West Virginia

College & Career Readiness Standard

Resource Booklet for Mathematics

Traditional Pathway

Grades 9-12

Based on WVBE Policy 2520.2B

Effective July 1, 2016

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Foreword

Dear West Virginia Educators,

As we move forward with the rollout of West Virginia's College- and Career-Readiness Standards for English Language Arts and Mathematics (West Virginia Board of Education Policies 2520.1A and 2520.2B, respectively), I am excited to share this standards-focused resource booklet with you. In this booklet you will find:

- Applicable West Virginia College- and Career-Readiness Standards for English Language Arts and/or Mathematics (effective July 1, 2016) for your grade/content area;
- Sample letters by grade level for families regarding the West Virginia College- and Career-Readiness Standards;
- Progression documents for English Language Arts and/or Mathematics; and
- The state-adopted definition of College and Career Readiness for West Virginia.

I know our goal of ensuring all West Virginia students graduate from high school with the skills, knowledge and dispositions to be considered truly college and career ready can become a reality if we focus on the development and success of all students. It is my sincere hope that you will utilize the resources found within this document to tailor your instruction and curricula to meet the needs of all the students you serve.

Last, I would like to thank you for your dedication to the lives and well-being of the students of our great state. I am humbled by the amazing work you do each day to ensure all students are college and career ready.

Sincerely,

Steven L. Paine, Ed.D
State Superintendent of Schools

College- and Career-Readiness in West Virginia

West Virginia's College- and Career-Readiness Standards have been developed with the goal of preparing students in a wide range of high-quality post-secondary opportunities. Specifically, college- and career-readiness refers to the knowledge, skills, and dispositions needed to be successful in higher education and/or training that lead to gainful employment. The West Virginia College- and Career-Readiness Standards establish a set of knowledge and skills that all individuals need to transition into higher education or the workplace, as both realms share many expectations. All students throughout their educational experience, should develop a full understanding of the career opportunities available, the education necessary to be successful in their chosen pathway, and a plan to attain their goals.

College- and Career-Readiness in the Mathematics Content Area

West Virginia's College- and Career-Readiness Standards for Mathematics are the culmination of an extended, broad-based effort to help ensure that all students are college- and career-ready upon completion of high school. The skills contained in the mathematics standards are essential for college- and career-readiness in a twenty-first-century, globally competitive society. The standards reflect a progression and key ideas determining how knowledge is organized and generated within the content area. Standards evolve from specifics to deeper structures inherent in the discipline. These deeper structures serve to connect the specifics. The standards follow such a design, stressing conceptual understanding of key ideas and continually returning to organizing principles such as place value or the properties of operations to structure those ideas. The sequence of topics and performances outlined in mathematics standards must respect the scientific research about how students learn and what is known about how their mathematical knowledge, skill, and understanding develop over time.

The West Virginia College- and Career-Readiness Standards are the result of a statewide public review of the state's educational standards. The West Virginia Department of Education (WVDE), West Virginia Board of Education (WVBE), and West Virginia University partnered in this initiative that began with a website, Academic Spotlight, which served as the platform for feedback collection. This website was active July through September of 2015. After the comment period closed, comments were evaluated by a team of diverse stakeholders, who made recommendations to WVBE based on the comments to meet the needs of West Virginia students. Additionally, during the month of September 2015, eight universities around the state hosted town hall meetings where citizens could pose questions about the standards to a panel of teachers, administrators, and representatives from higher education. The West Virginia College- and Career-Readiness Standards reflect the improvements brought to light by these two methods of public input.

Mathematics

The West Virginia College- and Career-Readiness Standards for Mathematics define what students should understand and be able to do in their study of mathematics. Asking a student to understand something means asking a teacher to assess whether the student has understood it. What does mathematical understanding look like? One hallmark of mathematical understanding is the ability to justify, in a way appropriate to the student's mathematical maturity, why a particular mathematical statement is true or where a mathematical rule comes from. There is a world of difference between a student who can summon a mnemonic device to expand a product such as $(a + b)(x + y)$ and a student who can explain where the mnemonic comes from. The student who can explain the rule understands the mathematics, and may have a better chance to succeed at a less familiar task such as expanding $(a + b + c)(x + y)$. Mathematical understanding and procedural skill are equally important, and both are assessable using mathematical tasks of sufficient richness.

The Standards begin with eight Mathematical Habits of Mind.

Mathematics: Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$ and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Connecting the Mathematical Habits of Mind to the Standards for Mathematical Content

The Mathematical Habits of Mind describe ways in which developing students of mathematics increasingly engage with the subject matter as they grow in mathematical maturity and expertise throughout the elementary, middle and high school years. Designers of curricula, assessments and professional development should all attend to the need to connect the mathematical habits of mind to mathematical content in mathematics instruction.

High School Mathematics

At the high school level, the standards are organized by conceptual category (number and quantity, algebra, functions, geometry, modeling and probability and statistics), showing the body of knowledge students should learn in each category to be college- and career-ready and to be prepared to study more advanced mathematics. There are two distinct course sequence pathways of the high school standards for the mathematics progression in grades 9-11:

- The Integrated Pathway with a course sequence of Math I, Math II, and Math III, each of which includes number, algebra, geometry, probability and statistics; and
- The Traditional Pathway with a course sequence of Algebra I, Geometry, and Algebra II, with some data, probability and statistics included in each course.

Each pathway organizes the identical standards into courses that provide a strong foundation for post-secondary success. As a result, the mathematics standards identified in Math I, Math II and Math III are identical to the standards identified in Algebra I, Geometry and Algebra II. The content is simply grouped differently among the three years. Local Education Agencies (LEA) must choose to implement either the Integrated or Traditional Pathway. Regardless of the pathway chosen for grades 9-11, the fourth course options for all students are the same.

West Virginia College- and Career-Readiness Standards for Mathematics

High School Algebra I for 8th Grade

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will focus on five critical units that deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. Students are introduced to methods for analyzing and using quadratic functions, including manipulating expressions for them, and solving quadratic equations. Students in 8th grade High School Algebra understand and apply the Pythagorean theorem, and use quadratic functions to model and solve problems. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from seventh grade, the following chart represents the mathematical understandings that will be developed:

Relationships between Quantities and Reasoning with Equations	**Linear and Exponential Relationships**
• Solve problems with a wide range of units and solve problems by thinking about units. (e.g., The Trans Alaska Pipeline System is 800 miles long and cost $8 billion to build. Divide one of these numbers by the other. What is the meaning of the answer? Greenland has a population of 56,700 and a land area of 2,175,600 square kilometers. By what factor is the population density of the United States, 80 persons per square mile, larger than the population density of Greenland?)	• Understand contextual relationships of variables and constants. (e.g., Annie is picking apples with her sister. The number of apples in her basket is described by $n = 22t + 12$, where t is the number of minutes Annie spends picking apples. What do the numbers 22 and 12 tell you about Annie's apple picking?)
Descriptive Statistics	**Expressions and Equations**
• Use linear regression techniques to describe the relationship between quantities and assess the fit of the model. (e.g., Use the high school and university grades for 250 students to create a model that can be used to predict a student's university GPA based on his high school GPA.)	• Interpret algebraic expressions and transforming them purposefully to solve problems. (e.g., In solving a problem about a loan with interest rate r and principal P, seeing the expression $P(1+r)^n$ as a product of P with a factor not depending on P.)
Quadratic Functions and Modeling	
• Solve real-world and mathematical problems by writing and solving nonlinear equations, such as quadratic equations ($ax^2 + bx + c = 0$).	

Numbering of Standards

The following Mathematics Standards are numbered continuously. The following ranges relate to the clusters found within Mathematics:

Relationships between Quantities and Reasoning with Equations	
Reason quantitatively and use units to solve problems.	Standards 1-3
Interpret the structure of expressions.	Standard 4
Create equations that describe numbers or relationships.	Standards 5-8
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 9
Solve equations and inequalities in one variable.	Standard 10
Linear and Exponential Relationships	
Extend the properties of exponents to rational exponents.	Standards 11-12
Analyze and solve linear equations and pairs of simultaneous linear equations.	Standard 13
Solve systems of equations.	Standards 14-15
Represent and solve equations and inequalities graphically.	Standards 16-18
Define, evaluate and compare functions.	Standards 19-21
Understand the concept of a function and use function notation.	Standards 22-24
Use functions to model relationships between quantities.	Standards 25-26
Interpret functions that arise in applications in terms of a context.	Standards 27-29
Analyze functions using different representations.	Standards 30-31
Build a function that models a relationship between two quantities.	Standards 32-33
Build new functions from existing functions.	Standard 34
Construct and compare linear, quadratic, and exponential models and solve problems.	Standards 35-37
Interpret expressions for functions in terms of the situation they model.	Standard 38
Descriptive Statistics	
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 39-41
Investigate patterns of association in bivariate data.	Standards 42-45
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standards 46-47
Interpret linear models.	Standards 48-50

Expressions and Equations	
Interpret the structure of equations.	Standards 51-52
Write expressions in equivalent forms to solve problems.	Standard 53
Perform arithmetic operations on polynomials.	Standard 54
Create equations that describe numbers or relationships.	Standards 55-57
Solve equations and inequalities in one variable.	Standard 58
Solve systems of equations.	Standard 59
Quadratic Functions and Modeling	
Use properties of rational and irrational numbers.	Standard 60
Understand and apply the Pythagorean theorem.	Standards 61-63
Interpret functions that arise in applications in terms of a context.	Standards 64-66
Analyze functions using different representations.	Standards 67-69
Build a function that models a relationship between two quantities.	Standard 70
Build new functions from existing functions.	Standards 71-72
Construct and compare linear, quadratic and exponential models and solve problems.	Standard 73

Relationships between Quantities

Cluster	Reason quantitatively and use units to solve problems
M.A18.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.A18.2	Define appropriate quantities for the purpose of descriptive modeling. Instructional Note: Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.
M.A18.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Cluster	Interpret the structure of expressions.
M.A18.4	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Limit to linear expressions and to exponential expressions with integer exponents.

Cluster	Create equations that describe numbers or relationships.
M.A18.5	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.
M.A18.6	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.
M.A18.7	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: Limit to linear equations and inequalities.
M.A18.8	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Limit to formulas with a linear focus.

Cluster	Understand solving equations as a process of reasoning and explain the reasoning.
M.A18.9	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Instructional Note: Students should focus on linear equations and be able to extend and apply their reasoning to other types of equations in future units and courses. Students will solve exponential equations in Algebra II.
Cluster	**Solve equations and inequalities in one variable.**
M.A18.10	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Instructional Note: Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.

Linear and Exponential Relationships

Cluster	Extend the properties of exponents to rational exponents.
M.A18.11	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5. Instructional Note: Address this standard before discussing exponential functions with continuous domains.
M.A18.12	Rewrite expressions involving radicals and rational exponents using the properties of exponents. Instructional Note: Address this standard before discussing exponential functions with continuous domains.

Cluster	Analyze and solve linear equations and pairs of simultaneous linear equations.
M.A18.13	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, 3x + 2y = 5 and 3x + 2y = 6 have no solution because 3x + 2y cannot simultaneously be 5 and 6. c. Solve real-world and mathematical problems leading to two linear equations in two variables. (e.g., Given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.) Instructional Note: While this content is likely subsumed by M.A18.10, 14, and 15, it could be used for scaffolding instruction to the more sophisticated content found there.

Cluster	Solve systems of equations.
M.A18.14	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. Instructional Note: Include cases where two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution).
M.A18.15	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Instructional Note: Include cases where two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution).

Cluster	Represent and solve equations and inequalities graphically.
M.A18.16	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Instructional Note: Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.
M.A18.17	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations). Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential and logarithmic functions. Instructional Note: Focus on cases where f(x) and g(x) are linear or exponential.
M.A18.18	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Cluster	**Define, evaluate and compare functions.** Instructional Note: While this content is likely subsumed by M.A18.22-24 and M.A18.30a it could be used for scaffolding instruction to the more sophisticated content found there.
M.A18.19	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
M.A18.20	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.)
M.A18.21	Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. (e.g., The function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.)

Cluster	**Understand the concept of a function and use function notation.**
M.A18.22	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Constrain examples to linear functions and exponential functions having integral domains.
M.A18.23	Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Constrain examples to linear functions and exponential functions having integral domains.
M.A18.24	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Constrain examples to linear functions and exponential functions having integral domains. Draw connection to M.A18.33, which requires students to write arithmetic and geometric sequences.)

Cluster	**Use functions to model relationships between quantities.** Instructional Note: While this content is likely subsumed by M.A18.27and M.A18.32a, it could be used for scaffolding instruction to the more sophisticated content found there.
M.A18.25	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
M.A18.26	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Cluster	**Interpret functions that arise in applications in terms of a context.**
M.A18.27	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on linear and exponential functions.
M.A18.28	Relate the domain of a function to its graph and where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Instructional Note: Focus on linear and exponential functions.
M.A18.29	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers. The Quadratic Functions and Modeling unit of this course and Algebra II course will address other function types.

