West Virginia

College & Career Readiness Standard

Resource Booklet for Mathematics

Grades 6-8

Based on WVBE Policy 2520.2B

Effective July 1, 2016

West Virginia Board of Education
2016-2017

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Steven L. Paine, Ex Officio
State Superintendent of Schools
West Virginia Department of Education

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Foreword

Dear West Virginia Educators,

As we move forward with the rollout of West Virginia's College- and Career-Readiness Standards for English Language Arts and Mathematics (West Virginia Board of Education Policies 2520.1A and 2520.2B, respectively), I am excited to share this standards-focused resource booklet with you. In this booklet you will find:

- Applicable West Virginia College- and Career-Readiness Standards for English Language Arts and/or Mathematics (effective July 1, 2016) for your grade/content area;
- Sample letters by grade level for families regarding the West Virginia College- and Career-Readiness Standards;
- Progression documents for English Language Arts and/or Mathematics; and
- The state-adopted definition of College and Career Readiness for West Virginia.

I know our goal of ensuring all West Virginia students graduate from high school with the skills, knowledge and dispositions to be considered truly college and career ready can become a reality if we focus on the development and success of all students. It is my sincere hope that you will utilize the resources found within this document to tailor your instruction and curricula to meet the needs of all the students you serve.

Last, I would like to thank you for your dedication to the lives and well-being of the students of our great state. I am humbled by the amazing work you do each day to ensure all students are college and career ready.

Sincerely,

Steven L. Paine, Ed.D
State Superintendent of Schools

College- and Career-Readiness in West Virginia

West Virginia's College- and Career-Readiness Standards have been developed with the goal of preparing students in a wide range of high-quality post-secondary opportunities. Specifically, college- and career-readiness refers to the knowledge, skills, and dispositions needed to be successful in higher education and/or training that lead to gainful employment. The West Virginia College- and Career-Readiness Standards establish a set of knowledge and skills that all individuals need to transition into higher education or the workplace, as both realms share many expectations. All students throughout their educational experience, should develop a full understanding of the career opportunities available, the education necessary to be successful in their chosen pathway, and a plan to attain their goals.

College- and Career-Readiness in the Mathematics Content Area

West Virginia's College- and Career-Readiness Standards for Mathematics are the culmination of an extended, broad-based effort to help ensure that all students are college- and career-ready upon completion of high school. The skills contained in the mathematics standards are essential for college- and career-readiness in a twenty-first-century, globally competitive society. The standards reflect a progression and key ideas determining how knowledge is organized and generated within the content area. Standards evolve from specifics to deeper structures inherent in the discipline. These deeper structures serve to connect the specifics. The standards follow such a design, stressing conceptual understanding of key ideas and continually returning to organizing principles such as place value or the properties of operations to structure those ideas. The sequence of topics and performances outlined in mathematics standards must respect the scientific research about how students learn and what is known about how their mathematical knowledge, skill, and understanding develop over time.

The West Virginia College- and Career-Readiness Standards are the result of a statewide public review of the state's educational standards. The West Virginia Department of Education (WVDE), West Virginia Board of Education (WVBE), and West Virginia University partnered in this initiative that began with a website, Academic Spotlight, which served as the platform for feedback collection. This website was active July through September of 2015. After the comment period closed, comments were evaluated by a team of diverse stakeholders, who made recommendations to WVBE based on the comments to meet the needs of West Virginia students. Additionally, during the month of September 2015, eight universities around the state hosted town hall meetings where citizens could pose questions about the standards to a panel of teachers, administrators, and representatives from higher education. The West Virginia College- and Career-Readiness Standards reflect the improvements brought to light by these two methods of public input.

Mathematics

The West Virginia College- and Career-Readiness Standards for Mathematics define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. What does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness. The Standards begin with eight Mathematical Habits of Mind.

Mathematics: Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$ and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Mathematical Habits of Mind to the Standards for Mathematical Content

The Mathematical Habits of Mind describe ways in which developing students of mathematics increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments and professional development should all attend to the need to connect the mathematical habits of mind to mathematical content in mathematics instruction.

West Virginia College- and Career-Readiness Standards for Mathematics – Grade 6

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in the sixth grade will focus on four critical areas: (1) connecting ratio and rate to whole number multiplication and division and using concepts of ratio and rate to solve problems; (2) completing understanding of division of fractions and extending the notion of number to the system of rational numbers, which includes negative numbers; (3) writing, interpreting and using expressions and equations; and (4) developing understanding of statistical thinking. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students in sixth grade will continue developing mathematical proficiency in developmentally-appropriate progressions of standards. Continuing the skill progressions from fifth grade, the following chart represents the mathematical understandings that will be developed in sixth grade:

Ratios and Proportional Reasoning	**The Number System**
• Understand ratios and rates, and solve problems involving proportional relationships (e.g., If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours?).	• Divide fractions and solve related word problems (e.g., How wide is a rectangular strip of land with length 3⁄4 mile and area 1⁄2 square mile?). • Use positive and negative numbers together to describe quantities; understand the ordering and absolute values of positive and negative numbers.
Expressions and Equations	**Geometry**
• Work with variables and expressions by generalizing the way numbers work (e.g., When adding numbers, the order doesn't matter, so x + y = y + x; likewise, properties of addition and multiplication can be used to rewrite 24x + 18y as 6(4x + 3y), or y + y + y as 3y). • Write equations to solve word problems and describe relationships between quantities (e.g., The distance D traveled by a train in time T might be expressed by an equation D = 85T, where D is in miles and T is in hours.).	• Reason about relationships between shapes to determine area, surface area, and volume.
Statistics and Probability	
• Create graphical representations of data and reason about statistical distributions.	

Numbering of Standards

The following Mathematics Standards are numbered continuously. The following ranges relate to the clusters found within Mathematics:

Ratios and Proportional Relationships	
Understand ratio concepts and use ratio reasoning to solve problems.	Standards 1-3
The Number System	
Apply and extend previous understandings of multiplication and division to divide fractions by fractions.	Standard 4
Compute fluently with multi-digit numbers and find common factors and multiples.	Standards 5-7
Apply and extend previous understandings of numbers to the system of rational numbers.	Standards 8-11
Expressions and Equations	
Apply and extend previous understandings of arithmetic to algebraic expressions.	Standards 12-15
Reason about and solve one-variable equations and inequalities.	Standards 16-19
Represent and analyze quantitative relationships between dependent and independent variables.	Standard 20
Geometry	
Solve real-world and mathematical problems involving area, surface area, and volume.	Standards 21-24
Statistics and Probability	
Develop understanding of statistical variability.	Standards 25-27
Summarize and describe distributions.	Standards 28-29

Ratios and Proportional Relationships

Cluster	Understand ratio concepts and use ratio reasoning to solve problems.
M.6.1	Understand the concept of a ratio and use ratio language to describe a ratio relationship between two quantities. (e.g., "The ratio of wings to beaks in the bird house at the zoo was 2:1, because for every 2 wings there was 1 beak." "For every vote candidate A received, candidate C received nearly three votes.")
M.6.2	Understand the concept of a unit rate a/b associated with a ratio a:b with $b \neq 0$, and use rate language in the context of a ratio relationship. (e.g., "This recipe has a ratio of 3 cups of flour to 4 cups of sugar, so there is 3/4 cup of flour for each cup of sugar." "We paid $75 for 15 hamburgers, which is a rate of $5 per hamburger.") Instructional Note: Expectations for unit rates in this grade are limited to non-complex fractions.
M.6.3	Use ratio and rate reasoning to solve real-world and mathematical problems, e.g., by reasoning about tables of equivalent ratios, tape diagrams, double number line diagrams, or equations. a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. (e.g., If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?) c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.

The Number System

Cluster	Apply and extend previous understandings of multiplication and division to divide fractions by fractions.
M.6.4	Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions by using visual fraction models and equations to represent the problem. (e.g., Create a story context for (2/3) ÷ (3/4) and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that (2/3) ÷ (3/4) = 8/9 because 3/4 of 8/9 is 2/3. (In general, (a/b) ÷ (c/d) = ad/bc.) How much chocolate will each person get if 3 people share 1/2 lb of chocolate equally? How many 3/4-cup servings are in 2/3 of a cup of yogurt? How wide is a rectangular strip of land with length 3/4 mi and area 1/2 square mi?)

Cluster	Compute fluently with multi-digit numbers and find common factors and multiples.
M.6.5	Fluently divide multi-digit numbers using the standard algorithm.
M.6.6	Fluently add, subtract, multiply and divide multi-digit decimals using the standard algorithm for each operation.
M.6.7	Find the greatest common factor of two whole numbers less than or equal to 100 and the least common multiple of two whole numbers less than or equal to 12. Use the distributive property to express a sum of two whole numbers 1–100 with a common factor as a multiple of a sum of two whole numbers with no common factor (e.g., express 36 + 8 as 4 (9 + 2)).

Cluster	Apply and extend previous understandings of numbers to the system of rational numbers.
M.6.8	Understand that positive and negative numbers are used together to describe quantities having opposite directions or values (e.g., temperature above/below zero, elevation above/below sea level, credits/debits, positive/negative electric charge); use positive and negative numbers to represent quantities in real-world contexts, explaining the meaning of 0 in each situation.
M.6.9	Understand a rational number as a point on the number line. Extend number line diagrams and coordinate axes familiar from previous grades to represent points on the line and in the plane with negative number coordinates. a. Recognize opposite signs of numbers as indicating locations on opposite sides of 0 on the number line; recognize that the opposite of the opposite of a number is the number itself, e.g., –(–3) = 3, and that 0 is its own opposite. b. Understand signs of numbers in ordered pairs as indicating locations in quadrants of the coordinate plane; recognize that when two ordered pairs differ only by signs, the locations of the points are related by reflections across one or both axes. c. Find and position integers and other rational numbers on a horizontal or vertical number line diagram; find and position pairs of integers and other rational numbers on a coordinate plane.
M.6.10	Understand ordering and absolute value of rational numbers. a. Interpret statements of inequality as statements about the relative position of two numbers on a number line diagram. (e.g., interpret –3 > –7 as a statement that –3 is located to the right of –7 on a number line oriented from left to right.) b. Write, interpret, and explain statements of order for rational numbers in real-world contexts (e.g., write -3^{o} C > -7^{o} C to express the fact that -3^{o} C is warmer than -7^{o} C). c. Understand the absolute value of a rational number as its distance from 0 on the number line; interpret absolute value as magnitude for a positive or negative quantity in a real-world situation. (e.g., for an account balance of –30 dollars, write |–30| = 30 to describe the size of the debt in dollars). d. Distinguish comparisons of absolute value from statements about order. (e.g., recognize that an account balance less than –30 dollars represents a debt greater than 30 dollars.)
M.6.11	Solve real-world and mathematical problems by graphing points in all four quadrants of the coordinate plane. Include use of coordinates and absolute value to find distances between points with the same first coordinate or the same second coordinate.

Expressions and Equations

Cluster	Apply and extend previous understandings of arithmetic to algebraic expressions.
M.6.12	Write and evaluate numerical expressions involving whole-number exponents.

M.6.13	Write, read and evaluate expressions in which letters stand for numbers. a. Write expressions that record operations with numbers and with letters standing for numbers. (e.g., Express the calculation, "Subtract y from 5" as 5 – y.) b. Identify parts of an expression using mathematical terms (sum, term, product, factor, quotient, coefficient); view one or more parts of an expression as a single entity. (e.g., Describe the expression 2 (8 + 7) as a product of two factors; view (8 + 7) as both a single entity and a sum of two terms.) c. Evaluate expressions at specific values of their variables. Include expressions that arise from formulas used in real-world problems. Perform arithmetic operations, including those involving whole number exponents, in the conventional order when there are no parentheses to specify a particular order: Order of Operations (e.g., use the formulas $V = s^3$ and $A = 6s^2$ to find the volume and surface area of a cube with sides of length s = 1/2).
M.6.14	Apply the properties of operations to generate equivalent expressions (e.g., apply the distributive property to the expression 3 (2 + x) to produce the equivalent expression 6 + 3x; apply the distributive property to the expression 24x + 18y to produce the equivalent expression 6 (4x + 3y); apply properties of operations to y + y + y to produce the equivalent expression 3y).
M.6.15	Identify when two expressions are equivalent; i.e., when the two expressions name the same number regardless of which value is substituted into them. (e.g., The expressions y + y + y and 3y are equivalent because they name the same number regardless of which number y stands for.)

Cluster	**Reason about and solve one-variable equations and inequalities.**
M.6.16	Understand solving an equation or inequality as a process of answering a question: which values from a specified set, if any, make the equation or inequality true? Use substitution to determine whether a given number in a specified set makes an equation or inequality true.
M.6.17	Use variables to represent numbers and write expressions when solving a real-world or mathematical problem; understand that a variable can represent an unknown number or depending on the purpose at hand, any number in a specified set.
M.6.18	Solve real-world and mathematical problems by writing and solving equations of the form x + p = q and px = q for cases in which p, q and x are all nonnegative rational numbers.
M.6.19	Write an inequality of the form x > c or x < c to represent a constraint or condition in a real-world or mathematical problem. Recognize that inequalities of the form x > c or x < c have infinitely many solutions; represent solutions of such inequalities on number line diagrams.

