

People who enjoy acquiring and sharing knowledge are derided as nerds, and a host of social pressures exist to stigmatize intellectualism. Women receive a double barrage, as they are subject to an additional set of social pressures (including femininity, competition, and a lack of role models) to influence them away from enjoying or excelling at mathematics. There exist instructional principles to promote gender equity that would increase fairness and benefit all students. Beyond that, mathematics instructors should discuss and make explicit the hidden social behaviors that stigmatize learning. For example, this jigsaw exercise asks students to compare and explore their attitudes towards entertainment and intellectual pursuits. Finally, explorations such as this one -- examining the reasons for studying mathematics in general -- can help students identify and defend their interest in mathematics.

Keep the Cart Afore the Horse

We have explored above several good reasons why, as individuals and as a society, we consider mathematics worthwhile. It is only right that students take stock to determine how and whether mathematics is personally relevant. Early in the year, ask students "Why study mathematics? Why take this course in particular? What do you hope to gain from it?" Many may list variants on "because my parents/counselor/principal is making me take it" or "because I need it for the TAKS/SAT/to get in to college" -- if so, why do these authority figures require mathematics, and are any of those reasons personally relevant? Beyond that, invite students to begin taking charge of their educational destiny and to begin asking for and seeking out meaning in their studies. For students and for teachers, knowing why we do learn helps to illuminate what and how we should learn.

Bibliography

[aMRtN] Paulos, John Allen. *A Mathematician Reads the Newspaper*, p164-168. Basic Books (1995). (see excerpt at http://www.uiowa.edu/~030116/prelaw/whymath.htm)

[Innumeracy] Paulos , John Allen. *Innumeracy*. Hill & Wang, New York (1988).

"Classical Education," *Wikipedia* (2005). Accessed May 18, 2005. (http://en.wikipedia.org/wiki/Classical_education)

"Quadrivium," *Wikipedia* (2005). Accessed May 18, 2005. (http://en.wikipedia.org/wiki/Quadrivium)

"Rousseau," *Wikipedia* (2005). Accessed May 18, 2005. (http://en.wikipedia.org/wiki/Jean-Jacques_Rousseau)

[EB] "education, history of." *Encyclopædia Britannica*. 2005. Encyclopædia Britannica Online
May 18, 2005 (http://search.eb.com.content.lib.utexas.edu:2048/eb/article?tocId=9105951)

"Plato FAQ: 'Let no one ignorant of geometry enter,'" Bernard Suzanne. (http://plato-dialogues.org/faq/faq009.htm)

[Republic] Plato. *The Republic*, "Chapter 7: On Shadows and Realities in Education," tr. Benjamin Jowett (1901). (http://www.mdx.ac.uk/www/study/xpla7.htm).

Plato. *Plato in Twelve Volumes*, Vols. 5 & 6 tr. Paul Shorey. Harvard University Press, Cambridge, MA (1969). (http://www.perseus.tufts.edu/cgi-bin/ptext?lookup=Plat.+Rep.+7.527b)

[Montesquieu] Arnold, M. *Culture and Anarchy* (1882). (http://www.library.utoronto.ca/utel/nonfiction_u/arnoldm_ca/ca_ch1.html)

[Vergerio] Vergerius, P. De Ingenuis Moribus from W.H. Woodward, ed., *Vittorino da Feltre and other Humanist Educators*. Cambridge University Press, Cambridge (1912), 93-118. (http://history.hanover.edu/texts/verg.html)

Adelman, C. Answers in the Toolbox (http://www.ed.gov/pubs/Toolbox/toolbox.html), a study about what contributes most to long-term bachelor's degree completion of students who attend 4-year colleges.

[HPL]: How People Learn, National Research Council (http://www.nap.edu/books/0309070368/html/51.html). Chapter 3 deals with learning and transfer.

Mulligan, C. (1988). Using polynomials to amaze. In A. Coxford & A. Shulte (Eds.), *The Ideas of Algebra, K-12* (206-211). Reston, VA: National Council of Teachers of Mathematics.

Anderson, K, R Brooks, and M Reavis. *Gender Equity for Mathematics and Science*, "Instructional Strategies." (http://www.woodrow.org/teachers/math/gender/11strategies.html)

[Holt] Holt, John. *How Children Fail* Da Capo, New York (1982).

[Schoenfeld 1988] Schoenfeld Alan H., "When Good Teaching Leads to Bad Results: The Disasters of 'Well-Taught' Mathematics Courses," *Educational Psychologist*, vol. 23, 1988, pp. 159, 164.

[Schoenfeld 1994] Schoenfeld, Alan H., "What do We Know about Mathematics Curricula?," *Journal of Mathematical Behavior* (1994), 13(1), pp. 55-80. (http://gse.berkeley.edu/faculty/AHSchoenfeld/WhatDoWeKnow/What_do_we_Know.html)

[Schoenfeld 2001] Schoenfeld, Alan H., "Reflections on an Impoverished Education," *Mathematics and Democracy: The Case for Quantitative Literacy*, ed. Lynn Arthur Steen. National Council on Education and the Disciplines (2001). (http://www.maa.org/ql/049-54.pdf)

[Steen 1990] Steen, Lynn Arthur, "Numeracy" (1990). (http://www.stolaf.edu/people/steen/Papers/numeracy.html)

[Steen 1997] Steen, Lynn Arthur, "The New Literacy" from *Why Numbers Count: Quantitative Literacy for Tomorrow's America* College Entrance Examination Board (1997). (http://www.stolaf.edu/other/extend/Numeracy/steen.html)

[Usiskin] Usiskin, Zalman. "Quantitative Literacy for the Next Generation," *Mathematics and Democracy: The Case for Quantitative Literacy*, ed. Lynn Arthur Steen. National Council on Education and the Disciplines (2001). (http://www.maa.org/ql/079-86.pdf) Cites in turn:

> Usiskin, Zalman. "From ‘Mathematics for Some’ to ‘Mathematics for All,’ " *Selected Lectures from the 7th International Congress on Mathematical Education*, ed. DF Robitaille, DH Wheeler, and C Kieran. Les Presses de L’Université Laval (1994).
> Usiskin, Zalman. "Mathematics as a Language," *Communication in Mathematics, K-12 and Beyond*, ed. Elliott and Kenney. National Council of Teachers of Mathematics (1996).