Cluster	**Analyze functions using different representations.**
M.A18.30	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph exponential and logarithmic functions, showing intercepts and end behavior and trigonometric functions, showing period, midline and amplitude. Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \cdot 2^n$.
M.A18.31	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \cdot 2^n$.

Cluster	Build a function that models a relationship between two quantities.
M.A18.32	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Instructional Note: Limit to linear and exponential functions.
M.A18.33	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Instructional Note: Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.

Cluster	Build new functions from existing functions.
M.A18.34	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.

Cluster	Construct and compare linear, quadratic, and exponential models and solve problems.
M.A18.35	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
M.A18.36	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship or two input-output pairs (include reading these from a table).
M.A18.37	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Limit to comparisons between linear and exponential models.

Cluster	Interpret expressions for functions in terms of the situation they model.
M.A18.38	Interpret the parameters in a linear or exponential function in terms of a context. Instructional Note: Limit exponential functions to those of the form $f(x) = b^x + k$.

Descriptive Statistics

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.
M.A18.39	Represent data with plots on the real number line (dot plots, histograms, and box plots).
M.A18.40	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Instructional Note: In grades 6 – 7, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.
M.A18.41	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Instructional Note: In grades 6 – 7, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.

Cluster	Investigate patterns of association in bivariate data. Instructional Note: While this content is likely subsumed by M.A18.47-50, it could be used for scaffolding instruction to the more sophisticated content found there.
M.A18.42	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
M.A18.43	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
M.A18.44	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. (e.g., In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.)
M.A18.45	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. (e.g., Collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?)

Cluster	Summarize, represent, and interpret data on two categorical and quantitative variables.
M.A18.46	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data.

M.A18.47	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Instructional Note: Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. Instructional Note: Focus should be on situations for which linear models are appropriate, but may be used to preview quadratic functions in the Quadratic Functions and Modeling Unit. c. Fit a linear function for scatter plots that suggest a linear association. Instructional Note: Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.

Cluster	**Interpret linear models.**
M.A18.48	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Instructional Note: Build on students' work with linear relationships and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.A18.49	Compute (using technology) and interpret the correlation coefficient of a linear fit. Instructional Note: Build on students' work with linear relationships and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.A18.50	Distinguish between correlation and causation. Instructional Note: The important distinction between a statistical relationship and a cause-and-effect relationship arises here.

Expressions and Equations

Cluster	**Interpret the structure of equations.**
M.A18.51	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Focus on quadratic and exponential expressions. For M.A18.51b, exponents are extended from integer found in the unit on Relationships between Quantities to rational exponents focusing on those that represent square roots and cube roots.
M.A18.52	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.

Cluster	Write expressions in equivalent forms to solve problems.
M.A18.53	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. Instructional Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.

Cluster	Perform arithmetic operations on polynomials.
M.A18.54	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Instructional Note: Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.

Cluster	Create equations that describe numbers or relationships.
M.A18.55	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Extend work on linear and exponential equations in the unit on Relationships between Quantities to include quadratic equations.
M.A18.56	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Extend work on linear and exponential equations in the unit on Relationships between Quantities to include quadratic equations.
M.A18.57	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Extend work on linear and exponential equations in the unit on Relationships between Quantities to include quadratic equations. Extend M.A18.57 to formulas involving squared variables.

Cluster	Solve equations and inequalities in one variable.
M.A18.58	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. Instructional Note: Students should learn of the existence of the complex number system, but will not solve quadratics with complex solutions until Algebra II.

Cluster	Solve systems of equations.
M.A18.59	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. (e.g., Find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.) Instructional Note: Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between $x^2 + y^2 = 1$ and $y = (x+1)/2$ leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple $3^2 + 4^2 = 5^2$.

Quadratic Functions and Modeling

Cluster	Use properties of rational and irrational numbers.
M.A18.60	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. Instructional Note: Connect to physical situations (e.g., finding the perimeter of a square of area 2).

Cluster	Understand and apply the Pythagorean theorem.
M.A18.61	Explain a proof of the Pythagorean Theorem and its converse
M.A18.62	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. Instructional Note: Discuss applications of the Pythagorean theorem and its connections to radicals, rational exponents, and irrational numbers.
M.A18.63	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. Instructional Note: Discuss applications of the Pythagorean theorem and its connections to radicals, rational exponents, and irrational numbers.

Cluster	Interpret functions that arise in applications in terms of a context.
M.A18.64	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studies in the unit on Linear and Exponential Functions.
M.A18.65	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studies in the unit on Linear and Exponential Functions.
M.A18.66	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studies in the unit on Linear and Exponential Functions.

Cluster	Analyze functions using different representations.
M.A18.67	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Instructional Note: Compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic function can be factored.
M.A18.68	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay. Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic function can be factored. This unit, and in particular in M.A18.68b, extends the work begun in Unit 2 on exponential functions with integral exponents.
M.A18.69	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic function can be factored.

Cluster	Build a function that models a relationship between two quantities.
M.A18.70	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Instructional Note: Focus on situations that exhibit a quadratic relationship.

Cluster	Build new functions from existing functions.
M.A18.71	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on quadratic functions, and consider including absolute value functions.

M.A18.72	Find inverse functions. a. Solve an equation of the form $f(x) = c$ for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$. Instructional Note: Focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2$, $x > 0$.

Cluster	**Construct and compare linear, quadratic and exponential models and solve problems.**
M.A18.73	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Compare linear and exponential growth to growth of quadratic growth.

West Virginia College- and Career-Readiness Standards for Mathematics

High School Algebra I

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will focus on five critical units that deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Relationships between Quantities and Reasoning with Equations	**Linear and Exponential Relationships**
• Solve problems with a wide range of units and solve problems by thinking about units. (e.g., The Trans Alaska Pipeline System is 800 miles long and cost $8 billion to build. Divide one of these numbers by the other. What is the meaning of the answer? Greenland has a population of 56,700 and a land area of 2,175,600 square kilometers. By what factor is the population density of the United States, 80 persons per square mile, larger than the population density of Greenland?)	• Understand contextual relationships of variables and constants. (e.g., Annie is picking apples with her sister. The number of apples in her basket is described by $n = 22t + 12$, where t is the number of minutes Annie spends picking apples. What do the numbers 22 and 12 tell you about Annie's apple picking?)
Descriptive Statistics	**Expressions and Equations**
• Use linear regression techniques to describe the relationship between quantities and assess the fit of the model. (e.g., Use the high school and university grades for 250 students to create a model that can be used to predict a student's university GPA based on his high school GPA.)	• Interpret algebraic expressions and transform them purposefully to solve problems. (e.g., In solving a problem about a loan with interest rate r and principal P, seeing the expression $P(1+r)^n$ as a product of P with a factor not depending on P.)
Quadratic Functions and Modeling	
• Solve real-world and mathematical problems by writing and solving nonlinear equations, such as quadratic equations ($ax^2 + bx + c = 0$).	

Numbering of Standards

The following Mathematics Standards are numbered continuously. The following ranges relate to the clusters found within Mathematics:

Relationships between Quantities and Reasoning with Equations	
Reason quantitatively and use units to solve problems.	Standards 1-3
Interpret the structure of expressions.	Standard 4
Create equations that describe numbers or relationships.	Standards 5-8
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 9
Solve equations and inequalities in one variable.	Standard 10
Linear and Exponential Relationships	
Extend the properties of exponents to rational exponents.	Standards 11-12
Solve systems of equations.	Standards 13-14
Represent and solve equations and inequalities graphically.	Standards 15-17
Understand the concept of a function and use function notation.	Standards 18-20
Interpret functions that arise in applications in terms of a context.	Standards 21-23
Analyze functions using different representations.	Standards 24-25
Build a function that models a relationship between two quantities.	Standards 26-27
Build new functions from existing functions.	Standards 28
Construct and compare linear, quadratic, and exponential models and solve problems.	Standards 29-31
Interpret expressions for functions in terms of the situation they model.	Standard 32
Descriptive Statistics	
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 33-35
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standards 36-37
Interpret linear models.	Standards 38-40
Expressions and Equations	
Interpret the structure of equations.	Standards 41-42
Write expressions in equivalent forms to solve problems.	Standard 43
Perform arithmetic operations on polynomials.	Standard 44
Create equations that describe numbers or relationships.	Standards 45-47
Solve equations and inequalities in one variable.	Standard 48
Solve systems of equations.	Standard 49

Quadratic Functions and Modeling	
Use properties of rational and irrational numbers.	Standard 50
Interpret functions that arise in applications in terms of a context.	Standards 51-53
Analyze functions using different representations.	Standards 54-56
Build a function that models a relationship between two quantities.	Standard 57
Build new functions from existing functions.	Standards 58-59
Construct and compare linear, quadratic and exponential models and solve problems.	Standard 60

Relationships between Quantities and Reasoning with Equations

Cluster	Reason quantitatively and use units to solve problems.
M.A1HS.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.A1HS.2	Define appropriate quantities for the purpose of descriptive modeling. Instructional Note: Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.
M.A1HS.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Cluster	Interpret the structure of expressions.
M.A1HS.4	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. (e.g., Interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Limit to linear expressions and to exponential expressions with integer exponents.

Cluster	Create equations that describe numbers or relationships.
M.A1HS.5	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.
M.A1HS.6	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.
M.A1HS.7	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: Limit to linear equations and inequalities.

M.A1HS.8	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Limit to formulas with a linear focus.

Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.**
M.A1HS.9	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Instructional Note: Students should focus on and master linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Algebra II.

Cluster	**Solve equations and inequalities in one variable.**
M.A1HS.10	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Instructional Note: Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.

Linear and Exponential Relationships

Cluster	**Extend the properties of exponents to rational exponents.**
M.A1HS.11	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. (e.g., We define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.) Instructional Note: Address this standard before discussing exponential functions with continuous domains.
M.A1HS.12	Rewrite expressions involving radicals and rational exponents using the properties of exponents. Instructional Note: Address this standard before discussing exponential functions with continuous domains.

Cluster	**Solve systems of equations**.
M.A1HS.13	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
M.A1HS.14	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Instructional Note: Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to standards in Geometry which require students to prove the slope criteria for parallel lines.

Cluster	Represent and solve equations and inequalities graphically.
M.A1HS.15	Recognize that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Instructional Note: Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.
M.A1HS.16	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations). Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential and logarithmic functions. Instructional Note: Focus on cases where $f(x)$ and $g(x)$ are linear or exponential.
M.A1HS.17	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Cluster	Understand the concept of a function and use function notation.
M.A1HS.18	Recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation $y = f(x)$. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear functions and exponential functions having integral domains.
M.A1HS.19	Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear functions and exponential functions having integral domains.
M.A1HS.20	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. (e.g., The Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n)+ f(n-1)$ for $n \geq 1$. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear functions and exponential functions having integral domains. Draw connection to M.A1HS.27, which requires students to write arithmetic and geometric sequences. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions.

Cluster	Interpret functions that arise in applications in terms of a context.
M.A1HS.21	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on linear and exponential functions.
M.A1HS.22	Relate the domain of a function to its graph and where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Instructional Note: Focus on linear and exponential functions.
M.A1HS.23	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on linear functions and exponential functions whose domain is a subset of the integers. The Unit on Quadratic Functions and Modeling in this course and the Algebra II course address other types of functions.

Cluster	Analyze functions using different representations.
M.A1HS.24	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph exponential and logarithmic functions, showing intercepts and end behavior and trigonometric functions, showing period, midline and amplitude. Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100^{2n}$)
M.A1HS.25	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100^{2n}$)

Cluster	Build a function that models a relationship between two quantities.
M.A1HS.26	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.) Instructional Note: Limit to linear and exponential functions.
M.A1HS.27	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Instructional Note: Limit to linear and exponential functions. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.

Cluster	Build new functions from existing functions.
M.A1HS.28	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.

Cluster	Construct and compare linear, quadratic, and exponential models and solve problems.
M.A1HS.29	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
M.A1HS.30	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship or two input-output pairs (include reading these from a table). Instructional Note: In constructing linear functions, draw on and consolidate previous work in Grade 8 on finding equations for lines and linear functions.
M.A1HS.31	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Limit to comparisons between exponential and linear models.

Cluster	Interpret expressions for functions in terms of the situation they model.
M.A1HS.32	Interpret the parameters in a linear or exponential function in terms of a context. Instructional Note: Limit exponential functions to those of the form $f(x) = b^x + k$.

Descriptive Statistics

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.
M.A1HS.33	Represent data with plots on the real number line (dot plots, histograms, and box plots).
M.A1HS.34	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Instructional Note: In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.
M.A1HS.35	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Instructional Note: In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.