Cluster	**Represent and analyze quantitative relationships between dependent and independent variables.**
M.6.20	Use variables to represent two quantities in a real-world problem that change in relationship to one another; write an equation to express one quantity, thought of as the dependent variable, in terms of the other quantity, thought of as the independent variable. Analyze the relationship between the dependent and independent variables using graphs and tables, and relate these to the equation. (e.g., In a problem involving motion at constant speed, list and graph ordered pairs of distances and times, and write the equation d = 65t to represent the relationship between distance and time.)

Geometry

Cluster	Solve real-world and mathematical problems involving area, surface area, and volume.
M.6.21	Find the area of right triangles, other triangles, special quadrilaterals and polygons by composing into rectangles or decomposing into triangles and other shapes; apply these techniques in the context of solving real-world and mathematical problems.
M.6.22	Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas V = l w h and V = B h to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.
M.6.23	Draw polygons in the coordinate plane given coordinates for the vertices; use coordinates to find the length of a side joining points with the same first coordinate or the same second coordinate. Apply these techniques in the context of solving real-world and mathematical problems.
M.6.24	Represent three-dimensional figures using nets made up of rectangles and triangles, and use the nets to find the surface area of these figures. Apply these techniques in the context of solving real-world and mathematical problems.

Statistics and Probability

Cluster	Develop understanding of statistical variability.
M.6.25	Recognize a statistical question as one that anticipates variability in the data related to the question and accounts for it in the answers. (e.g., "How old am I?" is not a statistical question, but "How old are the students in my school?" is a statistical question because one anticipates variability in students' ages.)
M.6.26	Through informal observation, understand that a set of data collected to answer a statistical question has a distribution which can be described by its center (mean/median), spread (range), and overall shape.
M.6.27	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number.

Cluster	Summarize and describe distributions.
M.6.28	Display numerical data in plots on a number line, including dot plots, histograms and box plots.
M.6.29	Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. d. Relating the choice of measures of center to the shape of the data distribution and the context in which the data were gathered.

West Virginia College- and Career-Readiness Standards for Mathematics – Grade 7

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in the seventh grade will focus on four critical areas: (1) developing understanding of and applying proportional relationships; (2) developing understanding of operations with rational numbers and working with expressions and linear equations; (3) solving problems involving scale drawings and informal geometric constructions and working with two- and three-dimensional shapes to solve problems involving area, surface area and volume; and (4) drawing inferences about populations based on samples. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students in seventh grade will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from sixth grade, the following chart represents the mathematical understandings that will be developed in seventh grade:

Ratios and Proportional Reasoning	**The Number System**
• Analyze proportional relationships (e.g., by graphing in the coordinate plane), and distinguish proportional relationships from other kinds of mathematical relationships (e.g., Buying 10 times as many items will cost you 10 times as much, but taking 10 times as many aspirin will not lower your fever 10 times as much.).	• Solve percent problems (e.g., tax, tips, and markups and markdowns). • Solve word problems that have a combination of whole numbers, fractions, and decimals (e.g., A woman making $25 per hour receives a 10% raise; she will make an additional 1⁄10 of his or her salary an hour, or $2.50, for a new salary of $27.50.)
Expressions and Equations	**Geometry**
• Solve equations such as 1⁄2 (x – 3) = 3⁄4 quickly and accurately, and write equations of this kind to solve word problems.	• Solve problems involving scale drawings.
Statistics and Probability	
• Use statistics to draw inferences and make comparisons (e.g., deciding which candidate is likely to win an election based on a survey).	

Numbering of Standards

The following Mathematics Standards are numbered continuously. The following ranges relate to the clusters found within Mathematics:

Ratios and Proportional Relationships	
Analyze proportional relationships and use them to solve real-world and mathematical problems.	Standards 1-3
The Number System	
Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.	Standards 4-6
Expressions and Equations	
Use properties of operations to generate equivalent expressions.	Standards 7-8
Solve real-life and mathematical problems using numerical and algebraic expressions and equations.	Standards 9-10
Geometry	
Draw, construct and describe geometrical figures and describe the relationships between them.	Standards 11-13
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.	Standards 14-16
Statistics and Probability	
Use random sampling to draw inferences about a population.	Standards 17-18
Draw informal comparative inferences about two populations.	Standards 19-22
Investigate chance processes and develop, use, and evaluate probability models.	Standards 23-26

Ratios and Proportional Relationships

Cluster	Analyze proportional relationships and use them to solve real-world and mathematical problems.
M.7.1	Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units. (e.g., If a person walks ½ mile in each ¼ hour, compute the unit rate as the complex fraction ½/¼ miles per hour, equivalently 2 miles per hour.)

M.7.2	Recognize and represent proportional relationships between quantities. a. Decide whether two quantities are in a proportional relationship (e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin). b. Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions of proportional relationships. c. Represent proportional relationships by equations. (e.g., If total cost t is proportional to the number n of items purchased at a constant price p, the relationship between the total cost and the number of items can be expressed as t = pn.) d. Explain what a point (x,y) on the graph of a proportional relationship means in terms of the situation. Focus special attention on the points (0,0) and (1,r) where r is the unit rate.
M.7.3	Use proportional relationships to solve multistep ratio and percent problems (e.g., simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and/or percent error).

The Number System

Cluster	**Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.**
M.7.4	Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram. a. Describe situations in which opposite quantities combine to make 0. (e.g., A hydrogen atom has 0 charge because its two constituents are oppositely charged.) b. Understand p + q as the number located a distance |q| from p, in the positive or negative direction, depending on whether q is positive or negative. (i.e., To add "p + q" on the number line, start at "0" and move to "p" then move |q| in the positive or negative direction depending on whether "q" is positive or negative.) Show that a number and its opposite have a sum of 0 (are additive inverses). Interpret sums of rational numbers by describing real-world contexts. c. Understand subtraction of rational numbers as adding the additive inverse, p – q = p + (–q). Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in real-world contexts. d. Apply properties of operations as strategies to add and subtract rational numbers.

M.7.5	Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers. a. Understand that multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as (–1)(–1) = 1 and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts. b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If p and q are integers, then –(p/q) = (–p)/q = p/(–q). Interpret quotients of rational numbers by describing real world contexts. c. Apply properties of operations as strategies to multiply and divide rational numbers. d. Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.
M.7.6	Solve real-world and mathematical problems involving the four operations with rational numbers. Instructional Note: Computations with rational numbers extend the rules for manipulating fractions to complex fractions.

Expressions and Equations

Cluster	**Use properties of operations to generate equivalent expressions.**
M.7.7	Apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients.
M.7.8	Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. (e.g., a + 0.05a = 1.05a means that “increase by 5%” is the same as “multiply by 1.05.”)

Cluster	**Solve real-life and mathematical problems using numerical and algebraic expressions and equations.**
M.7.9	Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. (e.g., If a woman making $25 an hour gets a 10% raise, she will make an additional 1/10 of her salary an hour, or $2.50, for a new salary of $27.50. If you want to place a towel bar 9 3/4 inches long in the center of a door that is 27 1/2 inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.)

M.7.10	Use variables to represent quantities in a real-world or mathematical problem and construct simple equations and inequalities to solve problems by reasoning about the quantities. a. Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p, q, and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach. (e.g., The perimeter of a rectangle is 54 cm. Its length is 6 cm. What is its width? An arithmetic solution similar to "54 – 6 – 6 divided by 2" may be compared with the reasoning involved in solving the equation $2w – 12 = 54$. An arithmetic solution similar to "54/2 – 6" may be compared with the reasoning involved in solving the equation $2(w – 6) = 54$.) b. Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p, q, and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem. (e.g., As a salesperson, you are paid $50 per week plus $3 per sale. This week you want your pay to be at least $100. Write an inequality for the number of sales you need to make, and describe the solutions.)

Geometry

Cluster	**Draw, construct and describe geometrical figures and describe the relationships between them.**
M.7.11	Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.
M.7.12	Draw (freehand, with ruler and protractor, and with technology) geometric shapes with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.
M.7.13	Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

Cluster	**Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.**
M.7.14	Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.
M.7.15	Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure.
M.7.16	Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

Statistics and Probability

Cluster	Use random sampling to draw inferences about a population.
M.7.17	Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.
M.7.18	Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. (e.g., Estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.)

Cluster	Draw informal comparative inferences about two populations.
M.7.19	Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.
M.7.20	Summarize numerical data sets in relation to their context, such as by: a. Reporting the number of observations. b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement. c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered. Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.
M.7.21	Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability. (e.g., The mean height of players on the basketball team is 10 cm greater than the mean height of players on the soccer team, about twice the variability (mean absolute deviation) on either team; on a dot plot, the separation between the two distributions of heights is noticeable.)
M.7.22	Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations. (e.g., Decide whether the words in a chapter of a seventh-grade science book are generally longer than the words in a chapter of a fourth-grade science book.)

Cluster	Investigate chance processes and develop, use, and evaluate probability models.
M.7.23	Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around 1/2 indicates an event that is neither unlikely nor likely and a probability near 1 indicates a likely event.
M.7.24	Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability. (e.g., When rolling a number cube 600 times, predict that a 3 or 6 would be rolled roughly 200 times, but probably not exactly 200 times.)

M.7.25	Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. (e.g., If a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.) b. Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process. (e.g., Find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?)
M.7.26	Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation. a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs. b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., “rolling double sixes”), identify the outcomes in the sample space which compose the event. c. Design and use a simulation to generate frequencies for compound events. (e.g., Use random digits as a simulation tool to approximate the answer to the question: If 40% of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?)

West Virginia College- and Career-Readiness Standards for Mathematics – Grade 8

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in the eighth grade will focus on three critical areas: 1) formulating and reasoning about expressions and equations, including modeling an association in bivariate data with a linear equation, and solving linear equations and systems of linear equations; (2) grasping the concept of a function and using functions to describe quantitative relationships; (3) analyzing two- and three-dimensional space and figures using distance, angle, similarity and congruence and understanding and applying the Pythagorean Theorem. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students in eighth grade will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from seventh grade, the following chart represents the mathematical understandings that will be developed in eighth grade:

<table>
<tr><th>The Number System</th><th>Expressions and Equations</th></tr>
<tr><td>• Understand that every number has a decimal expansion and use these to compare the size of irrational numbers.</td><td>• Work with positive and negative exponents, square root and cube root symbols, and scientific notation (e.g., Evaluate $\sqrt{36} + 64$; estimate world population as 7 x 109).
• Solve linear equations (e.g., $-x + 5(x + 1/3) = 2x - 8$); solve pairs of linear equations (e.g., $x + 6y = -1$ and $2x - 2y = 12$); and write equations to solve related word problems.</td></tr>
<tr><th>Functions</th><th>Geometry</th></tr>
<tr><td>• Understand slope and relate linear equations in two variables to lines in the coordinate plane.
• Understand functions as rules that assign a unique output number to each input number; use linear functions to model relationships.</td><td rowspan="3">• Understand congruence and similarity using physical models, transparencies, or geometry software (e.g., Given two congruent figures, show how to obtain one from the other by a sequence of rotations, translations, and/or reflections).</td></tr>
<tr><th>Statistics and Probability</th></tr>
<tr><td>• Analyze statistical relationships by using a best-fit line (a straight line that models an association between two quantities).</td></tr>
</table>

Numbering of Standards

The following Mathematics Standards are numbered continuously. The following ranges relate to the clusters found within Mathematics:

The Number System	
Know that there are numbers that are not rational, and approximate them by rational numbers.	Standards 1-2
Expressions and Equations	
Work with radicals and integer exponents.	Standards 3-6
Understand the connections between proportional relationships, lines, and linear equations.	Standards 7-8
Analyze and solve linear equations and pairs of simultaneous linear equations.	Standards 9-10
Functions	
Define, evaluate, and compare functions.	Standards 11-13
Use functions to model relationships between quantities.	Standards 14-15
Geometry	
Understand congruence and similarity using physical models, transparencies, or geometry software.	Standards 16-20
Understand and apply the Pythagorean Theorem.	Standards 21-23
Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.	Standard 24
Statistics and Probability	
Investigate patterns of association in bivariate data.	Standards 25-28

The Number System

Cluster	Know that there are numbers that are not rational, and approximate them by rational numbers.
M.8.1	Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number. Instructional Note: A decimal expansion that repeats the digit 0 is often referred to as a "terminating decimal."
M.8.2	Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram and estimate the value of expressions such as π^2. (e.g., By truncating the decimal expansion of $\sqrt{2}$, show that $\sqrt{2}$ is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.)