Cluster	Summarize, represent, and interpret data on two categorical and quantitative variables.
M.A1HS.36	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data.
M.A1HS.37	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. Instructional Note: Focus should be on situations for which linear models are appropriate. c. Fit a linear function for scatter plots that suggest a linear association. Instructional Note: Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.

Cluster	Interpret linear models.
M.A1HS.38	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Instructional Note: Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.A1HS.39	Compute (using technology) and interpret the correlation coefficient of a linear fit. Instructional Note: Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.A1HS.40	Distinguish between correlation and causation. Instructional Note: The important distinction between a statistical relationship and a cause-and-effect relationship is the focus.

Expressions and Equations

Cluster	Interpret the structure of equations.
M.A1HS.41	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Exponents are extended from the integer exponents found in the unit on Relationships between Quantities and Reasoning with Equations to rational exponents focusing on those that represent square or cube roots. Instructional Note: Focus on quadratic and exponential expressions.
M.A1HS.42	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$. Instructional Note: Focus on quadratic and exponential expressions.

Cluster	Write expressions in equivalent forms to solve problems.
M.A1HS.43	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. Instructional Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.

Cluster	Perform arithmetic operations on polynomials.
M.A1HS.44	Recognize that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Instructional Note: Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.

Cluster	Create equations that describe numbers or relationships.
M.A1HS.45	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Extend work on linear and exponential equations in the Relationships between Quantities and Reasoning with Equations unit to quadratic equations.
M.A1HS.46	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Extend work on linear and exponential equations in the Relationships between Quantities and Reasoning with Equations unit to quadratic equations.
M.A1HS.47	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R. Instructional Note: Extend work on linear and exponential equations in the Relationships between Quantities and Reasoning with Equations unit to quadratic equations. Extend this standard to formulas involving squared variables.

Cluster	Solve equations and inequalities in one variable.
M.A1HS.48	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b. Instructional Note: Students should learn of the existence of the complex number system, but will not solve quadratics with complex solutions until Algebra II.

Cluster	Solve systems of equations.
M.A1HS.49	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$. Instructional Note: Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between $x^2 + y^2 = 1$ and $y = (x+1)/2$ leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple $3^2 + 4^2 = 5^2$.

Quadratic Functions and Modeling

Cluster	Use properties of rational and irrational numbers.
M.A1HS.50	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. Instructional Note: Connect to physical situations (e.g., finding the perimeter of a square of area 2).

Cluster	Interpret functions that arise in applications in terms of a context.
M.A1HS.51	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in the Unit on Linear and Exponential Relationships.
M.A1HS.52	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in the Unit on Linear and Exponential Relationships.
M.A1HS.53	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in the Unit on Linear and Exponential Relationships.

Cluster	Analyze functions using different representations.
M.A1HS.54	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Instructional Note: Compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.

M.A1HS.55	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay. Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored. This standard extends the work begun in the Linear and Exponential Relationships unit on exponential functions with integer exponents.
M.A1HS.56	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. Instructional Note: Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.

Cluster	**Build a function that models a relationship between two quantities.**
M.A1HS.57	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Instructional Note: Focus on situations that exhibit a quadratic relationship.

Cluster	**Build new functions from existing functions.**
M.A1HS.58	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on quadratic functions, and consider including absolute value functions.
M.A1HS.59	Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$. Instructional Note: Focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2$, $x > 0$.

Cluster	**Construct and compare linear, quadratic and exponential models and solve problems.**
M.A1HS.60	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Compare linear and exponential growth to quadratic growth.

West Virginia College- and Career-Readiness Standards for Mathematics

High School Geometry

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Congruence, Proof, and Constructions	**Similarity, Proof, and Trigonometry**
• Prove theorems about triangles and other figures (e.g., that the sum of the measures of the angles in a triangle is 180^{o}). • Given a transformation, work backwards to discover the sequence that led to the transformation. • Given two quadrilaterals that are reflections of each other, find the line of that reflection.	• Apply knowledge of trigonometric ratios and the Pythagorean Theorem to determine distances in realistic situations. (e.g., Determine heights of inaccessible objects using various instruments, such as clinometers, hypsometers, transits, etc.)
Extending to Three Dimensions	**Connecting Algebra and Geometry Through Coordinates**
• Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	• Use a rectangular coordinate system and build on understanding of the Pythagorean Theorem to find distances. (e.g., Find the area and perimeter of a real-world shape using a coordinate grid and Google Earth.) • Analyze the triangles and quadrilaterals on the coordinate plane to determine their properties. (e.g., Determine whether a given quadrilateral is a rectangle).
Circles With and Without Coordinates	**Applications of Probability**
• Use coordinates and equations to describe geometric properties algebraically. (e.g., Write the equation for a circle in the plane with specified center and radius.)	• Work with probability and using ideas from probability in everyday situations. (e.g., Compare the chance that a person who smokes will develop lung cancer to the chance that a person who develops lung cancer smokes.)
Modeling with Geometry	
• Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision (e.g., estimate water and food needs in a disaster area, or use volume formulas and graphs to find an optimal size for an industrial package).	

Numbering of Standards

The following Mathematics Standards are numbered continuously. The following ranges relate to the clusters found within Mathematics:

Congruence, Proof, and Constructions	
Experiment with transformations in the plane.	Standards 1-5
Understand congruence in terms of rigid motions.	Standards 6-8
Prove geometric theorems.	Standards 9-11
Make geometric constructions.	Standards 12-13
Similarity, Proof, and Trigonometry	
Understand similarity in terms of similarity transformations.	Standards 14-16
Prove theorems involving similarity.	Standards 17-18
Define trigonometric ratios and solve problems involving right triangles.	Standards 19-21
Apply trigonometry to general triangles.	Standards 22-24
Extending to Three Dimensions	
Explain volume formulas and use them to solve problems.	Standards 25-26
Visualize the relation between two dimensional and three-dimensional objects.	Standard 27
Apply geometric concepts in modeling situations.	Standard 28
Connecting Algebra and Geometry Through Coordinates	
Use coordinates to prove simple geometric theorems algebraically.	Standards 29-32
Translate between the geometric description and the equation for a conic section.	Standard 33
Circles With and Without Coordinates	
Understand and apply theorems about circles.	Standards 34-37
Find arc lengths and areas of sectors of circles.	Standard 38
Translate between the geometric description and the equation for a conic section.	Standard 39
Use coordinates to prove simple geometric theorems algebraically.	Standard 40
Apply geometric concepts in modeling situations.	Standard 41
Applications of Probability	
Understand independence and conditional probability and use them to interpret data.	Standards 42-46
Use the rules of probability to compute probabilities of compound events in a uniform probability model.	Standards 47-50
Use probability to evaluate outcomes of decisions.	Standards 51-52
Modeling with Geometry	
Visualize relationships between two dimensional and three-dimensional objects and apply geometric concepts in modeling situations.	Standards 53-55

Congruence, Proof and Constructions

Cluster	Experiment with transformations in the plane.
M.GHS.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
M.GHS.2	Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.GHS.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.GHS.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.GHS.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, for example, graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle)

Cluster	Understand congruence in terms of rigid motions.
M.GHS.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.
M.GHS.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.

M.GHS.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.

Cluster	**Prove geometric theorems.**
M.GHS.9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.
M.GHS.10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of this standard may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for M.GHS.36.
M.GHS.11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

Cluster	**Make geometric constructions.**
M.GHS.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.
M.GHS.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.

Similarity, Proof, and Trigonometry

Cluster	Understand similarity in terms of similarity transformations.
M.GHS.14	Verify experimentally the properties of dilations given by a center and a scale factor. a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.
M.GHS.15	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
M.GHS.16	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Cluster	Prove theorems involving similarity.
M.GHS.17	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.
M.GHS.18	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Cluster	Define trigonometric ratios and solve problems involving right triangles.
M.GHS.19	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
M.GHS.20	Explain and use the relationship between the sine and cosine of complementary angles.
M.GHS.21	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Cluster	Apply trigonometry to general triangles.
M.GHS.22	Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
M.GHS.23	Prove the Laws of Sines and Cosines and use them to solve problems. Instructional Note: With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.
M.GHS.24	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles. Instructional Note: With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.

Extending to Three Dimensions

Cluster	Explain volume formulas and use them to solve problems.
M.GHS.25	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. Instructional Note: Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. Similarly, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.
M.GHS.26	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. Instructional Note: Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. Similarly, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.

Cluster	Visualize the relation between two dimensional and three-dimensional objects.
M.GHS.27	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Cluster	Apply geometric concepts in modeling situations.
M.GHS.28	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). Instructional Note: Focus on situations that require relating two- and three-dimensional objects, determining and using volume, and the trigonometry of general triangles.

Connecting Algebra and Geometry Through Coordinates

(This unit has a close connection with the unit, Circles With and Without Coordinates. Reasoning with triangles in this unit is limited to right triangles; e.g., derive the equation for a line through two points using similar right triangles. Relate work on parallel lines to work in High School Algebra I involving systems of equations having no solution or infinitely many solutions. M.GHS.32 provides practice with the distance formula and its connection with the Pythagorean Theorem.)

Cluster	Use coordinates to prove simple geometric theorems algebraically.
M.GHS.29	Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).
M.GHS.30	Prove the slope criteria for parallel and perpendicular lines and uses them to solve geometric problems. (e.g., Find the equation of a line parallel or perpendicular to a given line that passes through a given point.) Instructional Note: Relate work on parallel lines to work in High School Algebra I involving systems of equations having no solution or infinitely many solutions.
M.GHS.31	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.

M.GHS.32	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. This standard provides practice with the distance formula and its connection with the Pythagorean theorem.

Cluster	**Translate between the geometric description and the equation for a conic section.**
M.GHS.33	Derive the equation of a parabola given a focus and directrix. Instructional Note: The directrix should be parallel to a coordinate axis.

Circles With and Without Coordinates

Cluster	**Understand and apply theorems about circles.**
M.GHS.34	Prove that all circles are similar.
M.GHS.35	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
M.GHS.36	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
M.GHS.37	Construct a tangent line from a point outside a given circle to the circle.

Cluster	**Find arc lengths and areas of sectors of circles.**
M.GHS.38	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Instructional Note: Emphasize the similarity of all circles. Reason that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.

Cluster	**Translate between the geometric description and the equation for a conic section.**
M.GHS.39	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Cluster	**Use coordinates to prove simple geometric theorems algebraically.**
M.GHS.40	Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).) Instructional Note: Include simple proofs involving circles.

Cluster	**Apply geometric concepts in modeling situations.**
M.GHS.41	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). Instructional Note: Focus on situations in which the analysis of circles is required.

Applications of Probability

Cluster	Understand independence and conditional probability and use them to interpret data.
M.GHS.42	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
M.GHS.43	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
M.GHS.44	Recognize the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. Instructional Note: Build on work with two-way tables from Algebra I to develop understanding of conditional probability and independence.
M.GHS.45	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. Instructional Note: Build on work with two-way tables from Algebra I to develop understanding of conditional probability and independence.
M.GHS.46	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Cluster	Use the rules of probability to compute probabilities of compound events in a uniform probability model.
M.GHS.47	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
M.GHS.48	Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
M.GHS.49	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
M.GHS.50	Use permutations and combinations to compute probabilities of compound events and solve problems.

Cluster	Use probability to evaluate outcomes of decisions. Instructional Note: This unit sets the stage for work in Algebra II, where the ideas of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e. incomplete information) requires an understanding of probability concepts.
M.GHS.51	Use probabilities to make fair decisions (e.g., drawing by lots and/or using a random number generator).
M.GHS.52	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game).

Modeling with Geometry

Cluster	Visualize relationships between two dimensional and three-dimensional objects and apply geometric concepts in modeling situations.
M.GHS.53	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
M.GHS.54	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
M.GHS.55	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).