Expressions and Equations

Cluster	Work with radicals and integer exponents.
M.8.3	Know and apply the properties of integer exponents to generate equivalent numerical expressions. (e.g., $3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.)
M.8.4	Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.
M.8.5	Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other. (e.g., Estimate the population of the United States as 3×10^8 and the population of the world as 7×10^9, and determine that the world population is more than 20 times larger.)
M.8.6	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. (e.g., Use millimeters per year for seafloor spreading.) Interpret scientific notation that has been generated by technology.

Cluster	Understand the connections between proportional relationships, lines, and linear equations.
M.8.7	Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. (e.g., Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.)
M.8.8	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation $y = mx$ for a line through the origin and the equation $y = mx + b$ for a line intercepting the vertical axis at b.

Cluster	Analyze and solve linear equations and pairs of simultaneous linear equations.
M.8.9	Solve linear equations in one variable. a. Give examples of linear equations in one variable with one solution, infinitely many solutions or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form $x = a$, $a = a$, or $a = b$ results (where a and b are different numbers). b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

M.8.10	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically and estimate solutions by graphing the equations. Solve simple cases by inspection. (e.g., 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6.) c. Solve real-world and mathematical problems leading to two linear equations in two variables. (e.g., Given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.)

Functions

Cluster	**Define, evaluate, and compare functions.**
M.8.11	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output. Instructional Note: Function notation is not required in grade 8.
M.8.12	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.)
M.8.13	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. (e.g., The function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.)

Cluster	**Use functions to model relationships between quantities.**
M.8.14	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
M.8.15	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Geometry

Cluster	**Understand congruence and similarity using physical models, transparencies, or geometry software.**
M.8.16	Verify experimentally the properties of rotations, reflections and translations: a. Lines are taken to lines, and line segments to line segments of the same length. b. Angles are taken to angles of the same measure. c. Parallel lines are taken to parallel lines.
M.8.17	Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

M.8.18	Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates.
M.8.19	Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them.
M.8.20	Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles. (e.g., Arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.)

Cluster	**Understand and apply the Pythagorean Theorem.**
M.8.21	Explain a proof of the Pythagorean Theorem and its converse.
M.8.22	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.
M.8.23	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

Cluster	**Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.**
M.8.24	Know the formulas for the volumes of cones, cylinders and spheres and use them to solve real-world and mathematical problems.

Statistics and Probability

Cluster	**Investigate patterns of association in bivariate data.**
M.8.25	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association and nonlinear association.
M.8.26	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line.
M.8.27	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. (e.g., In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.)
M.8.28	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. (e.g., Collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?)

High School Mathematics

At the high school level, the standards are organized by conceptual category (number and quantity, algebra, functions, geometry, modeling and probability and statistics), showing the body of knowledge students should aquire in each category to be college- and career-ready and to be prepared to study more advanced mathematics. There are two distinct course sequence pathways of the high school standards for the mathematics progression in grades 9-11:

- The Integrated Pathway with a course sequence of Math I, Math II, and Math III, each of which includes number, algebra, geometry, probability and statistics; and
- The Traditional Pathway with a course sequence of Algebra I, Geometry, and Algebra II, with some data, probability and statistics included in each course.

Each pathway organizes the identical standards into courses that provide a strong foundation for post-secondary success. As a result, the mathematics standards identified in Math I, Math II and Math III are identical to the standards identified in Algebra I, Geometry and Algebra II. The content is simply grouped differently among the three years. Local Education Agencies (LEA) must choose to implement either the Integrated or Traditional Pathway. Regardless of the pathway chosen for grades 9-11, the fourth course options for all students are the same.

West Virginia College- and Career-Readiness Standards for Mathematics

Integrated Pathway
8th Grade High School Mathematics I

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will focus on six critical units that deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend. Students explore the role of rigid motions in congruence and similarity, are introduced to the Pythagorean Theorem, and examine volume relationships of cones, cylinders and spheres. Students in 8th Grade High School Mathematics I use properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades and develop connections between the algebraic and geometric ideas studied. Mathematical habits of mind, which, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from seventh grade, the following chart represents the mathematical understandings that will be developed:

Relationships between Quantities	**Linear and Exponential Relationships**
• Solve problems with a wide range of units and solve problems by thinking about units. (e.g., The Trans Alaska Pipeline System is 800 miles long and cost $8 billion to build. Divide one of these numbers by the other. What is the meaning of the answer? Greenland has a population of 56,700 and a land area of 2,175,600 square kilometers. By what factor is the population density of the United States, 80 persons per square mile, larger than the population density of Greenland?)	• Understand contextual relationships of variables and constants. (e.g., Annie is picking apples with her sister. The number of apples in her basket is described by n = 22t + 12, where t is the number of minutes Annie spends picking apples. What do the numbers 22 and 12 tell you about Annie's apple picking?)
Reasoning with Equations	**Descriptive Statistics**
• Translate between various forms of linear equations. (e.g., The perimeter of a rectangle is given by P = 2W + 2L. Solve for W and restate in words the meaning of this new formula in terms of the meaning of the other variables.). • Explore systems of equations, find and interpret their solutions. (e.g., The high school is putting on the musical Footloose. The auditorium has 300 seats. Student tickets are $3 and adult tickets are $5. The royalty for the musical is $1300. What combination of student and adult tickets do you need to fill the house and pay the royalty? How could you change the price of tickets so more students can go?)	• Use linear regression techniques to describe the relationship between quantities and assess the fit of the model. (e.g., Use the high school and university grades for 250 students to create a model that can be used to predict a student's university GPA based on his high school GPA.)
Congruence, Proof, and Constructions	**Connecting Algebra and Geometry through Coordinates**
• Given a transformation, work backwards to discover the sequence that led to the transformation. • Given two quadrilaterals that are reflections of each other, find the line of that reflection.	• Use a rectangular coordinate system and build on understanding of the Pythagorean Theorem to find distances. (e.g., Find the area and perimeter of a real-world shape using a coordinate grid and Google Earth.) • Analyze the triangles and quadrilaterals on the coordinate plane to determine their properties. (e.g., Determine whether a given quadrilateral is a rectangle.)

Numbering of Standards

The following Mathematics Standards are numbered continuously. The following ranges relate to the clusters found within Mathematics:

Relationships between Quantities	
Reason quantitatively and use units to solve problems.	Standards 1-3
Interpret the structure of expressions.	Standard 4
Create equations that describe numbers or relationships.	Standards 5-8
Linear and Exponential Relationships	
Represent and solve equations and inequalities graphically.	Standards 9-11
Define, evaluate, and compare functions.	Standards 12-14
Understand the concept of a function and use function notation.	Standards 15-17
Use functions to model relationships between quantities.	Standards 18-19
Interpret functions that arise in applications in terms of a context.	Standards 20-22
Analyze functions using different representations.	Standards 23-24
Build a function that models a relationship between two quantities.	Standards 25-26
Build new functions from existing functions.	Standard 27
Construct and compare linear, quadratic, and exponential models and solve problems.	Standards 28-30
Interpret expressions for functions in terms of the situation they model.	Standard 31
Reasoning with Equations	
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 32
Solve equations and inequalities in one variable.	Standard 33
Analyze and solve linear equations and pairs of simultaneous linear equations.	Standard 34
Solve systems of equations.	Standards 35-36
Descriptive Statistics	
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 37-39
Investigate patterns of association in bivariate data.	Standards 40-43
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standards 44-45
Interpret linear models.	Standards 46-48
Congruence, Proof, and Constructions	
Experiment with transformations in the plane.	Standards 49-53
Understand congruence in terms of rigid motions.	Standards 54-56
Make geometric constructions.	Standards 57-58
Understand and apply the Pythagorean theorem.	Standards 59-61
Connecting Algebra and Geometry through Coordinates	
Use coordinates to prove simple geometric theorems algebraically.	Standards 62-64

Relationships between Quantities

Cluster	Reason quantitatively and use units to solve problems.
M.1HS8.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.1HS8.2	Define appropriate quantities for the purpose of descriptive modeling. Instructional Note: Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.
M.1HS8.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Cluster	Interpret the structure of expressions.
M.1HS8.4	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Limit to linear expressions and to exponential expressions with integer exponents.

Cluster	Create equations that describe numbers or relationships.
M.1HS8.5	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions. Instructional Note: Limit to linear and exponential equations and in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.
M.1HS8.6	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Limit to linear and exponential equations and in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.
M.1HS8.7	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: Limit to linear equations and inequalities.
M.1HS8.8	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Limit to formulas with a linear focus.

Linear and Exponential Relationships

Cluster	Represent and solve equations and inequalities graphically.
M.1HS8.9	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Instructional Note: Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.

M.1HS8.10	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value exponential, and logarithmic functions. Instructional Note: Focus on cases where $f(x)$ and $g(x)$ are linear or exponential.
M.1HS8.11	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Cluster	**Define, evaluate, and compare functions.**
M.1HS8.12	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
M.1HS8.13	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.)
M.1HS8.14	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. (e.g., The function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.)

Cluster	**Understand the concept of a function and use function notation.**
M.1HS8.15	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Constrain to linear functions and exponential functions having integral domains.
M.1HS8.16	Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Constrain to linear functions and exponential functions having integral domains.
M.1HS8.17	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. (e.g., The Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n)+ f(n-1)$ for $n \geq 1$.) Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Constrain to linear functions and exponential functions having integral domains. Draw connection to M.1HS8.26, which requires students to write arithmetic and geometric sequences.

Cluster	Use functions to model relationships between quantities.
M.1HS8.18	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
M.1HS8.19	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Cluster	Interpret functions that arise in applications in terms of a context.
M.1HS8.20	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on linear and exponential functions.
M.1HS8.21	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Instructional Note: Focus on linear and exponential functions.
M.1HS8.22	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers. Mathematics II and III will address other function types.

Cluster	Analyze functions using different representations.
M.1HS8.23	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \times 2^n$.
M.1HS8.24	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \times 2^n$.

Cluster	Build a function that models a relationship between two quantities.
M.1HS8.25	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process or steps for calculation from a context. b. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.) Instructional Note: Limit to linear and exponential functions.
M.1HS8.26	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Instructional Note: Limit to linear and exponential functions. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.

Cluster	Build new functions from existing functions.
M.1HS8.27	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x+k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.

Cluster	Construct and compare linear, quadratic, and exponential models and solve problems.
M.1HS8.28	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
M.1HS8.29	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
M.1HS8.30	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Limit to comparisons between exponential and linear models.

Cluster	Interpret expressions for functions in terms of the situation they model.
M.1HS8.31	Interpret the parameters in a linear or exponential function in terms of a context. Instructional Note: Limit exponential functions to those of the form $f(x) = b^x + k$.

Reasoning with Equations

Cluster	Understand solving equations as a process of reasoning and explain the reasoning.
M.1HS8.32	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Instructional Note: Students should focus on linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Mathematics III.

Cluster	Solve equations and inequalities in one variable.
M.1HS8.33	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Instructional Note: Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.

Cluster	Solve equations and inequalities in one variable.
M.1HS8.34	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6. c. Solve real-world and mathematical problems leading to two linear equations in two variables. For example, given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair. Instructional Note: While this content is likely subsumed by M.1HS8.33, 35, and 36, it could be used for scaffolding instruction to the more sophisticated content found there.

Cluster	Solve systems of equations.
M.1HS8.35	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. Instructional Note: Include cases where two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution).
M.1HS8.36	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Instructional Note: Include cases where two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution).

Descriptive Statistics

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.
M.1HS8.37	Represent data with plots on the real number line (dot plots, histograms, and box plots).
M.1HS8.38	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Instructional Note: In grades 6 – 7, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.
M.1HS8.39	Interpret differences in shape, center and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Instructional Note: In grades 6 – 7, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.

Cluster	Investigate patterns of association in bivariate data. Instructional Note: While this content is likely subsumed by M.1HS8.45-48, it could be used for scaffolding instruction to the more sophisticated content found there.
M.1HS8.40	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association and nonlinear association.
M.1HS8.41	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line.
M.1HS8.42	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. (e.g., In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.)
M.1HS8.43	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. (e.g., Collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?)

Cluster	Summarize, represent, and interpret data on two categorical and quantitative variables.
M.1HS8.44	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data.

M.1HS8.45	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data. a. Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. b. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. c. Informally assess the fit of a function by plotting and analyzing residuals. (Focus should be on situations for which linear models are appropriate.) d. Fit a linear function for scatter plots that suggest a linear association. Instructional Note: Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.

Cluster	**Interpret linear models.**
M.1HS8.46	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Instructional Note: Build on students' work with linear relationships and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.1HS8.47	Compute (using technology) and interpret the correlation coefficient of a linear fit. Instructional Note: Build on students' work with linear relationships and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.1HS8.48	Distinguish between correlation and causation. Instructional Note: The important distinction between a statistical relationship and a cause-and-effect relationship arises here.