West Virginia College- and Career-Readiness Standards for Mathematics

High School Algebra II

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will build on their work with linear, quadratic, and exponential functions and extend their repertoire of functions to include polynomial, rational, and radical functions. (In this course rational functions are limited to those whose numerators are of degree at most 1 and denominators of degree at most 2; radical functions are limited to square roots or cube roots of at most quadratic polynomials.) Students will work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Polynomial, Rational, and Radical Relationships	**Trigonometric Functions**
• Derive the formula for the sum of a geometric series, and use the formula to solve problems. (e.g., Calculate mortgage payments.)	• Apply knowledge of trigonometric functions to determine distances in realistic situations. (e.g., Determine heights of inaccessible objects.)
Modeling with Functions	**Inferences and Conclusions from Data**
• Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision. (e.g., Estimate water and food needs in a disaster area, or use volume formulas and graphs to find an optimal size for an industrial package.) • Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Numbering of Standards

The following Mathematics Standards are numbered continuously. The following ranges relate to the clusters found within Mathematics:

Polynomial, Rational, and Radical Relationships	
Perform arithmetic operations with complex numbers.	Standards 1-2
Use complex numbers in polynomial identities and equations.	Standards 3-5
Interpret the structure of expressions.	Standards 6-7
Write expressions in equivalent forms to solve problems.	Standard 8
Perform arithmetic operations on polynomials.	Standard 9
Understand the relationship between zeros and factors of polynomials.	Standards 10-11
Use polynomial identities to solve problems.	Standards 12-13
Rewrite rational expressions.	Standards 14-15
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 16
Represent and solve equations and inequalities graphically.	Standard 17
Analyze functions using different representations.	Standard 18
Trigonometric Functions	
Extend the domain of trigonometric functions using the unit circle.	Standards 19-20
Model periodic phenomena with trigonometric functions.	Standard 21
Prove and apply trigonometric identities.	Standard 22
Modeling with Functions	
Create equations that describe numbers or relationships.	Standards 23-26
Interpret functions that arise in applications in terms of a context.	Standards 27-29
Analyze functions using different representations.	Standards 30-32
Build a function that models a relationship between two quantities.	Standard 33
Build new functions from existing functions.	Standards 34-35
Construct and compare linear, quadratic, and exponential models and solve problems.	Standard 36
Inferences and Conclusions from Data	
Summarize, represent, and interpret data on a single count or measurement variable.	Standard 37
Understand and evaluate random processes underlying statistical experiments.	Standards 38-39
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	Standards 40-43
Use probability to evaluate outcomes of decisions.	Standards 44-45

Polynomial, Rational, and Radical Relationships

Cluster	Perform arithmetic operations with complex numbers.
M.A2HS.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.
M.A2HS.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.

Cluster	Use complex numbers in polynomial identities and equations.
M.A2HS.3	Solve quadratic equations with real coefficients that have complex solutions. Instructional Note: Limit to polynomials with real coefficients.
M.A2HS.4	Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$. Instructional Note: Limit to polynomials with real coefficients.
M.A2HS.5	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Instructional Note: Limit to polynomials with real coefficients.

Cluster	Interpret the structure of expressions.
M.A2HS.6	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Extend to polynomial and rational expressions.
M.A2HS.7	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. Instructional Note: Extend to polynomial and rational expressions.

Cluster	Write expressions in equivalent forms to solve problems.
M.A2HS.8	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. Instructional Note: Consider extending this standard to infinite geometric series in curricular implementations of this course description.

Cluster	Perform arithmetic operations on polynomials.
M.A2HS.9	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Instructional Note: Extend beyond the quadratic polynomials found in Algebra I.

Cluster	Understand the relationship between zeros and factors of polynomials.
M.A2HS.10	Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).
M.A2HS.11	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.

Cluster	Use polynomial identities to solve problems.
M.A2HS.12	Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. Instructional Note: This cluster has many possibilities for optional enrichment, such as relating the example in M.A2HS.10 to the solution of the system $u^2 + v^2 = 1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x + y)^{n+1} = (x + y)(x + y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction.
M.A2HS.13	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. Instructional Note: This cluster has many possibilities for optional enrichment, such as relating the example in M.A2HS.10 to the solution of the system $u^2 + v^2 = 1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x + y)^{n+1} = (x + y)(x + y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction.

Cluster	Rewrite rational expressions.
M.A2HS.14	Rewrite simple rational expressions in different forms; write $a(x)/b(x)$ in the form $q(x) + r(x)/b(x)$, where $a(x)$, $b(x)$, $q(x)$, and $r(x)$ are polynomials with the degree of $r(x)$ less than the degree of $b(x)$, using inspection, long division, or, for the more complicated examples, a computer algebra system. Instructional Note: The limitations on rational functions apply to the rational expressions.
M.A2HS.15	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. Instructional Note: This standard requires the general division algorithm for polynomials.

Cluster	Understand solving equations as a process of reasoning and explain the reasoning.
M.A2HS.16	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. Instructional Note: Extend to simple rational and radical equations.

Cluster	Represent and solve equations and inequalities graphically.
M.A2HS.17	Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Instructional Note: Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Instructional Note: Include combinations of linear, polynomial, rational, radical, absolute value, and exponential functions.

Cluster	Analyze functions using different representations.
M.A2HS.18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. Instructional Note: Relate this standard to the relationship between zeros of quadratic functions and their factored forms.

Trigonometric Functions

Cluster	Extend the domain of trigonometric functions using the unit circle.
M.A2HS.19	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
M.A2HS.20	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.

Cluster	Model periodic phenomena with trigonometric functions.
M.A2HS.21	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.

Cluster	Prove and apply trigonometric identities.
M.A2HS.22	Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle. Instructional Note: Limit θ to angles between 0 and 90 degrees. Connect with the Pythagorean theorem and the distance formula. Extension of trigonometric functions to other angles through the unit circle is included in Mathematics III.

Modeling with Functions

Cluster	Create equations that describe numbers or relationships.
M.A2HS.23	Create equations and inequalities in one variable and use them to solve problems. Instructional Note: Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
M.A2HS.24	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: While functions will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. (e.g., Finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line).
M.A2HS.25	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: While functions will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line.
M.A2HS.26	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) While functions will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line. This example applies to earlier instances of this standard, not to the current course.

Cluster	Interpret functions that arise in applications in terms of a context.
M.A2HS.27	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Emphasize the selection of a model function based on behavior of data and context.
M.A2HS.28	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Note: Emphasize the selection of a model function based on behavior of data and context.
M.A2HS.29	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Note: Emphasize the selection of a model function based on behavior of data and context.

Cluster	Analyze functions using different representations.
M.A2HS.30	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. b. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.
M.A2HS.31	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.
M.A2HS.32	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.

Cluster	Build a function that models a relationship between two quantities.
M.A2HS.33	Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.) Instructional Note: Develop models for more complex or sophisticated situations than in previous courses.

Cluster	Build new functions from existing functions.
M.A2HS.34	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Use transformations of functions to find models as students consider increasingly more complex situations. Observe the effect of multiple transformations on a single graph and the common effect of each transformation across function types.
M.A2HS.35	Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. (e.g., $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.) Instructional Note: Use transformations of functions to find models as students consider increasingly more complex situations. Extend this standard to simple rational, simple radical, and simple exponential functions; connect this standard to M.A2HS.34.

Cluster	Construct and compare linear, quadratic, and exponential models and solve problems.
M.A2HS.36	For exponential models, express as a logarithm the solution to a $b^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. Instructional Note: Consider extending this unit to include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that log xy = log x +log y.

Inferences and Conclusions from Data

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.
M.A2HS.37	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. Instructional Note: While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution.

Cluster	Understand and evaluate random processes underlying statistical experiments.
M.A2HS.38	Understand statistics as a process for making inferences about population parameters based on a random sample from that population. Instructional Note: Include comparing theoretical and empirical results to evaluate the effectiveness of a treatment.
M.A2HS.39	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. (e.g., A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?)

Cluster	Make inferences and justify conclusions from sample surveys, experiments, and observational studies.
M.A2HS.40	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.
M.A2HS.41	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment. Focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.
M.A2HS.42	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment. Focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.
M.A2HS.43	Evaluate reports based on data. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.

Cluster	Use probability to evaluate outcomes of decisions.
M.A2HS.44	Use probabilities to make fair decisions (e.g., drawing by lots or using a random number generator). Instructional Note: Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results.
M.A2HS.45	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game). Instructional Note: Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results.

West Virginia College- and Career-Readiness Standards for Mathematics

Fourth Course Options

Fourth course options are available to students in either pathway:

- Advanced Mathematical Modeling
- Calculus
- High School Mathematics IV - Trigonometry/Pre-calculus
- STEM Readiness
- Transition Mathematics for Seniors
- AP® Calculus
- AP® Computer Science
- AP® Statistics

Additional course options include dual credit mathematics courses and advanced mathematics courses offered through WV Virtual School. School teams, including counselors, teachers and administrators, should confer with the student and his/her parents to decide what fourth year mathematics course best meets the needs of the student.

West Virginia College- and Career-Readiness Standards for Mathematics

Advanced Mathematical Modeling

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Primary focal points of Advanced Mathematical Modeling include the analysis of information using statistical methods and probability, modeling change and mathematical relationships, mathematical decision making in finance, and spatial and geometric modeling for decision-making. Students will learn to become critical consumers of the quantitative data that surround them every day, knowledgeable decision makers who use logical reasoning and mathematical thinkers who can use their quantitative skills to solve problems related to a wide range of situations. As students solve problems in various applied situations, they will develop critical skills for success in college and careers, including investigation, research, collaboration and both written and oral communication of their work. As students work with these topics, they will rely on mathematical processes, including problem-solving techniques, appropriate mathematical language and communication skills, connections within and outside mathematics and reasoning. Students will use multiple representations, technology, applications and modeling and numerical fluency in problem-solving contexts. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Developing College and Career Skills	**Finance**
• Develop and apply skills used in college and careers, including reasoning, planning and communication, to make decisions and solve problems in applied situations.	• Create and analyze mathematical models to make decisions related to earning, investing, spending and borrowing money.
Probability	**Statistics**
• Use basic rules of counting and probability to analyze and evaluate risk and return in the context of everyday situations.	• Make decisions based on understanding, analysis and critique of reported statistical information and summaries.
Modeling	**Networks**
• Analyze numerical data in everyday situations using a variety of quantitative measures and numerical processes.	• Use a variety of network models represented graphically to organize data in quantitative situations, make informed decisions, and solve problems.
Social Decision Making	**Geometry**
• Analyze the mathematics behind various methods of ranking and selection and consider the advantages/disadvantages of each method.	• Solve geometric problems involving inaccessible distances. • Use vectors to solve applied problems.

Numbering of Standards

The following Mathematics Standards are numbered continuously. The following ranges relate to the clusters found within Mathematics:

Developing College and Career Skills	
Math as a language.	Standards 1-2
Tools for problem solving.	Standard 3
Finance	
Understanding financial models.	Standards 4-6
Personal use of finance.	Standards 7-8
Probability	
Analyzing information using probability and counting.	Standards 9-10
Managing uncertainty.	Standards 11-12
Statistics	
Critiquing statistics.	Standards 13-16
Conducting statistical analysis.	Standards 17-21
Communicating statistical information.	Standards 22-23
Modeling	
Managing numerical data.	Standards 24-25
Modeling data and change with functions.	Standards 26-30
Networks	
Networking for decision making.	Standards 31-32
Social Decision Making	
Making decisions using ranking and voting.	Standards 33-34
Geometry	
Concrete geometric representation (physical modeling).	Standards 35-36
Abstract geometric representation (matrix modeling).	Standards 37-38

Developing College and Career Skills

Cluster	Math as a language
M.AMM.1	Demonstrate reasoning skills in developing, explaining and justifying sound mathematical arguments and analyze the soundness of mathematical arguments of others.
M.AMM.2	Communicate with and about mathematics orally and in writing as part of independent and collaborative work, including making accurate and clear presentations of solutions to problems.

Cluster	Tools for problem solving
M.AMM.3	Gather data, conduct investigations and apply mathematical concepts and models to solve problems in mathematics and other disciplines.

Finance

Cluster	Understanding financial models
M.AMM.4	Determine, represent and analyze mathematical models for loan amortization and the effects of different payments and/or finance terms (e.g., Auto, Mortgage, and/or Credit Card).
M.AMM.5	Determine, represent and analyze mathematical models for investments involving simple and compound interest with and without additional deposits. (e.g., Savings accounts, bonds, and/or certificates of deposit.)
M.AMM.6	Determine, represent, and analyze mathematical models for Inflation and the Consumer Price Index using concepts of rate of change and percentage growth.

Cluster	Personal use of finance
M.AMM.7	Research and analyze personal budgets based on given parameters (e.g., Fixed and discretionary expenses, insurance, gross vs. net pay, types of income, wage, salary, commission), career choice, geographic region, retirement and/or investment planning, etc.).
M.AMM.8	Research and analyze taxes including payroll, sales, personal property, real estate and income tax returns.

Probability

Cluster	Analyzing information using probability and counting
M.AMM.9	Use the Fundamental Counting Principle, Permutations and Combinations to determine all possible outcomes for an event; determine probability and odds of a simple event; explain the significance of the Law of Large Numbers.
M.AMM.10	Determine and interpret conditional probabilities and probabilities of compound events by constructing and analyzing representations, including tree diagrams, Venn diagrams, two-way frequency tables and area models, to make decisions in problem situations.

Cluster	Managing uncertainty
M.AMM.11	Use probabilities to make and justify decisions about risks in everyday life.
M.AMM.12	Calculate expected value to analyze mathematical fairness, payoff and risk.

Statistics

Cluster	Critiquing statistics
M.AMM.13	Identify limitations or lack of information in studies reporting statistical information, especially when studies are reported in condensed form.
M.AMM.14	Interpret and compare the results of polls, given a margin of error.
M.AMM.15	Identify uses and misuses of statistical analyses in studies reporting statistics or using statistics to justify particular conclusions, including assertions of cause and effect versus correlation.
M.AMM.16	Describe strengths and weaknesses of sampling techniques, data and graphical displays and interpretations of summary statistics and other results appearing in a study, including reports published in the media.

Cluster	Conducting statistical analysis
M.AMM.17	Identify the population of interest, select an appropriate sampling technique and collect data.
M.AMM.18	Identify the variables to be used in a study.
M.AMM.19	Determine possible sources of statistical bias in a study and how such bias may affect the ability to generalize the results.
M.AMM.20	Create data displays for given data sets to investigate, compare, and estimate center, shape, spread and unusual features.
M.AMM.21	Determine possible sources of variability of data, both those that can be controlled and those that cannot be controlled.

Cluster	Communicating statistical information
M.AMM.22	Report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays and interpreting results in terms of the question studied.
M.AMM.23	Communicate statistical results in both oral and written formats using appropriate statistical and nontechnical language.

Modeling

Cluster	Managing numerical data
M.AMM.24	Solve problems involving large quantities that are not easily measured.
M.AMM.25	Use arrays to efficiently manage large collections of data and add, subtract, and multiply matrices to solve applied problems.

Cluster	Modeling data and change with functions
M.AMM.26	Determine or analyze an appropriate model for problem situations - including linear, quadratic, power, exponential, logarithmic and logistic functions (e.g., stopping distance, period of a pendulum, population growth, Richter Scale, and/or Fujita Tornado Scale).
M.AMM.27	Determine or analyze an appropriate cyclical model for problem situations that can be modeled with trigonometric functions (e.g., predator-prey models, tide heights, diurnal cycle, and/or music).
M.AMM.28	Determine or analyze an appropriate piecewise model for problem situations (e.g., postal rates, phase change graphs, sales tax, and/or utility usage rates).
M.AMM.29	Solve problems using recursion or iteration (e.g., fractals, compound interest, population growth or decline, and/or radioactive decay).
M.AMM.30	Collect numerical bivariate data; use the data to create a scatter plot; determine whether or not a relationship exists; if so, select a function to model the data, justify the selection and use the model to make predictions.

Networks

Cluster	Networking for decision making
M.AMM.31	Solve problems involving scheduling or routing situations that can be represented by a vertex-edge graph; find critical paths, Euler paths, Hamiltonian paths, and minimal spanning trees (e.g., Konigsberg bridge problem, mail vs. Fed Ex delivery routes, kolam drawings of India, traveling salesman problem, and/or map coloring).
M.AMM.32	Construct, analyze, and interpret flow charts in order to develop and describe problem solving procedures.

Social Decision Making

Cluster	Making decisions using ranking and voting
M.AMM.33	Apply and analyze various ranking algorithms to determine an appropriate method for a given situation (e.g., fair division, apportionment, and/or search engine results).
M.AMM.34	Analyze various voting and selection processes to determine an appropriate method for a given situation (e.g., preferential vs. non-preferential methods, and/or weighted voting).

Geometry

Cluster	Concrete geometric representation (physical modeling)
M.AMM.35	Create and use two- and three-dimensional representations of authentic situations using paper techniques or dynamic geometric environments for computer-aided design and other applications.
M.AMM.36	Solve geometric problems involving inaccessible distances.

Cluster	Abstract geometric representation (matrix modeling)
M.AMM.37	Use vectors to represent and solve applied problems.
M.AMM.38	Use matrices to represent geometric transformations and solve applied problems.

West Virginia College- and Career-Readiness Standards for Mathematics

Calculus

All West Virginia teachers are responsible for classroom instruction that integrates content standards and objectives and mathematical habits of mind. Students will deepen and extend their understanding of functions, continuity, limits, differentiation, applications of derivatives, integrals, and applications of integration. Students will apply the Rule of Four (Numerical, Analytical, Graphical and Verbal) throughout the course and use available technology to enhance learning. Student will use graphing utilities to investigate concepts and to evaluate derivatives and integrals. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Algebra

- A utility company burns coal to generate electricity. The cost C in dollars of removing p% of the air pollutants emissions is $C = \frac{90000p}{100 - p}$, $0 \le p < 100$. Find the cost of removing (a) 10%, (b) 25%, and (c) 75% of the pollutants. Find the limit of C as $p \to 100^-$
- A management company is planning to build a new apartment complex. Knowing the maximum number of apartments the lot can hold and given a function for the maintenance costs, determine the number of apartments that will minimize the maintenance costs.
- The velocity v of the flow of blood at a distance r from the central axis of an artery of radius R is $v = k(R^2 - r^2)$ where k is the constant of proportionality. Find the average rate of flow of blood along a radius of the artery. (Use 0 and R as the limits of integration.)

Data Analysis and Probability

- The average data entry speeds S (words per minute) of a business student after t weeks of lessons are recorded in the following table.

t	5	10	15	20	25	30
S	28	56	79	90	93	94

  A model for the data is $S = \frac{100t^2}{65 + t}$, $t > 0$. Do you think that there is a limiting speed? If so, what is the limiting speed? If not, why?
- Identify a real life situation that involves quantities that change over time and develop a method to collect and analyze related data. Develop a continuous function to model the data and generalize the results to make a conclusion.
- A sheet of typing paper is ruled with parallel lines that are 2 inches apart. A two-inch needle is tossed randomly onto the sheet of paper. The probability that the needle will touch a line is $P = \frac{2}{\pi}\int_0^{\pi/2} \sin\theta \, d\theta$ where θ is the acute angle between the needle and any one of the parallel lines. Find the probability.

Geometry

- The radius of a right circular cylindrical balloon is given by $\sqrt{t + 2}$ and its height is $\frac{1}{2}\sqrt{t}$, where t is time in seconds and the dimensions are in inches. Find the rate of change of the volume with respect to time.
- Given 50 meters of framing material, construct a window that will let in the most light if the middle of the window is a rectangle and the top and bottom of the window are semi-circles.
- The graph of f consists of the three line segments joining the point (0,0), (2–2), (6,2), and (8,3). The function F is defined as follows $F(x) = \int_0^x f(t)\,dt$. Find the total enclosed areas generated by f and the x-axis. Determine the points of inflection of F on the interval (0,8).

Numbering of Standards

The following Mathematics Standards are numbered continuously. The following ranges relate to the clusters found within Mathematics:

Algebra	
Understand the key concepts, connections and applications of functions, limits, continuity, derivatives, and integrals represented in multiple ways.	Standards 1-19
Geometry	
Apply the key concepts, connections and applications of limits, continuity, derivatives, and integration for a wide variety of regions.	Standards 20-22
Data Analysis and Probability	
Apply the key concepts and applications of limits, continuity, derivatives, and integration to analyze functions that represent a collection of data.	Standard 23

Algebra

Cluster	Understand the key concepts, connections and applications of functions, limits, continuity, derivatives, and integrals represented in multiple ways.
M.C.1	Use abstract notation to apply properties of algebraic, trigonometric, exponential, logarithmic and composite functions, as well as their inverses, represented graphically, numerically, analytically, and verbally; and demonstrate an understanding of the connections among these representations.
M.C.2	Demonstrate a conceptual understanding of the definition of a limit via the analysis of continuous and discontinuous functions represented using multiple representations (e.g. graphs and tables).
M.C.3	Use the properties of limits including addition, product, quotient, composition, and squeeze/sandwich theorem to calculate the various forms of limits: one-sided limits, limits at infinity, infinite limits, limits that do not exist, and special limits such as $\lim_{x\to 0} \frac{\sin x}{x} = 1$ and $\lim_{x\to 0} \frac{1-\cos x}{x} = 0$
M.C.4	Apply the definition of continuity to determine where a function is continuous or discontinuous including continuity at a point, continuity over an interval, application of the Intermediate Value Theorem, and graphical interpretation of continuity and discontinuity.
M.C.5	Investigate and apply the definition of the derivative graphically, numerically, and analytically at a point, conceptually interpreting the derivative as an instantaneous rate of change and the slope of the tangent line.
M.C.6	Discriminate between the average rate of change and the instantaneous rate of change using real-world problems.
M.C.7	Recognize when the Extreme Value Theorem indicates that function extrema exist.

M.C.8	Quickly recall and apply rules of differentiation including the constant multiple rule, sum rule, the difference rule, the product rule, the quotient rule, the power rule, and the chain rule as applied to algebraic, trigonometric, exponential, logarithmic, and inverse trigonometric functions using techniques of both explicit and implicit differentiation.
M.C.9	Apply Rolle's Theorem and the Mean Value Theorem to real-world problems.
M.C.10	Construct and use mathematical models to solve optimization, related-rates, velocity, and acceleration problems.
M.C.11	Determine antiderivatives that follow from derivatives of basic functions and apply substitution of variables.
M.C.12	Evaluate definite integrals using basic integration properties such as addition, subtraction, constant multipliers, the power rule, substitution, and change of limits.
M.C.13	Characterize the definite integral as the total change of a function over an interval and use this to solve real-world problems.
M.C.14	Apply the Fundamental Theorem of Calculus to evaluate definite integrals and to formulate a cumulative area function and interpret the function as it relates to the integrand.
M.C.15	Use limits to deduce asymptotic behavior of the graph of a function.
M.C.16	Compare and contrast the limit definition (not delta epsilon) of continuity and the graphical interpretation of the continuity of a function at a point; recognize different types of discontinuities.
M.C.17	Develop tangent lines as best linear approximations to functions near specific points; explain this conceptually; and construct these tangent lines; and apply this concept to Newton's Method.
M.C.18	Investigate and explain the relationships among the graphs of a function, its derivative and its second derivative; construct the graph of a function using the first and second derivatives including extrema, points of inflection, and asymptotic behavior.
M.C.19	Approximate areas under a curve using Riemann sums by applying and comparing left, right, and midpoint methods for a finite number of subintervals.

Geometry

Cluster	**Apply the key concepts, connections and applications of limits, continuity, derivatives, and integration for a wide variety of regions.**
M.C.20	Justify why differentiability implies continuity, and classify functional cases when continuity does not imply differentiability.
M.C.21	Calculate a definite integral using Riemann sums by evaluating an infinite limit of a sum using summation notation and rules for summation.
M.C.22	Use integration to solve problems that involve linear displacement, total distance, position, velocity, acceleration and area between curves by looking at both functions of x and functions of y; utilize units to interpret the physical nature of the calculus process.

Data Analysis and Probability

Cluster	Apply the key concepts and applications of limits, continuity, derivatives, and integration to analyze functions that represent a collection of data.
M.C.23	Identify a real life situation that involves quantities that change over time; pose a question; make a hypothesis as to the answer; develop, justify, and implement a method to collect, organize, and analyze related data; extend the nature of collected, discrete data to that of a continuous function that describes the known data set; generalize the results to make a conclusion; compare the hypothesis and the conclusion; present the project numerically, analytically, graphically and verbally using the predictive and analytic tools of calculus.

West Virginia College- and Career-Readiness Standards for Mathematics

High School Mathematics IV – Trigonometry/Pre-calculus

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will generalize and abstract learning accumulated through previous courses as the final springboard to calculus. Students will take an extensive look at the relationships among complex numbers, vectors, and matrices. They will build on their understanding of functions, analyze rational functions using an intuitive approach to limits and synthesize functions by considering compositions and inverses. Students will expand their work with trigonometric functions and their inverses and complete the study of the conic sections begun in previous courses. They will enhance their understanding of probability by considering probability distributions and have previous experiences with series augmented. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Building Relationships among Complex Numbers, Vectors, and Matrices	**Analysis and Synthesis of Functions**
• Represent abstract situations involving vectors symbolically.	• Write a function that describes a relationship between two quantities. (e.g., if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.)
Trigonometric and Inverse Trigonometric Functions of Real Numbers	**Derivations in Analytic Geometry**
• Make sense of the symmetry, periodicity, and special values of trigonometric functions using the unit circle. • Prove trigonometric identities and apply them problem solving situations.	• Make sense of the derivations of the equations of an ellipse and a hyperbola.
Modeling with Probability	**Series and Informal Limits**
• Develop a probability distribution. (e.g., Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.)	• Apply mathematical induction to prove summation formulas.