Congruence, Proof, and Constructions

Cluster	**Experiment with transformations in the plane.**
M.1HS8.49	Know precise definitions of angle, circle, perpendicular line, parallel line and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
M.1HS8.50	Represent transformations in the plane using, example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.1HS8.51	Given a rectangle, parallelogram, trapezoid or regular polygon, describe the rotations and reflections that carry it onto itself. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).

M.1HS8.52	Develop definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.1HS8.53	Given a geometric figure and a rotation, reflection or translation draw the transformed figure using, e.g., graph paper, tracing paper or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).

Cluster	**Understand congruence in terms of rigid motions.**
M.1HS8.54	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.
M.1HS8.55	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.
M.1HS8.56	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.

Cluster	**Make geometric constructions.**
M.1HS8.57	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.
M.1HS8.58	Construct an equilateral triangle, a square and a regular hexagon inscribed in a circle. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.

Cluster	Understand and apply the Pythagorean theorem.
M.1HS8.59	Explain a proof of the Pythagorean theorem and its converse.
M.1HS8.60	Apply the Pythagorean theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. Instructional Note: Discuss applications of the Pythagorean theorem and its connections to radicals, rational exponents, and irrational numbers.
M.1HS8.61	Apply the Pythagorean theorem to find the distance between two points in a coordinate system. Instructional Note: Discuss applications of the Pythagorean theorem and its connections to radicals, rational exponents, and irrational numbers.

Connecting Algebra and Geometry through Coordinates

Cluster	Use coordinates to prove simple geometric theorems algebraically.
M.1HS8.62	Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).) Instructional Note: Reasoning with triangles in this unit is limited to right triangles (e.g., derive the equation for a line through two points using similar right triangles).
M.1HS8.63	Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems. (e.g., Find the equation of a line parallel or perpendicular to a given line that passes through a given point.) Instructional Note: Reasoning with triangles in this unit is limited to right triangles (e.g., derive the equation for a line through two points using similar right triangles). Relate work on parallel lines to work on M.1HS8.35 involving systems of equations having no solution or infinitely many solutions.
M.1HS8.64	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, (e.g., using the distance formula). Instructional Note: Reasoning with triangles in this unit is limited to right triangles (e.g., derive the equation for a line through two points using similar right triangles). This standard provides practice with the distance formula and its connection with the Pythagorean theorem.

West Virginia College- and Career-Readiness Standards for Mathematics

Traditional Pathway
High School Algebra I for 8th Grade

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will focus on five critical units that deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. Students are introduced to methods for analyzing and using quadratic functions, including manipulating expressions for them, and solving quadratic equations. Students in High School Algebra I for 8th Grade understand and apply the Pythagorean theorem, and use quadratic functions to model and solve problems. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from seventh grade, the following chart represents the mathematical understandings that will be developed:

<table>
<tr><th>Relationships between Quantities and Reasoning with Equations</th><th>Linear and Exponential Relationships</th></tr>
<tr><td>• Solve problems with a wide range of units and solve problems by thinking about units. (e.g., The Trans Alaska Pipeline System is 800 miles long and cost $8 billion to build. Divide one of these numbers by the other. What is the meaning of the answer? Greenland has a population of 56,700 and a land area of 2,175,600 square kilometers. By what factor is the population density of the United States, 80 persons per square mile, larger than the population density of Greenland?)</td><td>• Understand contextual relationships of variables and constants. (e.g., Annie is picking apples with her sister. The number of apples in her basket is described by n = 22t + 12, where t is the number of minutes Annie spends picking apples. What do the numbers 22 and 12 tell you about Annie's apple picking?)</td></tr>
<tr><th>Descriptive Statistics</th><th>Expressions and Equations</th></tr>
<tr><td>• Use linear regression techniques to describe the relationship between quantities and assess the fit of the model. (e.g., Use the high school and university grades for 250 students to create a model that can be used to predict a student's university GPA based on his high school GPA.)</td><td rowspan="3">• Interpret algebraic expressions and transforming them purposefully to solve problems. (e.g., In solving a problem about a loan with interest rate r and principal P, seeing the expression $P(1+r)^n$ as a product of P with a factor not depending on P.)</td></tr>
<tr><th>Quadratic Functions and Modeling</th></tr>
<tr><td>• Solve real-world and mathematical problems by writing and solving nonlinear equations, such as quadratic equations ($ax^2 + bx + c = 0$).</td></tr>
</table>

Numbering of Standards

The following Mathematics Standards are numbered continuously. The following ranges relate to the clusters found within Mathematics:

Relationships between Quantities and Reasoning with Equations	
Reason quantitatively and use units to solve problems.	Standards 1-3
Interpret the structure of expressions.	Standard 4
Create equations that describe numbers or relationships.	Standards 5-8
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 9
Solve equations and inequalities in one variable.	Standard 10
Linear and Exponential Relationships	
Extend the properties of exponents to rational exponents.	Standards 11-12
Analyze and solve linear equations and pairs of simultaneous linear equations.	Standard 13
Solve systems of equations.	Standards 14-15
Represent and solve equations and inequalities graphically.	Standards 16-18
Define, evaluate and compare functions.	Standards 19-21
Understand the concept of a function and use function notation.	Standards 22-24
Use functions to model relationships between quantities.	Standards 25-26
Interpret functions that arise in applications in terms of a context.	Standards 27-29
Analyze functions using different representations.	Standards 30-31
Build a function that models a relationship between two quantities.	Standards 32-33
Build new functions from existing functions.	Standard 34
Construct and compare linear, quadratic, and exponential models and solve problems.	Standards 35-37
Interpret expressions for functions in terms of the situation they model.	Standard 38
Descriptive Statistics	
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 39-41
Investigate patterns of association in bivariate data.	Standards 42-45
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standards 46-47
Interpret linear models.	Standards 48-50

Expressions and Equations	
Interpret the structure of equations.	Standards 51-52
Write expressions in equivalent forms to solve problems.	Standard 53
Perform arithmetic operations on polynomials.	Standard 54
Create equations that describe numbers or relationships.	Standards 55-57
Solve equations and inequalities in one variable.	Standard 58
Solve systems of equations.	Standard 59
Quadratic Functions and Modeling	
Use properties of rational and irrational numbers.	Standard 60
Understand and apply the Pythagorean theorem.	Standards 61-63
Interpret functions that arise in applications in terms of a context.	Standards 64-66
Analyze functions using different representations.	Standards 67-69
Build a function that models a relationship between two quantities.	Standard 70
Build new functions from existing functions.	Standards 71-72
Construct and compare linear, quadratic and exponential models and solve problems.	Standard 73

Relationships between Quantities

Cluster	Reason quantitatively and use units to solve problems.
M.A18.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.A18.2	Define appropriate quantities for the purpose of descriptive modeling. Instructional Note: Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.
M.A18.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Cluster	Interpret the structure of expressions.
M.A18.4	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Limit to linear expressions and to exponential expressions with integer exponents.

Cluster	Create equations that describe numbers or relationships.
M.A18.5	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.
M.A18.6	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.
M.A18.7	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: Limit to linear equations and inequalities.
M.A18.8	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Limit to formulas with a linear focus.

Cluster	Understand solving equations as a process of reasoning and explain the reasoning.
M.A18.9	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Instructional Note: Students should focus on linear equations and be able to extend and apply their reasoning to other types of equations in future units and courses. Students will solve exponential equations in Algebra II.

Cluster	Solve equations and inequalities in one variable.
M.A18.10	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Instructional Note: Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.

Linear and Exponential Relationships

Cluster	Extend the properties of exponents to rational exponents.
M.A18.11	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5. Instructional Note: Address this standard before discussing exponential functions with continuous domains.
M.A18.12	Rewrite expressions involving radicals and rational exponents using the properties of exponents. Instructional Note: Address this standard before discussing exponential functions with continuous domains.

Cluster	Analyze and solve linear equations and pairs of simultaneous linear equations.
M.A18.13	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6. c. Solve real-world and mathematical problems leading to two linear equations in two variables. (e.g., Given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.) Instructional Note: While this content is likely subsumed by M.A18.10, 14, and 15, it could be used for scaffolding instruction to the more sophisticated content found there.

Cluster	Solve systems of equations.
M.A18.14	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. Instructional Note: Include cases where two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution).
M.A18.15	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Instructional Note: Include cases where two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution).

Cluster	Represent and solve equations and inequalities graphically.
M.A18.16	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Instructional Note: Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.
M.A18.17	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations). Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential and logarithmic functions. Instructional Note: Focus on cases where f(x) and g(x) are linear or exponential.
M.A18.18	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Cluster	Define, evaluate and compare functions. Instructional Note: While this content is likely subsumed by M.A18.22-24 and M.A18.30a it could be used for scaffolding instruction to the more sophisticated content found there.
M.A18.19	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

M.A18.20	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.)
M.A18.21	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. (e.g., The function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.)

Cluster	**Understand the concept of a function and use function notation.**
M.A18.22	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Constrain examples to linear functions and exponential functions having integral domains.
M.A18.23	Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Constrain examples to linear functions and exponential functions having integral domains.
M.A18.24	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n)+ f(n-1)$ for $n \geq 1$. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Constrain examples to linear functions and exponential functions having integral domains. Draw connection to M.A18.33, which requires students to write arithmetic and geometric sequences.)

Cluster	**Use functions to model relationships between quantities.** Instructional Note: While this content is likely subsumed by M.A18.27and M.A18.32a, it could be used for scaffolding instruction to the more sophisticated content found there.
M.A18.25	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
M.A18.26	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Cluster	Interpret functions that arise in applications in terms of a context.
M.A18.27	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on linear and exponential functions.
M.A18.28	Relate the domain of a function to its graph and where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Instructional Note: Focus on linear and exponential functions.
M.A18.29	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers. The Quadratic Functions and Modeling unit of this course and Algebra II course will address other function types.

Cluster	Analyze functions using different representations.
M.A18.30	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph exponential and logarithmic functions, showing intercepts and end behavior and trigonometric functions, showing period, midline and amplitude. Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \bullet 2^n$.
M.A18.31	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \bullet 2^n$.

Cluster	Build a function that models a relationship between two quantities.
M.A18.32	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Instructional Note: Limit to linear and exponential functions.
M.A18.33	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Instructional Note: Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.

Cluster	Build new functions from existing functions.
M.A18.34	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.

Cluster	Construct and compare linear, quadratic, and exponential models and solve problems.
M.A18.35	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
M.A18.36	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship or two input-output pairs (include reading these from a table).
M.A18.37	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Limit to comparisons between linear and exponential models.

Cluster	Interpret expressions for functions in terms of the situation they model.
M.A18.38	Interpret the parameters in a linear or exponential function in terms of a context. Instructional Note: Limit exponential functions to those of the form $f(x) = b^x + k$.

Descriptive Statistics

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.
M.A18.39	Represent data with plots on the real number line (dot plots, histograms, and box plots).
M.A18.40	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Instructional Note: In grades 6 – 7, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.
M.A18.41	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Instructional Note: In grades 6 – 7, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.

Cluster	Investigate patterns of association in bivariate data.
M.A18.42	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
M.A18.43	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
M.A18.44	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. (e.g., In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.)
M.A18.45	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. (e.g., Collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?)

Cluster	Summarize, represent, and interpret data on two categorical and quantitative variables.
M.A18.46	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data.
M.A18.47	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Instructional Note: Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. Instructional Note: Focus should be on situations for which linear models are appropriate, but may be used to preview quadratic functions in the Quadratic Functions and Modeling Unit. c. Fit a linear function for scatter plots that suggest a linear association. Instructional Note: Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.

Cluster	Interpret linear models.
M.A18.48	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Instructional Note: Build on students' work with linear relationships and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.A18.49	Compute (using technology) and interpret the correlation coefficient of a linear fit. Instructional Note: Build on students' work with linear relationships and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.

M.A18.50	Distinguish between correlation and causation. Instructional Note: The important distinction between a statistical relationship and a cause-and-effect relationship arises here.

Expressions and Equations

Cluster	**Interpret the structure of equations.**
M.A18.51	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Focus on quadratic and exponential expressions. For M.A18.51b, exponents are extended from integer found in the unit on Relationships between Quantities to rational exponents focusing on those that represent square roots and cube roots.
M.A18.52	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.

Cluster	**Write expressions in equivalent forms to solve problems.**
M.A18.53	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. Instructional Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.

Cluster	**Perform arithmetic operations on polynomials.**
M.A18.54	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Instructional Note: Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.

Cluster	**Create equations that describe numbers or relationships.**
M.A18.55	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Extend work on linear and exponential equations in the unit on Relationships between Quantities to include quadratic equations.

M.A18.56	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Extend work on linear and exponential equations in the unit on Relationships between Quantities to include quadratic equations.
M.A18.57	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Extend work on linear and exponential equations in the unit on Relationships between Quantities to include quadratic equations. Extend M.A18.57 to formulas involving squared variables.

Cluster	**Solve equations and inequalities in one variable.**
M.A18.58	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. Instructional Note: Students should learn of the existence of the complex number system, but will not solve quadratics with complex solutions until Algebra II.

Cluster	**Solve systems of equations.**
M.A18.59	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. (e.g., Find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.) Instructional Note: Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between $x^2 + y^2 = 1$ and $y = (x+1)/2$ leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple $3^2 + 4^2 = 5^2$.

Quadratic Functions and Modeling

Cluster	**Use properties of rational and irrational numbers.**
M.A18.60	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. Instructional Note: Connect to physical situations (e.g., finding the perimeter of a square of area 2).

Cluster	**Understand and apply the Pythagorean theorem.**
M.A18.61	Explain a proof of the Pythagorean Theorem and its converse
M.A18.62	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. Instructional Note: Discuss applications of the Pythagorean theorem and its connections to radicals, rational exponents, and irrational numbers.
M.A18.63	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. Instructional Note: Discuss applications of the Pythagorean theorem and its connections to radicals, rational exponents, and irrational numbers.

Cluster	Interpret functions that arise in applications in terms of a context.
M.A18.64	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studies in the unit on Linear and Exponential Functions.
M.A18.65	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studies in the unit on Linear and Exponential Functions.
M.A18.66	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studies in the unit on Linear and Exponential Functions.

Cluster	Analyze functions using different representations.
M.A18.67	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Instructional Note: Compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic function can be factored.
M.A18.68	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay. Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic function can be factored. This unit, and in particular in M.A18.68b, extends the work begun in Unit 2 on exponential functions with integral exponents.
M.A18.69	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic function can be factored.

Cluster	Build a function that models a relationship between two quantities.
M.A18.70	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Instructional Note: Focus on situations that exhibit a quadratic relationship.

Cluster	Build new functions from existing functions.
M.A18.71	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on quadratic functions, and consider including absolute value functions.
M.A18.72	Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or f(x) = (x+1)/(x-1) for $x \neq 1$. Instructional Note: Focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2$, $x > 0$.

Cluster	Construct and compare linear, quadratic and exponential models and solve problems.
M.A18.73	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Compare linear and exponential growth to growth of quadratic growth.

Appendix A
Standards vs. Curriculum

COLLEGE- & CAREER-READINESS

STANDARDS	CURRICULUM
What's the Difference?	
Standards are what we want students to know, understand and be able to do; **Standards** represent goals.	The **Curriculum** is an intentional learning plan to ensure students achieve the goals of the standards; the **Curriculum** represents the learning experience.
Standards and Curriculum *A STANDARD is a goal. The CURRICULUM is a means to achieve the goal.*	
Example 1 • 3rd Grade Mathematics Goal **Standard:** M.3.8 *Solve two-step word problems using the four operations, represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.*	**Curriculum:** Teacher locates instructional materials, plans and facilitates learning activities and assesses the students' mastery of the standard.
Example 2 • 6th Grade English Language Arts Goal **Standard:** ELA.6.18 *By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grade 6-8 text complexity range proficiently, with scaffolding as needed at the high end of the range.*	
Who is Responsible?	
West Virginia Board of Education West Virginia Department of Education	County boards of education, administrators and teachers

West Virginia College & Career Readiness Standards

West Virginia EDUCATION
Steven L. Paine, Ed.D.
State Superintendent of Schools

Appendix B
Sample Parent Letter (Grade 6)

(Insert School Address)

(Insert Date)

Dear Parent or Guardian,

I look forward to being your child's mathematics teacher for sixth grade! The first years of the middle grades can be an exciting, intimidating, and overwhelming experience – all at once. I want to welcome your child to my classroom and give you a preview of what to expect in mathematics for the upcoming school year. As always, if you have any questions or concerns, please feel free to contact me using the information below.

Providing students with the skills they need to be college- and career-ready is the ultimate goal of the educational standards in West Virginia. Students in sixth grade will continue enhancing skills through a developmentally-appropriate progression of standards, building on what they learned in fifth grade and preparing them for what they will need in seventh grade. The following summary highlights some of the mathematical understandings students will develop this school year.

Ratios and Proportional Reasoning	**The Number System**
• Understand ratios and rates, and solve problems involving proportional relationships (e.g., If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours?).	• Divide fractions and solve related word problems (e.g., How wide is a rectangular strip of land with length 3/4 mile and area 1/2 square mile?). • Use positive and negative numbers together to describe quantities; understand the ordering and absolute values of positive and negative numbers.
Expressions and Equations	**Geometry**
• Work with variables and expressions by generalizing the way numbers work (e.g., When adding numbers, the order doesn't matter, so x + y = y + x; likewise, properties of addition and multiplication can be used to rewrite 24x + 18y as 6(4x + 3y), or y + y + y as 3y). • Write equations to solve word problems and describe relationships between quantities (e.g., The distance D traveled by a train in time T might be expressed by an equation D = 85T, where D is in miles and T is in hours.).	• Reason about relationships between shapes to determine area, surface area, and volume.
Statistics and Probability	
• Create graphical representations of data and reason about statistical distributions.	

With your assistance and continued support, your student will have a successful school year and a smooth transition to middle school. I am available by phone and email if you have any questions or concerns or if you would like to set up a conference.

Sincerely,

Email:
Phone:

Sample Parent Letter (Grade 7)

(Insert School Address)

(Insert Date)

Dear Parent or Guardian,

I look forward to being your child's mathematics teacher for seventh grade! The middle grades can be an exciting, intimidating, and overwhelming experience – all at once. I want to welcome your child to my classroom and give you a preview of what to expect in mathematics for the upcoming school year. As always, if you have any questions or concerns, please feel free to contact me using the information below.

Providing students with the skills they need to be college- and career-ready is the ultimate goal of the educational standards in West Virginia. Students in seventh grade will continue enhancing skills through a developmentally-appropriate progression of standards, building on what they learned in sixth grade and preparing them for what they will need in eighth grade. The following summary highlights some of the mathematical understandings students will develop this school year.

Ratios and Proportional Reasoning	**The Number System**
• Analyze proportional relationships (e.g., by graphing in the coordinate plane), and distinguish proportional relationships from other kinds of mathematical relationships (e.g., Buying 10 times as many items will cost you 10 times as much, but taking 10 times as many aspirin will not lower your fever 10 times as much.).	• Solve percent problems (e.g., tax, tips, and markups and markdowns). • Solve word problems that have a combination of whole numbers, fractions, and decimals (e.g., A woman making $25 per hour receives a 10% raise; she will make an additional 1⁄10 of his or her salary an hour, or $2.50, for a new salary of $27.50.)
Expressions and Equations	**Geometry**
• Solve equations such as 1⁄2 (x – 3) = 3⁄4 quickly and accurately, and write equations of this kind to solve word problems.	• Solve problems involving scale drawings.
Statistics and Probability	
• Use statistics to draw inferences and make comparisons (e.g., deciding which candidate is likely to win an election based on a survey).	

With your assistance and continued support, your student will have a successful school year. I am available by phone and email if you have any questions or concerns or if you would like to set up a conference.

Sincerely,

Email:
Phone:

Sample Parent Letter (Grade 8)

(Insert School Address)

(Insert Date)

Dear Parent or Guardian,

I look forward to being your child's mathematics teacher for eighth grade! The middle grades can be an exciting, intimidating, and overwhelming experience – all at once. I want to welcome your child to my classroom and give you a preview of what to expect in mathematics for the upcoming school year. As always, if you have any questions or concerns, please feel free to contact me using the information below.

Providing students with the skills they need to be college- and career-ready is the ultimate goal of the educational standards in West Virginia. Students in eighth grade will continue enhancing skills through a developmentally-appropriate progression of standards, building on what they learned in seventh grade and preparing them for what they will need in ninth grade. The following summary highlights some of the mathematical understandings students will develop this school year.

<table>
<tr><th>The Number System</th><th>Expressions and Equations</th></tr>
<tr><td>• Understand that every number has a decimal expansion and use these to compare the size of irrational numbers.</td><td>• Work with positive and negative exponents, square root and cube root symbols, and scientific notation (e.g., Evaluate √36 + 64; estimate world population as 7 x 109).
• Solve linear equations (e.g., –x + 5(x + 1⁄3) = 2x – 8); solve pairs of linear equations (e.g., x + 6y = –1 and 2x – 2y = 12); and write equations to solve related word problems.</td></tr>
<tr><th>Functions</th><th>Geometry</th></tr>
<tr><td>• Understand slope and relate linear equations in two variables to lines in the coordinate plane.
• Understand functions as rules that assign a unique output number to each input number; use linear functions to model relationships.</td><td rowspan="2">• Understand congruence and similarity using physical models, transparencies, or geometry software (e.g., Given two congruent figures, show how to obtain one from the other by a sequence of rotations, translations, and/or reflections).</td></tr>
<tr><th>Statistics and Probability</th></tr>
<tr><td>• Analyze statistical relationships by using a best-fit line (a straight line that models an association between two quantities).</td><td></td></tr>
</table>

With your assistance and continued support, your student will have a successful school year. I am available by phone and email if you have any questions or concerns or if you would like to set up a conference.

Sincerely,

Email:
Phone:

Sample Parent Letter
8th Grade High School Mathematics I

(Insert School Address)

(Insert Date)

Dear Parent or Guardian,

I look forward to being your child's teacher for the 8th Grade High School Mathematics I course! Middle school and this opportunity to take a high school mathematics course can be an exciting, intimidating, and overwhelming experience – all at once. I want to welcome your child to my classroom and give you a preview of what to expect in mathematics for the upcoming school year. As always, if you have any questions or concerns, please feel free to contact me using the information below.

Providing students with the skills they need to be college- and career-ready is the ultimate goal of the educational standards in West Virginia. Students in this course will continue enhancing skills through a developmentally-appropriate progression of standards, building on what they learned in seventh grade and preparing them for what they will need in the High School Mathematics II course. To support student progress from seventh grade mathematics to this high school course, several important topics from eighth grade mathematics will also need to be addressed. The following summary highlights some of the mathematical understandings students will develop this school year.

Relationships between Quantities	**Linear and Exponential Relationships**
• Solve problems with a wide range of units and solve problems by thinking about units. (e.g., The Trans Alaska Pipeline System is 800 miles long and cost $8 billion to build. Divide one of these numbers by the other. What is the meaning of the answer? Greenland has a population of 56,700 and a land area of 2,175,600 square kilometers. By what factor is the population density of the United States, 80 persons per square mile, larger than the population density of Greenland?)	• Understand contextual relationships of variables and constants. (e.g., Annie is picking apples with her sister. The number of apples in her basket is described by $n = 22t + 12$, where t is the number of minutes Annie spends picking apples. What do the numbers 22 and 12 tell you about Annie's apple picking?)
Reasoning with Equations	**Descriptive Statistics**
• Translate between various forms of linear equations. (e.g., The perimeter of a rectangle is given by $P = 2W + 2L$. Solve for W and restate in words the meaning of this new formula in terms of the meaning of the other variables.). • Explore systems of equations, find and interpret their solutions. (e.g., The high school is putting on the musical Footloose. The auditorium has 300 seats. Student tickets are $3 and adult tickets are $5. The royalty for the musical is $1300. What combination of student and adult tickets do you need to fill the house and pay the royalty? How could you change the price of tickets so more students can go?)	• Use linear regression techniques to describe the relationship between quantities and assess the fit of the model. (e.g., Use the high school and university grades for 250 students to create a model that can be used to predict a student's university GPA based on his high school GPA.)
Congruence, Proof, and Constructions	**Connecting Algebra and Geometry through Coordinates**
• Given a transformation, work backwards to discover the sequence that led to the transformation. • Given two quadrilaterals that are reflections of each other, find the line of that reflection.	• Use a rectangular coordinate system and build on understanding of the Pythagorean Theorem to find distances. (e.g., Find the area and perimeter of a real-world shape using a coordinate grid and Google Earth.) • Analyze the triangles and quadrilaterals on the coordinate plane to determine their properties. (e.g., Determine whether a given quadrilateral is a rectangle.)

With your assistance and continued support, your student will have a successful school year and a smooth transition to high school mathematics. I am available by phone and email if you have any questions or concerns or if you would like to set up a conference.

Sincerely,

Email:
Phone:

Sample Parent Letter
High School Algebra I for 8th Grade

(Insert School Address)

(Insert Date)

Dear Parent or Guardian,

I look forward to being your child's teacher for the High School Algebra I for 8th Grade course! Middle school and this opportunity to take a high school mathematics course can be an exciting, intimidating, and overwhelming experience – all at once. I want to welcome your child to my classroom and give you a preview of what to expect in mathematics for the upcoming school year. As always, if you have any questions or concerns, please feel free to contact me using the information below.

Providing students with the skills they need to be college- and career-ready is the ultimate goal of the educational standards in West Virginia. Students in this course will continue enhancing skills through a developmentally-appropriate progression of standards, building on what they learned in seventh grade and preparing them for what they will need in the high school Geometry and Algebra II courses. To support student progress from seventh grade mathematics to this high school course, several important topics from eighth grade mathematics will also need to be addressed. The following summary highlights some of the mathematical understandings students will develop this school year.