Numbering of Standards

The following Mathematics Standards are numbered continuously. The following ranges relate to the clusters found within Mathematics:

Building Relationships among Complex Numbers, Vectors, and Matrices	
Perform arithmetic operations with complex numbers.	Standard 1
Represent complex numbers and their operations on the complex plane.	Standards 2-4
Represent and model with vector quantities.	Standards 5-7
Perform operations on vectors.	Standards 8-9
Perform operations on matrices and use matrices in applications.	Standards 10-16
Solve systems of equations.	Standards 17-18
Analysis and Synthesis of Functions	
Analyze functions using different representations.	Standard 19
Build a function that models a relationship between two quantities.	Standard 20
Build new functions from existing functions.	Standards 21-22
Trigonometric and Inverse Trigonometric Functions of Real Numbers	
Extend the domain of trigonometric functions using the unit circle.	Standards 23-24
Model periodic phenomena with trigonometric functions.	Standards 25-27
Prove and apply trigonometric identities.	Standard 28
Apply transformations of function to trigonometric functions.	Standard 29
Derivations in Analytic Geometry	
Translate between the geometric description and the equation for a conic section.	Standard 30
Explain volume formulas and use them to solve problems.	Standard 31
Modeling with Probability	
Calculate expected values and use them to solve problems.	Standards 32-35
Use probability to evaluate outcomes of decisions.	Standard 36
Series and Informal Limits	
Use sigma notations to evaluate finite sums.	Standards 37-38
Extend geometric series to infinite geometric series.	Standards 39-40

Building Relationships among Complex Numbers, Vectors, and Matrices

Cluster	Perform arithmetic operations with complex numbers
M.4HSTP.1	Find the conjugate of a complex number; use conjugates to find moduli (magnitude) and quotients of complex numbers. Instructional Note: In Math II students extended the number system to include complex numbers and performed the operations of addition, subtraction, and multiplication.

Cluster	Represent complex numbers and their operations on the complex plane.
M.4HSTP.2	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
M.4HSTP.3	Represent addition, subtraction, multiplication and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. (e.g., $(-1 + \sqrt{3}\, i)^3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.
M.4HSTP.4	Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints.

Cluster	Represent and model with vector quantities.
M.4HSTP.5	Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments and use appropriate symbols for vectors and their magnitudes (e.g., v, |v|, ||v||, v). Instructional Note: This is the student's first experience with vectors. The vectors must be represented both geometrically and in component form with emphasis on vocabulary and symbols.
M.4HSTP.6	Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal point.
M.4HSTP.7	Solve problems involving velocity and other quantities that can be represented by vectors.

Cluster	Perform operations on vectors.
M.4HSTP.8	Add and subtract vectors. a. Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes. b. Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum. c. Understand vector subtraction v – w as v + (–w), where –w is the additive inverse of w, with the same magnitude as w and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order and perform vector subtraction component-wise.
M.4HSTP.9	Multiply a vector by a scalar. a. Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as c(vx, vy) = (cvx, cvy). b. Compute the magnitude of a scalar multiple c$\mathbf{v}$ using $\|c\mathbf{v}\| = \|c\| \cdot \|\mathbf{v}\|$. Compute the direction of c$\mathbf{v}$ knowing that when |c|v ≠ 0, the direction of c$\mathbf{v}$ is either along $\mathbf{v}$ (for c > 0) or against $\mathbf{v}$ (for c < 0).

Cluster	Perform operations on matrices and use matrices in applications.
M.4HSTP.10	Use matrices to represent and manipulate data (e.g., to represent payoffs or incidence relationships in a network).
M.4HSTP.11	Multiply matrices by scalars to produce new matrices (e.g., as when all of the payoffs in a game are doubled.
M.4HSTP.12	Add, subtract and multiply matrices of appropriate dimensions.
M.4HSTP.13	Understand that, unlike multiplication of numbers, matrix multiplication for square matrices is not a commutative operation, but still satisfies the associative and distributive properties. Instructional Note: This is an opportunity to view the algebraic field properties in a more generic context, particularly noting that matrix multiplication is not commutative.
M.4HSTP.14	Understand that the zero and identity matrices play a role in matrix addition and multiplication similar to the role of 0 and 1 in the real numbers. The determinant of a square matrix is nonzero if and only if the matrix has a multiplicative inverse.
M.4HSTP.15	Multiply a vector (regarded as a matrix with one column) by a matrix of suitable dimensions to produce another vector. Work with matrices as transformations of vectors.
M.4HSTP.16	Work with 2 × 2 matrices as transformations of the plane and interpret the absolute value of the determinant in terms of area. Instructional Note: Matrix multiplication of a 2 x 2 matrix by a vector can be interpreted as transforming points or regions in the plane to different points or regions. In particular a matrix whose determinant is 1 or -1 does not change the area of a region.

Cluster	Solve systems of equations
M.4HSTP.17	Represent a system of linear equations as a single matrix equation in a vector variable.
M.4HSTP.18	Find the inverse of a matrix if it exists and use it to solve systems of linear equations (using technology for matrices of dimension 3 × 3 or greater). Instructional Note: Students have earlier solved two linear equations in two variables by algebraic methods.

Analysis and Synthesis of Functions

Cluster	Analyze functions using different representations.
M.4HSTP.19	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior. Instructional Note: This is an extension of graphical analysis from Math III or Algebra II that develops the key features of graphs with the exception of asymptotes. Students examine vertical, horizontal, and oblique asymptotes by considering limits. Students should note the case when the numerator and denominator of a rational function share a common factor. Utilize an informal notion of limit to analyze asymptotes and continuity in rational functions. Although the notion of limit is developed informally, proper notation should be followed.

Cluster	Build a function that models a relationship between two quantities.
M.4HSTP.20	Write a function that describes a relationship between two quantities, including composition of functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

Cluster	Build new functions from existing functions.
M.4HSTP.21	Find inverse functions. Instructional Note: This is an extension of concepts from Math III where the idea of inverse functions was introduced. a. Verify by composition that one function is the inverse of another. b. Read values of an inverse function from a graph or a table, given that the function has an inverse. Instructional Note: Students must realize that inverses created through function composition produce the same graph as reflection about the line y = x.) c. Produce an invertible function from a non-invertible function by restricting the domain. Instructional Note: Systematic procedures must be developed for restricting domains of non-invertible functions so that their inverses exist.)
M.4HSTP.22	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Trigonometric and Inverse Trigonometric Functions of Real Numbers

Cluster	Extend the domain of trigonometric functions using the unit circle.
M.4HSTP.23	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number. Instructional Note: Students use the extension of the domain of the trigonometric functions developed in Math III to obtain additional special angles and more general properties of the trigonometric functions.
M.4HSTP.24	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Cluster	Model periodic phenomena with trigonometric functions.
M.4HSTP.25	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.

M.4HSTP.26	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context. Instructional Note: Students should draw analogies to the work with inverses in the previous unit.
M.4HSTP.27	Solve more general trigonometric equations. (e.g., $2 \sin^2 x + \sin x - 1 = 0$ can be solved using factoring.

Cluster	**Prove and apply trigonometric identities.**
M.4HSTP.28	Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.

Cluster	**Apply transformations of function to trigonometric functions.**
M.4HSTP.29	Graph trigonometric functions showing key features, including phase shift. Instructional Note: In Math III, students graphed trigonometric functions showing period, amplitude and vertical shifts.)

Derivations in Analytic Geometry

Cluster	**Translate between the geometric description and the equation for a conic section.**
M.4HSTP.30	Derive the equations of ellipses and hyperbolas given the foci, using the fact that the sum or difference of distances from the foci is constant. Instructional Note: In Math II students derived the equations of circles and parabolas. These derivations provide meaning to the otherwise arbitrary constants in the formulas.)

Cluster	**Explain volume formulas and use them to solve problems.**
M.4HSTP.31	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures. Instructional Note: Students were introduced to Cavalieri's principle in Math II.

Modeling with Probability

Cluster	**Calculate expected values and use them to solve problems.**
M.4HSTP.32	Define a random variable for a quantity of interest by assigning a numerical value to each event in a sample space; graph the corresponding probability distribution using the same graphical displays as for data distributions. Instructional Note: Although students are building on their previous experience with probability in middle grades and in Math II and III, this is their first experience with expected value and probability distributions.
M.4HSTP.33	Calculate the expected value of a random variable; interpret it as the mean of the probability distribution.
M.4HSTP.34	Develop a probability distribution for a random variable defined for a sample space in which theoretical probabilities can be calculated; find the expected value. (e.g., Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.)

M.4HSTP.35	Develop a probability distribution for a random variable defined for a sample space in which probabilities are assigned empirically; find the expected value. For example, find a current data distribution on the number of TV sets per household in the United States, and calculate the expected number of sets per household. How many TV sets would you expect to find in 100 randomly selected households? Instructional Note: It is important that students can interpret the probability of an outcome as the area under a region of a probability distribution graph.

Cluster	**Use probability to evaluate outcomes of decisions.**
M.4HSTP.36	Weigh the possible outcomes of a decision by assigning probabilities to payoff values and finding expected values. a. Find the expected payoff for a game of chance. (e.g., Find the expected winnings from a state lottery ticket or a game at a fast food restaurant.) b. Evaluate and compare strategies on the basis of expected values. (e.g., Compare a high-deductible versus a low-deductible automobile insurance policy using various, but reasonable, chances of having a minor or a major accident.)

Series and Informal Limits

Cluster	**Use sigma notations to evaluate finite sums.**
M.4HSTP.37	Develop sigma notation and use it to write series in equivalent form. For example, write $\sum_{i=1}^{n}(3i^2 + 7)$ as $3\sum_{i=1}^{n} i^2 + 7\sum_{i=1}^{n} 1$.
M.4HSTP.38	Apply the method of mathematical induction to prove summation formulas. For example, verify that $\sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}$. Instructional Note: Some students may have encountered induction in Math III in proving the Binomial Expansion Theorem, but for many this is their first experience.

Cluster	**Extend geometric series to infinite geometric series.**
M.4HSTP.39	Develop intuitively that the sum of an infinite series of positive numbers can converge and derive the formula for the sum of an infinite geometric series. Instructional Note: In Math I, students described geometric sequences with explicit formulas. Finite geometric series were developed in Math III.
M.4HSTP.40	Apply infinite geometric series models. For example, find the area bounded by a Koch curve. Instructional Note: Rely on the intuitive concept of limit developed in unit 2 to justify that a geometric series converges if and only if the ratio is between -1 and 1.

West Virginia College- and Career-Readiness Standards for Mathematics

STEM Readiness

All West Virginia teachers are responsible for classroom instruction that integrates content standards and objectives and mathematical habits of mind. This course is designed for students who have completed the Math III (LA) course and subsequently decided they are interested in pursuing a STEM career. It includes standards that would have been covered in Mathematics III (STEM) but not in Mathematics III (LA) (i.e. standards that are marked with a "+"), selected topics from the Mathematics IV course, and topics drawing from standards covered in Mathematics I and Mathematics II as needed for coherence. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

<table>
<tr><th>Arithmetic and Algebra of Complex Numbers</th><th>Polynomial, Rational, and Radical Relationships</th></tr>
<tr><td>• Understand that the arithmetic and algebra of expressions involving rational numbers is governed by the same rules as the arithmetic and algebra of real numbers.</td><td>• Derive the formula for the sum of a geometric series, and use the formula to solve problems. (e.g., Calculate mortgage payments.)</td></tr>
<tr><th>Probability for Decisions</th><th>Trigonometry of General Triangles</th></tr>
<tr><td>• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.</td><td rowspan="3">• Apply knowledge of the Law of Sines and the Law of Cosines to determine distances in realistic situations. (e.g., Determine heights of inaccessible objects.)</td></tr>
<tr><th>Functions and Modeling</th></tr>
<tr><td>• Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision. (e.g., Estimate water and food needs in a disaster area, or use volume formulas and graphs to find an optimal size for an industrial package.)</td></tr>
</table>

Numbering of Standards

The following Mathematics Standards are numbered continuously. The following ranges relate to the clusters found within Mathematics:

Arithmetic and Algebra of Complex Numbers	
Perform arithmetic operations with complex numbers.	Standards 1-3
Represent complex numbers and their operations on the complex plane.	Standards 4-6
Use complex numbers in polynomial identities and equations.	Standards 7-9
Polynomial, Rational, and Radical Relationships	
Use polynomial identities to solve problems.	Standard 10
Rewrite rational expressions.	Standard 11
Probability for Decisions	
Use probability to evaluate outcomes of decisions.	Standards 12-13
Trigonometry of General Triangles	
Apply trigonometry to general triangles.	Standards 14-16
Functions and Modeling	
Analyze functions using different representations.	Standards 17-19
Building a function that models a relationship between two quantities.	Standards 20-21
Build new functions from existing functions.	Standards 22-26
Extend the domain of trigonometric functions using the unit circle.	Standards 27-28
Model periodic phenomena using trigonometric functions.	Standards 29-30
Prove and apply trigonometric identities.	Standard 31

Arithmetic and Algebra of Complex Numbers

Cluster	**Perform arithmetic operations with complex numbers**
M.SRM.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.
M.SRM.2	Use the relation $i^2 = -1$ and the commutative, associative and distributive properties to add, subtract and multiply complex numbers.
M.SRM.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Cluster	**Represent complex numbers and their operations on the complex plane**
M.SRM.4	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) and explain why the rectangular and polar forms of a given complex number represent the same number.