Relationships between Quantities and Reasoning with Equations	**Linear and Exponential Relationships**
• Solve problems with a wide range of units and solve problems by thinking about units. (e.g., The Trans Alaska Pipeline System is 800 miles long and cost $8 billion to build. Divide one of these numbers by the other. What is the meaning of the answer? Greenland has a population of 56,700 and a land area of 2,175,600 square kilometers. By what factor is the population density of the United States, 80 persons per square mile, larger than the population density of Greenland?)	• Understand contextual relationships of variables and constants. (e.g., Annie is picking apples with her sister. The number of apples in her basket is described by $n = 22t + 12$, where t is the number of minutes Annie spends picking apples. What do the numbers 22 and 12 tell you about Annie's apple picking?)
Descriptive Statistics	**Expressions and Equations**
• Use linear regression techniques to describe the relationship between quantities and assess the fit of the model. (e.g., Use the high school and university grades for 250 students to create a model that can be used to predict a student's university GPA based on his high school GPA.)	• Interpret algebraic expressions and transforming them purposefully to solve problems. (e.g., In solving a problem about a loan with interest rate r and principal P, seeing the expression $P(1+r)^n$ as a product of P with a factor not depending on P.)
Quadratic Functions and Modeling	
• Solve real-world and mathematical problems by writing and solving nonlinear equations, such as quadratic equations ($ax^2 + bx + c = 0$).	

With your assistance and continued support, your student will have a successful school year and a smooth transition to high school mathematics. I am available by phone and email if you have any questions or concerns or if you would like to set up a conference.

Sincerely,

Email:
Phone:

Appendix C
Mathematics Standards Progressions

Skill Progressions in West Virginia College- and Career-Readiness Standards for Mathematics

The following pages outline the skill progressions found in the West Virginia College- and Career Readiness Standards for Mathematics. In Mathematics, the sequence of topics follow a programmatic progression that are reflected in the domains. Domains have been organized into programmatic levels where grade-level clusters provide detail about the skill progressions. The language of the clusters illustrates the advancing rigor and complexity of the expectations for what students should know, understand, and be able to do.

This document is intended to be a resource to foster and support discussion among teachers or how best to personalize and differentiate instruction for their students. The progression of skills toward college- and career-readiness that are outlined here can be used to scaffold instruction, assist with remediation, and to develop instructional plans that meet the specific needs of each student.

Mathematics Progressions – Grade 6 through Grade 8

Domain: Number – Ratios and Proportional Relationships

Grade	Clusters
6	• Understand ratio concepts and use reasoning to solve problems.
7	• Analyzing proportional relationships and use them to solve real-world and mathematical problems.
8	• Not a primary focus of Grade 8

Domain: Number – The Number System

Grade	Clusters
6	• Apply and extend previous understandings of multiplication and division to divide fractions by fractions. • Compute fluently with multi-digit numbers and find common factors and multiples. • Apply previous understandings of numbers to the system of rational numbers.
7	• Apply and extend previous understandings of operations with fractions to add, subtract, multiply, and divide rational numbers.
8	• Know that there are numbers that are not rational, and approximate them by rational numbers.

Domain: Number – Expressions and Equations

Grade	Clusters
6	• Apply and extend previous understandings of arithmetic to algebraic expressions. • Reason about and solve one-variable equations and inequalities. • Represent and analyze quantitative relationships between dependent and independent variables.
7	• Use properties of operations to generate equivalent expressions. • Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

8	• Work with radicals and integer exponents. • Understand the connections between proportional relationships, lines, and linear equations. • Analyze and solve linear equations and pairs of simultaneous linear equations.

Domain: Number – Functions

Grade	Clusters
6	• Initial focus begins in Grade 8
7	• Initial focus begins in Grade 8
8	• Define, evaluate, and compare functions. • Use functions to model relationships between quantities.

Domain: Number – Geometry

Grade	Clusters
6	• Solve real-world and mathematical problems involving area, surface area, and volume.
7	• Draw, construct, and describe geometrical figures and describe the relationships between them. • Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.
8	• Understand congruence and similarity using physical models, transparencies, or geometric software. • Understand and apply the Pythagorean Theorem. • Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.

Domain: Number – Statistics and Probability

Grade	Clusters
6	• Develop understanding of statistical variability. • Summarize and describe distributions.
7	• Use random sampling to draw inferences about a population. • Draw informal comparative inferences about two populations. • Investigate change processes and develop, use, and evaluate probability models.
8	• Investigate patterns of association in bivariate data.

Mathematics Progressions – High School Integrated Pathway

Domain: The Real Number System

Grade	Clusters
8th Grade High School Mathematics I	• Initial focus begins in High School Mathematics II
High School Mathematics I	• Initial focus begins in High School Mathematics II
High School Mathematics II	• Extend the properties of exponents to rational exponents. • Use properties of rational and irrational numbers.
High School Mathematics III	• Not a primary focus of High School Mathematics III

Domain: Quantities

Grade	Clusters
8th Grade High School Mathematics I	• Reason quantitatively and use units to solve problems. (Foundation for work with expressions, equations, and functions.)
High School Mathematics I	• Reason quantitatively and use units to solve problems. (Foundation for work with expressions, equations, and functions.)
High School Mathematics II	• Not a primary focus of High School Mathematics II
High School Mathematics III	• Not a primary focus of High School Mathematics III

Domain: The Complex Number System

Grade	Clusters
8th Grade High School Mathematics I	• Initial focus begins in High School Mathematics II
High School Mathematics I	• Initial focus begins in High School Mathematics II
High School Mathematics II	• Perform arithmetic operations with complex numbers. (i^2 as the highest power of i) • Use complex numbers in polynomial identities and equations (Quadratics with real coefficients)
High School Mathematics III	• Use complex numbers in polynomial identities and equations. (Polynomials with real coefficients)

Domain: Seeing Structure in Expressions

Grade	Clusters
8th Grade High School Mathematics I	• Interpret the structure of expressions. (Linear expressions and exponential expressions with integer exponents)
High School Mathematics I	• Interpret the structure of expressions. (Linear expressions and exponential expressions with integer exponents)
High School Mathematics II	• Interpret the structure of expressions. (Quadratic and exponential) • Write expressions in equivalent forms to solve problems. (Quadratic and exponential)
High School Mathematics III	• Interpret the structure of expressions. (Polynomial and rational) • Write expressions in equivalent forms to solve problems.

Domain: Arithmetic with Polynomials and Rational Expressions

Grade	Clusters
8th Grade High School Mathematics I	• Initial focus begins in High School Mathematics II
High School Mathematics I	• Initial focus begins in High School Mathematics II
High School Mathematics II	• Perform arithmetic operations on polynomials. (Polynomials that simplify to quadratics.)

High School Mathematics III	• Perform arithmetic operations on polynomials. (Beyond quadratics) • Understand the relationship between zeros and factors of polynomials. • Use polynomial identities to solve problems. • Rewrite rational expressions. (Linear and quadratic denominators)

Domain: Creating Equations

Grade	Clusters
8th Grade High School Mathematics I	• Create equations that describe numbers or relationships. (Linear and exponential [integer inputs only])
High School Mathematics I	• Create equations that describe numbers or relationships. (Linear and exponential [integer inputs only])
High School Mathematics II	• Create equations that describe numbers or relationships. (Include formulas involving quadratic terms)
High School Mathematics III	• Create equations that describe numbers or relationships. (Equations using all available types of expressions, including simple root functions.)

Domain: Reasoning with Equations and Inequalities

Grade	Clusters
8th Grade High School Mathematics I	• Understand solving equations as a process of reasoning and explain the reasoning. (Master linear, learn as a general principle) • Solve equations and inequalities in one variable. (Linear inequalities; literal equations that are linear in the variables being solved for; exponential of a form, such as $2x = 1/16$) • Analyze and solve linear equations and pairs of simultaneous linear equations. (Systems of linear equations) • Solve systems of equations (Linear systems) • Represent and solve equations and inequalities graphically. (Linear and exponential; learn as a general principle)
High School Mathematics I	• Understand solving equations as a process of reasoning and explain the reasoning. (Master linear, learn as a general principle) • Solve equations and inequalities in one variable. (Linear inequalities; literal equations that are linear in the variables being solved for; exponential of a form, such as $2x = 1/16$) • Analyze and solve linear equations and pairs of simultaneous linear equations. (Systems of linear equations) • Solve systems of equations (Linear systems) • Represent and solve equations and inequalities graphically. (Linear and exponential; learn as a general principle)
High School Mathematics II	• Solve equations and inequalities in one variable. (Quadratics with real coefficients) • Solve systems of equations. (Linear-quadratic systems)
High School Mathematics III	• Understand solving equations as a process of reasoning and explain the reasoning. (Simple radical and rational) • Represent and solve equations and inequalities graphically. (Combine polynomial, rational, radical, absolute value, and exponential functions)

Domain: Interpreting Functions

Grade	Clusters
8th Grade High School Mathematics I	• Define, evaluate, and compare functions. • Understand the concept of a function and use function notation. (Learn as a general principle. Focus on linear and exponential (integer domains) and on arithmetic and geometric sequences.) • Use function to model relationships between quantities. • Interpret functions that arise in applications in terms of a context. (Linear and exponential [linear domain]) • Analyze functions using different representations. (Linear and exponential)
High School Mathematics I	• Understand the concept of a function and use function notation. (Learn as a general principle. Focus on linear and exponential [integer domains] and on arithmetic and geometric sequences.) • Interpret functions that arise in applications in terms of a context. (Linear and exponential [linear domain]) • Analyze functions using different representations. (Linear and exponential)
High School Mathematics II	• Interpret functions that arise in applications in terms of a context. (Include rational, square root and cube root; emphasize selection of appropriate models) • Analyze functions using different representations. (Linear, exponential, quadratic, absolute value, step, piecewise-defined)
High School Mathematics III	• Interpret functions that arise in applications in terms of a context. (Quadratic) • Analyze functions using different representations. (Include rational and radical; focus on using key features to guide selection of appropriate types of model function)

Domain: Building Functions

Grade	Clusters
8th Grade High School Mathematics I	• Build a function that models a relationship between two quantities. (Linear and exponential integer inputs]) • Build new functions from existing functions. (Linear and exponential; focus on vertical translations for exponential)
High School Mathematics I	• Build a function that models a relationship between two quantities. (Linear and exponential [integer inputs]) • Build new functions from existing functions. (Linear and exponential; focus on vertical translations for exponential)
High School Mathematics II	• Build a function that models a relationship between two quantities. (Quadratic and exponential) • Build new functions from existing functions. (Quadratic, all exponential, absolute value)
High School Mathematics III	• Build a function that models a relationship between two quantities. (Include all types of functions studied) • Build new functions from existing functions. (Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types)

Domain: Linear, Quadratic, and Exponential Models

Grade	Clusters
8th Grade High School Mathematics I	• Construct and compare linear, quadratic, and exponential models and solve problems. (Linear and exponential) • Interpret expressions for functions in terms of the situation they model. (Linear and exponential of form $f(x) = b^x + k$)

High School Mathematics I	• Construct and compare linear, quadratic, and exponential models and solve problems. (Linear and exponential) • Interpret expressions for functions in terms of the situation they model. (Linear and exponential of form $f(x) = b^x + k$)
High School Mathematics II	• Construct and compare linear, quadratic, and exponential models and solve problems. (Include quadratic)
High School Mathematics III	• Construct and compare linear, quadratic, and exponential models and solve problems. (Logarithms as solutions for exponentials)

Domain: Trigonometric Functions

Grade	Clusters
8th Grade High School Mathematics I	• Initial focus begins in High School Mathematics II
High School Mathematics I	• Initial focus begins in High School Mathematics II
High School Mathematics II	• Prove and apply trigonometric identities.
High School Mathematics III	• Extend the domain of trigonometric functions using the unit circle. • Model periodic phenomena with trigonometric functions.

Domain: Congruence

Grade	Clusters
8th Grade High School Mathematics I	• Experiment with transformations in the plane. • Understand congruence in terms of rigid motions. (Build on rigid motions as a familiar starting point for development of concept of geometric proof) • Make geometric constructions. (Formalize and explain processes)
High School Mathematics I	• Experiment with transformations in the plane. • Understand congruence in terms of rigid motions. (Build on rigid motions as a familiar starting point for development of concept of geometric proof) • Make geometric constructions. (Formalize and explain processes)
High School Mathematics II	• Prove geometric theorems. (Focus on validity of underlying reasoning while using variety of ways of writing proofs)
High School Mathematics III	• Not a primary focus of High School Mathematics III

Domain: Similarity, Right Triangles, and Trigonometry

Grade	Clusters
8th Grade High School Mathematics I	• Initial focus begins in High School Mathematics II
High School Mathematics I	• Initial focus begins in High School Mathematics II
High School Mathematics II	• Understand similarity in terms of similarity transformations. • Prove theorems involving similarity. (Focus on validity of underlying reasoning while using variety of formats) • Define trigonometric ratios and solve problems involving right triangles.
High School Mathematics III	• Apply trigonometry to general triangles.