M.SRM.5	Represent addition, subtraction, multiplication and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. (e.g., $(-1 + \sqrt{3}\, i)^3 = 8$ because $(-1 + \sqrt{3}\, i)$ has modulus 2 and argument 120°.)
M.SRM.6	Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints.

Cluster	**Use complex numbers in polynomial identities and equations**
M.SRM.7	Solve quadratic equations with real coefficients that have complex solutions.
M.SRM.8	Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.
M.SRM.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Polynomials, Rational, and Radical Relationships

Cluster	**Use polynomial identities to solve problems.**
M.SRM.10	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Cluster	**Rewrite rational expressions**
M.SRM.11	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication and division by a nonzero rational expression; add, subtract, multiply and divide rational expressions.

Probability for Decisions

Cluster	**Use probability to evaluate outcomes of decisions.**
M.SRM.12	Use probabilities to make fair decisions (e.g. drawing by lot or using a random number generator).
M.SRM.13	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game).

Trigonometry of General Triangles

Cluster	**Apply trigonometry to general triangles.**
M.SRM.14	Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
M.SRM.15	Prove the Laws of Sines and Cosines and use them to solve problems.
M.SRM.16	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems or resultant forces).

Functions and Modeling

Cluster	Analyze functions using different representations.
M.SRM.17	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
M.SRM.18	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior.
M.SRM.19	Graph exponential and logarithmic functions, showing intercepts and end behavior and trigonometric functions, showing period, midline, and amplitude.

Cluster	Building a function that models a relationship between two quantities.
M.SRM.20	Write a function that describes a relationship between two quantities.
M.SRM.21	Compose functions. (e.g., If T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.)

Cluster	Build new functions from existing functions.
M.SRM.22	Find inverse functions.
M.SRM.23	Verify by composition that one function is the inverse of another.
M.SRM.24	Read values of an inverse function from a graph or a table, given that the function has an inverse.
M.SRM.25	Produce an invertible function from a non-invertible function by restricting the domain.
M.SRM.26	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Cluster	Extend the domain of trigonometric functions using the unit circle.
M.SRM.27	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
M.SRM.28	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Cluster	Model periodic phenomena using trigonometric functions.
M.SRM.29	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
M.SRM.30	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Cluster	Prove and apply trigonometric identities.
M.SRM.31	Prove the addition and subtraction formulas for sine, cosine and tangent and use them to solve problems.

West Virginia College- and Career-Readiness Standards for Mathematics

Transition Mathematics for Seniors

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Transition Mathematics for Seniors prepares students for their entry-level credit-bearing liberal studies mathematics course at the post-secondary level. Students will solidify their quantitative literacy by enhancing numeracy and problem solving skills as they investigate and use the fundamental concepts of algebra, geometry, and introductory trigonometry. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progression of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Number and Quantity: The Real Number System The Complex Number System	**Algebra:** Seeing Structure in Expressions Arithmetic with Polynomials and Rational Expressions Creating Equations Reasoning with Equations and Inequalities
• Develop an understanding of basic operations, equivalent representations, and properties of the real and complex number systems.	• Create equations or inequalities that model physical situations. • Solve systems of equations, with an emphasis on efficiency of solution as well as reasonableness of answers, given physical limitations.
Functions: Interpreting Functions Building Functions	**Geometry:** Geometric Measuring and Dimension Expressing Geometric Properties with Equations Modeling with Geometry
• Develop knowledge and understanding of the concept of functions as they use, analyze, represent and interpret functions and their applications.	• Use coordinates and to prove geometric properties algebraically.
Statistics and Probability: Interpreting Categorical and Quantitative Data Making Inferences and Justifying Conclusions	
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	

Numbering of Standards

The following Mathematics Standards are numbered continuously. The following ranges relate to the clusters found within Mathematics:

Number and Quantity – The Real Number System	
Extend the properties of exponents to rational exponents.	Standards 1-2
Number and Quantity – The Complex Number System	
Use complex numbers in polynomial identities and equations.	Standard 3
Algebra – Seeing Structure in Expressions	
Interpret the structure of expressions.	Standard 4
Write expressions in equivalent forms to solve problems.	Standards 5-6
Understand the connections between proportional relationship, lines, and linear equations.	Standards 7-9
Algebra – Arithmetic with Polynomials and Rational Expressions	
Perform arithmetic operations on polynomials.	Standard 10
Algebra – Creating Equations	
Create equations that describe numbers or relationships.	Standards 11-14
Algebra – Reasoning with Equations and Inequalities	
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 15
Solve equations and inequalities in one variable.	Standards 16-18
Solve systems of equations.	Standards 19-21
Represent and solve equations and inequalities graphically.	Standards 22-23
Functions – Interpreting Functions	
Understand the concept of a function and use function notation.	Standard 24
Interpret functions that arise in applications in terms of the context.	Standards 25-28
Analyze functions using different representations.	Standards 29-35
Functions – Building Functions	
Build a function that models a relationship between two quantities.	Standards 36-37
Geometry – Geometric Measuring and Dimension	
Explain volume formulas and use them to solve problems.	Standards 38-39
Geometry – Expressing Geometric Properties with Equations	
Use coordinates to prove simple geometric theorems algebraically.	Standards 40-41

Geometry – Modeling with Geometry	
Apply geometric concepts in modeling situations.	Standard 42
Statistics and Probability – Interpreting Categorical and Quantitative Data	
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standards 43-46
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 47-51
Statistics and Probability – Making Inferences and Justifying Conclusions	
Understand and evaluate random processes underlying statistical experiments.	Standard 52

Number and Quantity – The Real Number System

Cluster	Extend the properties of exponents to rational exponents.
M.TMS.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.TMS.2	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Number and Quantity – The Complex Number System

Cluster	Use complex numbers in polynomial identities and equations.
M.TMS.3	Solve quadratic equations with real coefficients that have complex solutions.

Algebra – Seeing Structure in Expressions

Cluster	Interpret the structure of expressions.
M.TMS.4	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$.

Cluster	Write expressions in equivalent forms to solve problems.
M.TMS.5	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
M.TMS.6	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems.

Cluster	Understand the connections between proportional relationship, lines, and linear equations.
M.TMS.7	Graph proportional relationships, interpreting the unit rates as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.
M.TMS.8	Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plan; derive the equation y = mx for a line through the origin and the equation y = mx + b for a line intercepting the vertical axis at b.
M.MTS.9	Solve linear equations in one variable.

Algebra – Arithmetic with Polynomials and Rational Expressions

Cluster	Perform arithmetic operations on polynomials.
M.TMS.10	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract and multiply polynomials.

Algebra – Creating Equations

Cluster	Create equations that describe numbers or relationships.
M.TMS.11	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions.
M.TMS.12	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales.
M.TMS.13	Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or nonviable options in a modeling context. For example, represent inequalities describing nutritional and cost constraints on combinations of different foods.
M.TMS.14	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Algebra – Reasoning with Equations and Inequalities

Cluster	Understand solving equations as a process of reasoning and explain the reasoning.
M.TMS.15	Solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise.

Cluster	Solve equations and inequalities in one variable.
M.TMS.16	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters.

M.TMS.17	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
M.TMS.18	Solve quadratic equations in one variable. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.

Cluster	**Solve systems of equations.**
M.TMS.19	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.
M.TMS.20	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically.
M.TMS.21	Explain why the x-coordinates of the points where the graphs of the equation y = f(x) and y = g(x) intersect are the solution of the equation f(x) = g (x); find the solution approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations).

Cluster	**Represent and solve equations and inequalities graphically.**
M.TMS.22	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables.
M.TMS.23	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Functions – Interpreting Functions

Cluster	**Understand the concept of a function and use function notation.**
M.TMS.24	Understand a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).

Cluster	**Interpret functions that arise in applications in terms of the context.**
M.TMS.25	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms.
M.TMS.26	Interpret the parameters in a linear or exponential function in terms of a context.
M.TMS.27	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity.
M.TMS.28	Distinguish between situations that can be modeled with linear functions and with exponential functions.

Cluster	Analyze functions using different representations.
M.TMS.29	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line, give examples of functions that are not linear.
M.TMS.30	Describe qualitatively the functional relationship between two quantities by analyzing a graph.
M.TMS.31	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs.
M.TMS.32	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
M.TMS.33	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasingly linearly, quadratically, or (more generally) as a polynomial function.
M.TMS.34	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.
M.TMS.35	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).

Functions - Building Functions

Cluster	Build a function that models a relationship between two quantities.
M.TMS.36	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
M.TMS.37	Write a function that describes a relationship between two quantities. a. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. b. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

Geometry – Geometric Measuring and Dimension

Cluster	Explain volume formulas and use them to solve problems.
M.TMS.38	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.
M.TMS.39	Give an informal argument using Cavalieri's principle for the formulas for the volume of a sphere and other solid figures.

Geometry – Expressing Geometric Properties with Equations

Cluster	Use coordinates to prove simple geometric theorems algebraically
M.TMS.40	Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).
M.TMS.41	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, (e.g., using the distance formula).

Geometry – Modeling with Geometry

Cluster	Apply geometric concepts in modeling situations.
M.TMS.42	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with topographic grid systems based on ratios).

Statistics and Probability - Interpreting Categorical & Quantitative Data

Cluster	Summarize, represent, and interpret data on two categorical and quantitative variables.
M.TMS.43	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. Interpret linear models.
M.TMS.44	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.
M.TMS.45	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
M.TMS.46	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.
M.TMS.47	Represent data with plots on the real number line (dot plots, histograms, and box plots).
M.TMS.48	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
M.TMS.49	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
M.TMS.50	Computer (using technology) and interpret the correlation coefficient of a linear fit.
M.TMS.51	Distinguish between correlation and causation.

Statistics and Probability - Interpreting Categorical & Quantitative Data

Cluster	Understand and evaluate random processes underlying statistical experiments
M.TMS.52	Understand statistics as a process for making inferences about population parameters based on a random sample from that population.

Appendix A
Standards vs. Curriculum

COLLEGE- & CAREER-READINESS

STANDARDS

CURRICULUM

What's the Difference?

Standards are what we want students to know, understand and be able to do; **Standards** represent goals.

The **Curriculum** is an intentional learning plan to ensure students achieve the goals of the standards; the **Curriculum** represents the learning experience.

Standards and Curriculum

A STANDARD is a goal. The CURRICULUM is a means to achieve the goal.

Example 1 • 3rd Grade Mathematics Goal

Standard: M.3.8

Solve two-step word problems using the four operations, represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding.

Example 2 • 6th Grade English Language Arts Goal

Standard: ELA.6.18

By the end of the year, read and comprehend literature, including stories, dramas, and poems, in the grade 6-8 text complexity range proficiently, with scaffolding as needed at the high end of the range.

Curriculum:
Teacher locates instructional materials, plans and facilitates learning activities and assesses the students' mastery of the standard.

Who is Responsible?