Domain: Circles

Grade	Clusters
8th Grade High School Mathematics I	• Initial focus begins in High School Mathematics II
High School Mathematics I	• Initial focus begins in High School Mathematics II
High School Mathematics II	• Understand and apply theorems about circles. • Find arc lengths and area of sectors of circles. (Radian introduced only as a unit of measure)
High School Mathematics III	• Not a primary focus of High School Mathematics III

Domain: Expressing Geometric Properties with Equations

Grade	Clusters
8th Grade High School Mathematics I	• Use coordinates to prove simple geometric theorems algebraically. (Include distance formula; relate to Pythagorean Theorem)
High School Mathematics I	• Use coordinates to prove simple geometric theorems algebraically. (Include distance formula; relate to Pythagorean Theorem)
High School Mathematics II	• Translate between the geometric description and the equation for a conic section. • Use coordinates to prove simple geometric theorems algebraically. (Include simple circle theorems)
High School Mathematics III	• Not a primary focus of High School Mathematics III

Domain: Geometric Measurement and Dimension

Grade	Clusters
8th Grade High School Mathematics I	• Understand and apply the Pythagorean Theorem. (Connect to radicals, rational exponents, and irrational numbers)
High School Mathematics I	• Not a primary focus of Mathematics I
High School Mathematics II	• Explain volume formulas and use them to solve problems.
High School Mathematics III	• Visualize the relation between two-dimensional and three-dimensional objects.

Domain: Interpreting Categorical and Quantitative Data

Grade	Clusters
8th Grade High School Mathematics I	• Summarize, represent, and interpret data on a single count or measurement variable. • Investigate patterns of association in bivariate data. • Summarize, represent, and interpret data on two categorical and quantitative variables. (Linear focus; discuss general principle) • Interpret linear models.

High School Mathematics I	• Summarize, represent, and interpret data on a single count or measurement variable. • Investigate patterns of association in bivariate data. • Summarize, represent, and interpret data on two categorical and quantitative variables. (Linear focus; discuss general principle) • Interpret linear models.
High School Mathematics II	• Summarize, represent, and interpret data on two categorical and quantitative variables.
High School Mathematics III	• Not a primary focus of High School Mathematics III

Domain: Making Inferences and Justifying Conclusions

Grade	Clusters
8th Grade High School Mathematics I	• Initial focus begins in Mathematics III
High School Mathematics I	• Initial focus begins in Mathematics III
High School Mathematics II	• Initial focus begins in Mathematics III
High School Mathematics III	• Understand and evaluate random processes underlying statistical experiments. • Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Domain: Conditional Probability and the Rules of Probability

Grade	Clusters
8th Grade High School Mathematics I	• Initial focus begins in Mathematics II
High School Mathematics I	• Initial focus begins in Mathematics II
High School Mathematics II	• Understand independence and conditional probability and use them to interpret data. (Link to data simulations or experiments) • Use the rules of probability to compute probabilities of compound events in a uniform probability model.
High School Mathematics III	• Not a primary focus of High School Mathematics III

Domain: Using Probability to Make Decisions

Grade	Clusters
8th Grade High School Mathematics I	• Initial focus begins in Mathematics II
High School Mathematics I	• Initial focus begins in Mathematics II
High School Mathematics II	• Use probability to evaluate outcomes of decisions. (Introductory; apply counting rules)
High School Mathematics III	• Use probability to evaluate outcomes of decisions. (Include more complex situations)

Mathematics Progressions – High School Traditional Pathway

Domain: The Real Number System

Grade	Clusters
High School Algebra I for 8th Grade	• Extend the properties of exponents to rational exponents. • Use properties of rational and irrational numbers.
High School Algebra I	• Extend the properties of exponents to rational exponents. • Use properties of rational and irrational numbers.
High School Geometry	• Not a primary focus of High School Geometry
High School Algebra II	• Not a primary focus of High School Algebra II

Domain: Quantities

Grade	Clusters
High School Algebra I for 8th Grade	• Reason quantitatively and use units to solve problems. (Foundation for work with expressions, equations, and functions.)
High School Algebra I	• Reason quantitatively and use units to solve problems. (Foundation for work with expressions, equations, and functions.)
High School Geometry	• Not a primary focus of High School Geometry
High School Algebra II	• Not a primary focus of High School Algebra II

Domain: The Complex Number System

Grade	Clusters
High School Algebra I for 8th Grade	• Initial focus begins in High School Algebra II
High School Algebra I	• Initial focus begins in High School Algebra II
High School Geometry	• Initial focus begins in High School Algebra II
High School Algebra II	• Perform arithmetic operations with complex numbers. • Use complex numbers in polynomial identities and equations. (Polynomials with real coefficients)

Domain: Seeing Structure in Expressions

Grade	Clusters
High School Algebra I for 8th Grade	• Interpret the structure of expressions. (Linear, exponential, quadratic) • Write expressions in equivalent forms to solve problems. (Quadratic and exponential)
High School Algebra I	• Interpret the structure of expressions. (Linear, exponential, quadratic) • Write expressions in equivalent forms to solve problems. (Quadratic and exponential)
High School Geometry	• Not a primary focus of High School Geometry
High School Algebra II	• Interpret the structure of expressions. (Polynomial and rational) • Write expressions in equivalent forms to solve problems.

Domain: Arithmetic with Polynomials and Rational Expressions

Grade	Clusters
High School Algebra I for 8th Grade	• Perform arithmetic operations on polynomials. (Linear and quadratic.)
High School Algebra I	• Perform arithmetic operations on polynomials. (Polynomials that simplify to quadratics.)
High School Geometry	• Not a primary focus of High School Geometry
High School Algebra II	• Perform arithmetic operations on polynomials. (Beyond quadratics) • Understand the relationship between zeros and factors of polynomials. • Use polynomial identities to solve problems. • Rewrite rational expressions. (Linear and quadratic denominators)

Domain: Creating Equations

Grade	Clusters
High School Algebra I for 8th Grade	• Create equations that describe numbers or relationships. (Linear, quadratic, and exponential [integer inputs only])
High School Algebra I	• Create equations that describe numbers or relationships. (Linear, quadratic, and exponential [integer inputs only])
High School Geometry	• Not a primary focus of High School Geometry
High School Algebra II	• Create equations that describe numbers or relationships. (Equations using all available types of expressions, including simple root functions.)

Domain: Reasoning with Equations and Inequalities

Grade	Clusters
High School Algebra I for 8th Grade	• Understand solving equations as a process of reasoning and explain the reasoning. (Master linear, learn as a general principle) • Solve equations and inequalities in one variable. (Linear inequalities; literal equations that are linear in the variables being solved for; quadratics with real solutions) • Analyze and solve linear equations and pairs of simultaneous linear equations. • Solve systems of equations (Linear-linear and linear-quadratic) • Represent and solve equations and inequalities graphically. (Linear and exponential; learn as a general principle)
High School Algebra I	• Understand solving equations as a process of reasoning and explain the reasoning. (Master linear, learn as a general principle) • Solve equations and inequalities in one variable. (Linear inequalities; literal equations that are linear in the variables being solved for; quadratics with real solutions) • Solve systems of equations (Linear-linear and linear-quadratic) • Represent and solve equations and inequalities graphically. (Linear and exponential; learn as a general principle)
High School Geometry	• Not a primary focus of High School Geometry
High School Algebra II	• Understand solving equations as a process of reasoning and explain the reasoning. (Simple radical and rational) • Represent and solve equations and inequalities graphically. (Combine polynomial, rational, radical, absolute value, and exponential functions)

Domain: Interpreting Functions

Grade	Clusters
High School Algebra I for 8th Grade	• Define, evaluate, and compare functions. • Understand the concept of a function and use function notation. (Learn as a general principle. Focus on linear and exponential [integer domains] and on arithmetic and geometric sequences.) • Use functions to model relationships between quantities. • Interpret functions that arise in applications in terms of a context. (Linear, exponential, and quadratic) • Analyze functions using different representations. (Linear, exponential, quadratic, absolute value, step, piecewise-defined)
High School Algebra I	• Understand the concept of a function and use function notation. (Learn as a general principle. Focus on linear and exponential [integer domains] and on arithmetic and geometric sequences.) • Interpret functions that arise in applications in terms of a context. (Linear, exponential, and quadratic) • Analyze functions using different representations. (Linear, exponential, quadratic, absolute value, step, piecewise-defined)
High School Geometry	• Not a primary focus of High School Geometry
High School Algebra II	• Interpret functions that arise in applications in terms of a context. (Emphasize selection of appropriate models) • Analyze functions using different representations. (Focus on using key features to guide selection of appropriate types of model function)

Domain: Building Functions

Grade	Clusters
High School Algebra I for 8th Grade	• Build a function that models a relationship between two quantities. (Linear, exponential, and quadratic) • Build new functions from existing functions. (Linear, exponential, quadratic, and absolute value)
High School Algebra I	• Build a function that models a relationship between two quantities. (Linear, exponential, and quadratic) • Build new functions from existing functions. (Linear, exponential, quadratic, and absolute value)
High School Geometry	• Not a primary focus of High School Geometry
High School Algebra II	• Build a function that models a relationship between two quantities. (Include all types of functions studied) • Build new functions from existing functions. (Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types)

Domain: Linear, Quadratic, and Exponential Models

Grade	Clusters
High School Algebra I for 8th Grade	• Construct and compare linear, quadratic, and exponential models and solve problems. • Interpret expressions for functions in terms of the situation they model. (Linear and exponential of form $f(x) = b^x + k$)
High School Algebra I	• Construct and compare linear, quadratic, and exponential models and solve problems. • Interpret expressions for functions in terms of the situation they model. (Linear and exponential of form $f(x) = b^x + k$)
High School Geometry	• Not a primary focus of High School Geometry
High School Algebra II	• Construct and compare linear, quadratic, and exponential models and solve problems. (Logarithms as solutions for exponentials)

Domain: Trigonometric Functions

Grade	Clusters
High School Algebra I for 8th Grade	• Initial focus begins in High School Algebra II
High School Algebra I	• Initial focus begins in High School Algebra II
High School Geometry	• Initial focus begins in High School Algebra II
High School Algebra II	• Extend the domain of trigonometric functions using the unit circle. • Model periodic phenomena with trigonometric functions. • Prove and apply trigonometric identities.

Domain: Congruence

Grade	Clusters
High School Algebra I for 8th Grade	• Initial focus begins in High School Geometry
High School Algebra I	• Initial focus begins in High School Geometry
High School Geometry	• Experiment with transformations in the plane. • Understand congruence in terms of rigid motions. (Build on rigid motions as a familiar starting point for development of concept of geometric proof) • Prove geometric theorems. (Focus on validity of underlying reasoning while using variety of ways of writing proofs) • Make geometric constructions. (Formalize and explain processes)
High School Algebra II	• Not a primary focus of High School Algebra II

Domain: Similarity, Right Triangles, and Trigonometry

Grade	Clusters
High School Algebra I for 8th Grade	• Initial focus begins in High School Geometry
High School Algebra I	• Initial focus begins in High School Geometry
High School Geometry	• Understand similarity in terms of similarity transformations. • Prove theorems involving similarity. (Focus on validity of underlying reasoning while using variety of formats) • Define trigonometric ratios and solve problems involving right triangles. • Apply trigonometry to general triangles.
High School Algebra II	• Not a primary focus of High School Algebra II

Domain: Circles

Grade	Clusters
High School Algebra I for 8th Grade	• Initial focus begins in High School Geometry
High School Algebra I	• Initial focus begins in High School Geometry
High School Geometry	• Understand and apply theorems about circles. • Find arc lengths and area of sectors of circles. (Radian introduced only as a unit of measure)
High School Algebra II	• Not a primary focus of High School Algebra II

Domain: Expressing Geometric Properties with Equations

Grade	Clusters
High School Algebra I for 8th Grade	• Initial focus begins in High School Geometry
High School Algebra I	• Initial focus begins in High School Geometry
High School Geometry	• Translate between the geometric description and the equation for a conic section. • Use coordinates to prove simple geometric theorems algebraically. (Include distance formula; relate to Pythagorean Theorem)
High School Algebra II	• Not a primary focus of High School Algebra II

Domain: Geometric Measurement and Dimension

Grade	Clusters
High School Algebra I for 8th Grade	• Understand and apply the Pythagorean Theorem. (Connect to radicals, rational exponents, and irrational numbers)
High School Algebra I	• Not a primary focus of High School Algebra I
High School Geometry	• Explain volume formulas and use them to solve problems. • Visualize the relation between two-dimensional and three-dimensional objects. • Apply geometric concepts in modeling situations.
High School Algebra II	• Not a primary focus of High School Algebra II

Domain: Interpreting Categorical and Quantitative Data

Grade	Clusters
High School Algebra I for 8th Grade	• Summarize, represent, and interpret data on a single count or measurement variable. • Investigate patterns of association in bivariate data. • Summarize, represent, and interpret data on two categorical and quantitative variables. (Linear focus; discuss general principle) • Interpret linear models.
High School Algebra I	• Summarize, represent, and interpret data on a single count or measurement variable. • Investigate patterns of association in bivariate data. • Summarize, represent, and interpret data on two categorical and quantitative variables. (Linear focus; discuss general principle) • Interpret linear models.
High School Geometry	• Not a primary focus of High School Geometry
High School Algebra II	• Summarize, represent, and interpret data on two categorical and quantitative variables.

Domain: Making Inferences and Justifying Conclusions

Grade	Clusters
High School Algebra I for 8th Grade	• Initial focus begins in High School Algebra II
High School Algebra I	• Initial focus begins in High School Algebra II
High School Geometry	• Initial focus begins in High School Algebra II
High School Algebra II	• Understand and evaluate random processes underlying statistical experiments. • Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Domain: Conditional Probability and the Rules of Probability

Grade	Clusters
High School Algebra I for 8th Grade	• Initial focus begins in High School Geometry
High School Algebra I	• Initial focus begins in High School Geometry
High School Geometry	• Understand independence and conditional probability and use them to interpret data. (Link to data simulations or experiments) • Use the rules of probability to compute probabilities of compound events in a uniform probability model.
High School Algebra II	• Not a primary focus of High School Algebra II

Domain: Using Probability to Make Decisions

Grade	Clusters
High School Algebra I for 8th Grade	• Initial focus begins in High School Geometry
High School Algebra I	• Initial focus begins in High School Geometry
High School Geometry	• Use probability to evaluate outcomes of decisions. (Introductory; apply counting rules)
High School Algebra II	• Use probability to evaluate outcomes of decisions. (Include more complex situations)

Appendix D
Mathematics: Grade-level and Course Overview

Grade/Course	Number of Standards
Mathematics – Kindergarten	22
Mathematics – Grade 1	21
Mathematics – Grade 2	26
Mathematics – Grade 3	25
Mathematics – Grade 4	28
Mathematics – Grade 5	26
Mathematics – Grade 6	29
Mathematics – Grade 7	26
Mathematics – Grade 8	28

Integrated Pathway		Traditional Pathway	
Grade/Course	**Number of Standards**	**Grade/Course**	**Number of Standards**
8th Grade High School Mathematics I	64	High School Algebra I for 8th Grade	73
High School Mathematics I	51	High School Algebra I	60
High School Mathematics II	61	High School Geometry	55
High School Mathematics III	48	High School Algebra II	45

Fourth Course Options	
Grade/Course	**Number of Standards**
Advanced Mathematical Modeling	38
Calculus	23
High School Mathematics IV - Trigonometry/Pre-calculus	40
STEM Readiness	31
Transition Mathematics for Seniors	52

AP® Calculus
AP® Computer Science
AP® Statistics

Additional course options include dual credit mathematics courses and advanced mathematics courses offered through WV Virtual School. School teams, including counselors, teachers and administrators, should confer with the student and his/her parents to decide what fourth year mathematics course best meets the needs of the student.

Appendix E
West Virginia's Comprehensive Assessment System

West Virginia's Comprehensive Assessment System

A Comprehensive Assessment System

When some people hear the phrase "state assessment," they think of the test students have to take at the end of the school year. While summative tests are important and provide valuable information about student performance, a comprehensive assessment system goes well beyond the end-of-the-year test. In addition to the summative assessment given to students in the spring of the school year, West Virginia's Comprehensive Assessment System also includes access to interim and diagnostic assessments, as well as formative assessment resources in the Digital Library. All of these are critical to support meaningful instruction and help students in the successful demonstration of knowledge and skills based upon rigorous college and career readiness standards.

Summative Assessment

West Virginia's summative test is known as the West Virginia General Summative Assessment (WVGSA). The English language arts (ELA) and mathematics assessments are computer adaptive, which means each student's individual test adjusts based on how he or she responds. For example, if a student answers questions correctly, the next set of questions the student receives will be more difficult; if a student answers questions incorrectly, the next set of questions the student receives will be easier. A computer adaptive test allows a more precise, valid and reliable score for each student. A more detailed look at the WVGSA is provided on the next page. The state also administers an alternate summative assessments to students with significant cognitive disabilities. For more information about the alternate assessment, contact the Office of Assessment.

Interim/Diagnostic Assessments

The interim and diagnostic assessments are optional tests that allow educators to measure student progress throughout the year and support instruction of the standards. There are two types of interim assessments—the Interim Comprehensive Assessment (ICA) and the Interim Assessment Block (IAB). The ICAs mirror the end-of-year summative test. The item types and formats, including performance tasks, are similar to those students will encounter on the summative test. The IABs are shorter and focus on sets of targets. The diagnostic assessments were created by West Virginia teachers to focus on specific skills aligned to various targets.

Formative Assessment

Formative assessment is a deliberate, ongoing process used during instruction to ensure students are making progress toward specific learning goals by providing actionable feedback. Formative assessment resources are available in the Digital Library. These resources are designed to help teachers integrate formative assessment processes instructionally.

Reports

A variety of useful reports are available for the WVGSA, as well as the interim and diagnostic assessments. These reports provide valuable information to districts, schools, teachers, students and parents. The summative assessment reports include roster performance on each target and student performance on each claim and in each proficiency level. Students and parents receive individual student reports that indicate students' levels of performance. In the future, we expect to add a separate writing score report that will show how students scored on the following writing traits—purpose/organization, elaboration/evidence, and conventions. A variety reports also are available for the interim and diagnostic assessments, including item-level analysis reporting. West Virginia will continue to work toward improving reports so teachers and parents will have the information they need.

Measuring Critical Thinking

West Virginia's balanced assessment system measures critical thinking and higher-level problem solving skills through a variety of innovative test questions. Students are required to write persuasively and use evidence to support their answers. The state's assessments also measure research, listening, and communicating reasoning skills, which are not assessed by other tests.

Supports for All Students

West Virginia provides accommodations, designated supports and universal tools to ensure all students have access to the assessments. The state offers assessments in a variety of forms, including braille, large print and Spanish. For more options and information, see the West Virginia *Guidelines for Participation in Statewide Assessments* at the link at the bottom of this page.

For more information on West Virginia's statewide assessment system, call 304.558.2546 or visit the following website: http://wvde.state.wv.us/assessment/.

West Virginia General Summative Assessment (WVGSA)

Overview:
The West Virginia General Summative Assessment (WVGSA) is the state test administered at the end of each school year. Results from the test provide information about a student's academic strengths, as well as any areas that need improvement, in each assessed content area.

Test Length
The length of time a student spends taking the WVGSA depends on what grade the student is in and could be affected by the student's ability and effort. One advantage of the WVGSA is that it is an untimed test. As long as students are actively engaged in taking the test, students can take as much time as they need. An analysis of last year's test scores shows that the more time students spend working on the test, the better they perform. On average, each student will spend no more than a total of 6 to 8 hours taking the test with sessions spread over 4 to 5 days, depending on the school's testing schedule and the student's grade level. Each district sets its own overall testing window and school testing schedule.

Grades 3-8:
Students in Grades 3-8 take the English language arts (ELA) and mathematics tests. Students in Grades 4 and 6 also take the science test. The tests are aligned to the state-approved grade-level standards for each content area and provide teachers, students and parents with information on how well students are progressing toward being college and career ready when they graduate.

High School
Students in Grades 9-11 take the ELA and mathematics tests. Students in Grade 10 also take the science test. The tests are aligned to the state-approved grade-level high school standards for each content area. West Virginia also offers a Grade 12 College and Career Readiness Assessment, a retest of the Grade 11 WVGSA.

West Virginia students deserve to graduate prepared for the world that awaits them. High school is a critical time to ensure students are on track to graduate with the skills and knowledge to be college and career ready. To ensure students are making progress and meeting college and career readiness benchmarks, West Virginia administers ELA and math tests for Grades 9, 10 and 11 using a comprehensive high school item bank that includes thousands of possible questions written to various assessment targets and at various levels of difficulty.

Achievement levels have been established at each grade level so students and parents can see where students are performing based on grade-level expectations.

- **Grade 9:**
  Students are tested in ELA and math. Students get information about their progress based on West Virginia's 9th grade ELA and math achievement levels. The ELA test is aligned to specific ELA content and targets that span the breadth of high school ELA standards. The math test is aligned to the specific math content students should know by the end of their 9th grade year.

- **Grade 10:**
  Students are tested in ELA, math and science. Students get information about their progress based on West Virginia's 10th grade ELA, math and science achievement levels. The ELA test is aligned to specific ELA content and targets that span the breadth of high school ELA standards. The math test is aligned to the specific math content students should know by the end of their 10th grade year.

- **Grade 11:**
  Students are tested in ELA and math on the high school standards and get information about their progress based on 11th grade achievement levels. If students score at a 3 or higher in ELA or math, West Virginia colleges and universities recognize the student is prepared for credit-bearing courses; thus, they can skip remedial courses and enroll in credit bearing classes. This saves students time and money. Additionally, the student's scores help schools determine whether students would benefit from placement in Grade 12 transition courses to help them achieve college and career readiness.

- **Grade 12 College and Career Readiness Assessment:**
  Seniors enrolled in a Grade 12 transition course will take the Grade 12 College and Career Readiness Assessment (CCRA) unless they can provide evidence they have earned an acceptable benchmark on the Grade 11 WVGSA, ACT or SAT. Students only have to take the content area test for which they do not have a college and career ready indicator. Seniors not enrolled in a Grade 12 transition course have the option to take the Grade 12 CCRA. They can use their ACT or SAT scores to show college readiness.

Appendix F

A SNAPSHOT of Assessments and Assessment Processes for West Virginia Schools

	FORMATIVE ASSESSMENT PROCESS *(occurs daily in grades Pre-K-12; is a fundamental component of high-quality teaching and learning)*	**INTERIM/ DIAGNOSTIC ASSESSMENTS** *(occur periodically in grades Pre-K-12; are optional)*	**STATE SUMMATIVE ASSESSMENT** *(occurs yearly in grades 3-8 and grade 11 in English language arts and mathematics, and in science in grades 5,8,and 10)*
What is it?	A daily process teachers and students use that links evidence of learning to standards in order to personalize learning for all students. (Evidence of learning can include work samples, observations, anecdotal information, graded work, etc…)	Non-secure assessments used to obtain data educators can use to help identify: » strengths and weaknesses of their classes and individual students » necessary adjustments to instruction	A standardized test designed to provide a snapshot of student progress toward college and career readiness in the tested content areas
Who selects the assessment?	Is a teacher-driven process; not an isolated event	Educators	State
Who participates in it?	All educators and students in grades Pre-K-12	Students in grades Pre-K-12	All students in grades 3-8 and grade 11
When does it occur?	Daily, during high-quality instruction; the formative assessment process is NOT an event	Periodically, throughout the school year as applicable	At the end of the year or at the end of a course of study
What is done with the results?	Evidence of learning is collected and discussed by teachers and students; evidence is organized in a way that helps teachers tailor their instruction and articulate learning to families	Districts, schools, and educators use results to evaluate student achievement and learning	Long-range planning based on results can occur at the district or state levels; used in state accountability system
How much time does the assessment take?	Is an ongoing, daily process teachers use to personalize learning for all students	1 hour average	4.5 hour average for the WV General Summative Assessment (average across all grades levels and includes ELA, math, and science)

West Virginia DEPARTMENT OF EDUCATION

Appendix G
Overview of the West Virginia TREE
(Teacher Resources for Educational Excellence)

WEST VIRGINIA **TREE**
one voice. one focus. one stop.

https://wvde.state.wv.us/apps/tree/

West Virginia's online platform for educators is a one stop, grade- and/or content-specific site highlighting WV content standards, resources, and links that are essential to ensure high-quality educational programming. The resources include grade specific lessons, professional learning, and guidance documents crafted to help enhance teaching practice and guide the classroom teacher in the art of teaching. The links connect teachers with information regarding:

- Grade- and/or content-specific content standards, linked to resources to support use
- College and career readiness in West Virginia
- The formative assessment process
- Summative assessment login and resources (grades 3-12)
- Opportunities for professional learning
- Working with children with special needs
- Educator effectiveness and licensure (certification and evaluation)
- Guidance documents
- Programmatic level foundations for learning
- Additional resources

The WV TREE is designed with the teacher's busy schedule in mind, one stop, one focus, and tailored for the professional educator. This 'one stop' ensures teachers will not have to scour the WVDE website to find needed resources.

The WV TREE is a fluid website, with resources and content added on a regular basis. Additionally, future plans for the TREE include a site specific to principals, county chief instructional leaders, as well as counselors.

Notes

Notes

Steven L. Paine, Ed.D.
West Virginia Superintendent of Schools