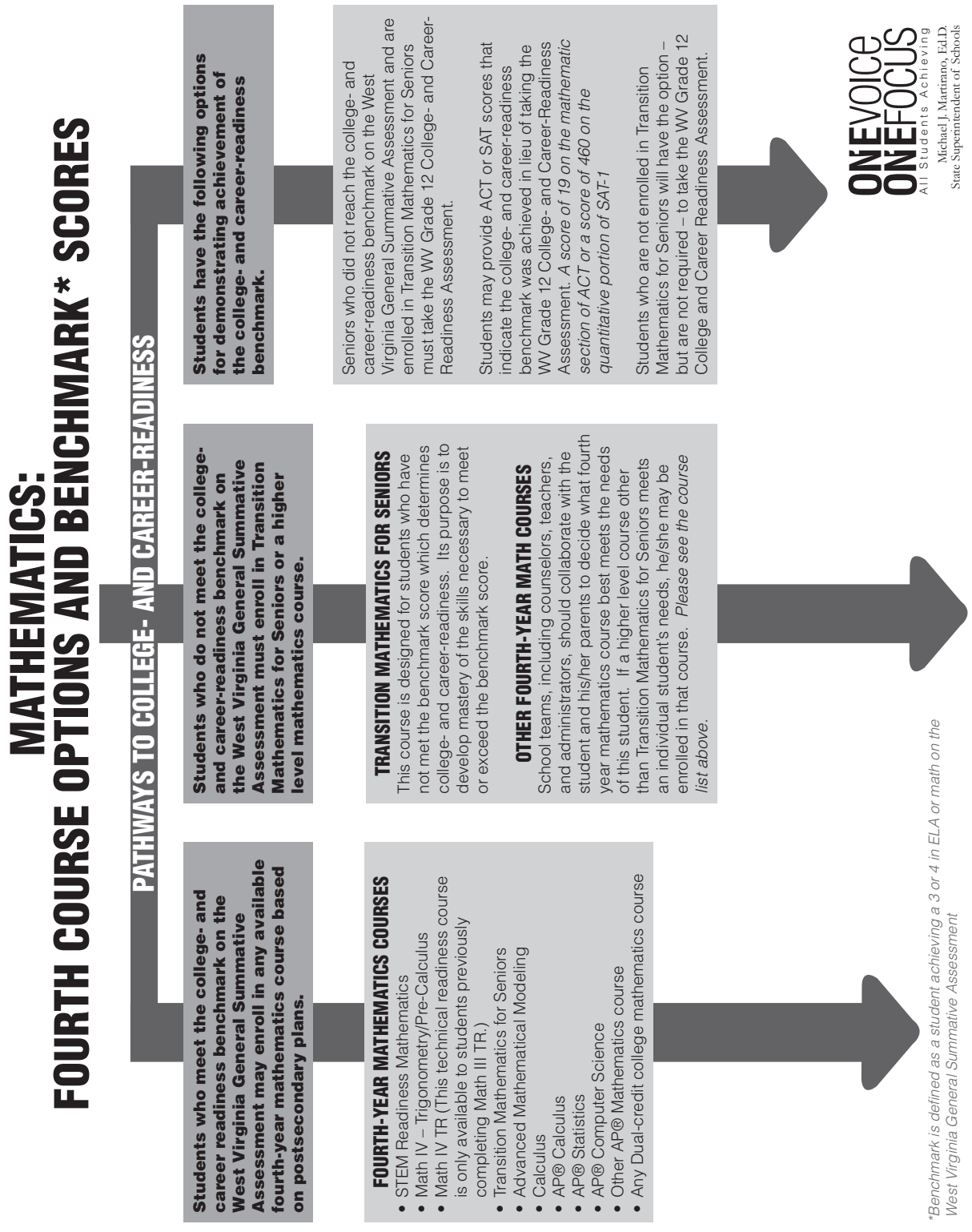
West Virginia Board of Education
West Virginia Department of Education

County boards of education, administrators and teachers

West Virginia College & Career Readiness Standards

West Virginia Department of EDUCATION
Steven L. Paine, Ed.D.
State Superintendent of Schools

Appendix B
Fourth Course Options and Benchmark Scores



Appendix C
Sample Parent Letter
High School Algebra I for 8th Grade

(Insert School Address)

(Insert Date)

Dear Parent or Guardian,

I look forward to being your child's teacher for the High School Algebra I for 8th Grade course! This opportunity to take a high school mathematics course can be an exciting, intimidating, and overwhelming experience – all at once. I want to welcome your child to my classroom and give you a preview of what to expect in mathematics for the upcoming school year. As always, if you have any questions or concerns, please feel free to contact me using the information below.

Providing students with the skills they need to be college- and career-ready is the ultimate goal of the educational standards in West Virginia. Students in this course will continue enhancing skills through a developmentally-appropriate progression of standards, building on what they learned in seventh grade and preparing them for what they will need in the high school Geometry and Algebra II courses. To support student progress from seventh grade mathematics to this high school course, several important topics from eighth grade mathematics will also need to be addressed. The following summary highlights some of the mathematical understandings students will develop this school year.

Relationships between Quantities and Reasoning with Equations	**Linear and Exponential Relationships**
• Solve problems with a wide range of units and solve problems by thinking about units. (e.g., The Trans Alaska Pipeline System is 800 miles long and cost $8 billion to build. Divide one of these numbers by the other. What is the meaning of the answer? Greenland has a population of 56,700 and a land area of 2,175,600 square kilometers. By what factor is the population density of the United States, 80 persons per square mile, larger than the population density of Greenland?)	• Understand contextual relationships of variables and constants. (e.g., Annie is picking apples with her sister. The number of apples in her basket is described by $n = 22t + 12$, where t is the number of minutes Annie spends picking apples. What do the numbers 22 and 12 tell you about Annie's apple picking?)
Descriptive Statistics	**Expressions and Equations**
• Use linear regression techniques to describe the relationship between quantities and assess the fit of the model. (e.g., Use the high school and university grades for 250 students to create a model that can be used to predict a student's university GPA based on his high school GPA.)	• Interpret algebraic expressions and transforming them purposefully to solve problems. (e.g., In solving a problem about a loan with interest rate r and principal P, seeing the expression $P(1+r)^n$ as a product of P with a factor not depending on P.)
Quadratic Functions and Modeling	
• Solve real-world and mathematical problems by writing and solving nonlinear equations, such as quadratic equations ($ax^2 + bx + c = 0$).	

With your assistance and continued support, your student will have a successful school year. I am available by phone and email if you have any questions or concerns or if you would like to set up a conference.

Sincerely,

Email:
Phone:

Sample Parent Letter
High School Algebra I

(Insert School Address)

(Insert Date)

Dear Parent or Guardian,

I look forward to being your child's teacher for Algebra I course! High school can be an exciting, intimidating, and overwhelming experience – all at once. I want to welcome your child to my classroom and give you a preview of what to expect in mathematics for the upcoming school year. As always, if you have any questions or concerns, please feel free to contact me using the information below.

Providing students with the skills they need to be college- and career-ready is the ultimate goal of the educational standards in West Virginia. Students in this course will continue enhancing skills through a developmentally-appropriate progression of standards, building on what they learned in eighth grade and preparing them for what they will need in the Geometry and Algebra II courses. The following summary highlights some of the mathematical understandings students will develop this school year.

Relationships between Quantities and Reasoning with Equations	**Linear and Exponential Relationships**
• Solve problems with a wide range of units and solve problems by thinking about units. (e.g., The Trans Alaska Pipeline System is 800 miles long and cost $8 billion to build. Divide one of these numbers by the other. What is the meaning of the answer? Greenland has a population of 56,700 and a land area of 2,175,600 square kilometers. By what factor is the population density of the United States, 80 persons per square mile, larger than the population density of Greenland?)	• Understand contextual relationships of variables and constants. (e.g., Annie is picking apples with her sister. The number of apples in her basket is described by $n = 22t + 12$, where t is the number of minutes Annie spends picking apples. What do the numbers 22 and 12 tell you about Annie's apple picking?)
Descriptive Statistics	**Expressions and Equations**
• Use linear regression techniques to describe the relationship between quantities and assess the fit of the model. (e.g., Use the high school and university grades for 250 students to create a model that can be used to predict a student's university GPA based on his high school GPA.)	• Interpret algebraic expressions and transform them purposefully to solve problems. (e.g., In solving a problem about a loan with interest rate r and principal P, seeing the expression $P(1+r)^n$ as a product of P with a factor not depending on P.)
Quadratic Functions and Modeling	
• Solve real-world and mathematical problems by writing and solving nonlinear equations, such as quadratic equations ($ax^2 + bx + c = 0$).	

With your assistance and continued support, your student will have a successful school year. I am available by phone and email if you have any questions or concerns or if you would like to set up a conference.

Sincerely,

Email:
Phone:

Sample Parent Letter
High School Geometry

(Insert School Address)

(Insert Date)

Dear Parent or Guardian,

I look forward to being your child's teacher for the Geometry course! High school can be an exciting, intimidating, and overwhelming experience – all at once. I want to welcome your child to my classroom and give you a preview of what to expect in mathematics for the upcoming school year. As always, if you have any questions or concerns, please feel free to contact me using the information below.

Providing students with the skills they need to be college- and career-ready is the ultimate goal of the educational standards in West Virginia. Students in this course will continue enhancing skills through a developmentally-appropriate progression of standards, building on what they learned in prior courses and preparing them for what they will need in the Algebra II course. The following summary highlights some of the mathematical understandings students will develop this school year.

Congruence, Proof, and Constructions	**Similarity, Proof, and Trigonometry**
• Prove theorems about triangles and other figures (e.g., that the sum of the measures of the angles in a triangle is 180°). • Given a transformation, work backwards to discover the sequence that led to the transformation. • Given two quadrilaterals that are reflections of each other, find the line of that reflection.	• Apply knowledge of trigonometric ratios and the Pythagorean Theorem to determine distances in realistic situations. (e.g., Determine heights of inaccessible objects using various instruments, such as clinometers, hypsometers, transits, etc.)
Extending to Three Dimensions	**Connecting Algebra and Geometry Through Coordinates**
• Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	• Use a rectangular coordinate system and build on understanding of the Pythagorean Theorem to find distances. (e.g., Find the area and perimeter of a real-world shape using a coordinate grid and Google Earth.) • Analyze the triangles and quadrilaterals on the coordinate plane to determine their properties. (e.g., Determine whether a given quadrilateral is a rectangle).
Circles With and Without Coordinates	**Applications of Probability**
• Use coordinates and equations to describe geometric properties algebraically. (e.g., Write the equation for a circle in the plane with specified center and radius.)	• Work with probability and using ideas from probability in everyday situations. (e.g., Compare the chance that a person who smokes will develop lung cancer to the chance that a person who develops lung cancer smokes.)
Modeling with Geometry	
• Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision (e.g., estimate water and food needs in a disaster area, or use volume formulas and graphs to find an optimal size for an industrial package).	

With your assistance and continued support, your student will have a successful school year. I am available by phone and email if you have any questions or concerns or if you would like to set up a conference.

Sincerely,

Email:
Phone:

Sample Parent Letter
High School Algebra II

(Insert School Address)

(Insert Date)

Dear Parent or Guardian,

I look forward to being your child's teacher for the Algebra II course! High school can be an exciting, intimidating, and overwhelming experience – all at once. I want to welcome your child to my classroom and give you a preview of what to expect in mathematics for the upcoming school year. As always, if you have any questions or concerns, please feel free to contact me using the information below.

Providing students with the skills they need to be college- and career-ready is the ultimate goal of the educational standards in West Virginia. Students in this course will continue enhancing skills through a developmentally-appropriate progression of standards, building on what they learned in prior courses and preparing them for what they will need in future courses. The following summary highlights some of the mathematical understandings students will develop this school year.

Polynomial, Rational, and Radical Relationships	**Trigonometric Functions**
• Derive the formula for the sum of a geometric series, and use the formula to solve problems. (e.g., Calculate mortgage payments.)	• Apply knowledge of trigonometric functions to determine distances in realistic situations. (e.g., Determine heights of inaccessible objects.)
Modeling with Functions	**Inferences and Conclusions from Data**
• Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision. (e.g., Estimate water and food needs in a disaster area, or use volume formulas and graphs to find an optimal size for an industrial package.) • Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

With your assistance and continued support, your student will have a successful school year. I am available by phone and email if you have any questions or concerns or if you would like to set up a conference.

Sincerely,

Email:
Phone:

Sample Parent Letter
Advanced Mathematical Modeling

(Insert School Address)

(Insert Date)

Dear Parent or Guardian,

I look forward to being your child's teacher for the Advanced Mathematical Modeling course! High school can be an exciting, intimidating, and overwhelming experience – all at once. I want to welcome your child to my classroom and give you a preview of what to expect in mathematics for the upcoming school year. As always, if you have any questions or concerns, please feel free to contact me using the information below.

Providing students with the skills they need to be college- and career-ready is the ultimate goal of the educational standards in West Virginia. Students in this course will continue enhancing skills through a developmentally-appropriate progression of standards, building on what they learned in prior courses and preparing them for what they will need in future courses. The following summary highlights some of the mathematical understandings students will develop this school year.

Developing College and Career Skills	**Finance**
• Develop and apply skills used in college and careers, including reasoning, planning and communication, to make decisions and solve problems in applied situations.	• Create and analyze mathematical models to make decisions related to earning, investing, spending and borrowing money.
Probability	**Statistics**
• Use basic rules of counting and probability to analyze and evaluate risk and return in the context of everyday situations.	• Make decisions based on understanding, analysis and critique of reported statistical information and summaries.
Modeling	**Networks**
• Analyze numerical data in everyday situations using a variety of quantitative measures and numerical processes.	• Use a variety of network models represented graphically to organize data in quantitative situations, make informed decisions, and solve problems.
Social Decision Making	**Geometry**
• Analyze the mathematics behind various methods of ranking and selection and consider the advantages/disadvantages of each method.	• Solve geometric problems involving inaccessible distances. • Use vectors to solve applied problems.

With your assistance and continued support, your student will have a successful school year. I am available by phone and email if you have any questions or concerns or if you would like to set up a conference.

Sincerely,

Email:
Phone:

Sample Parent Letter (Calculus)

(Insert School Address)

(Insert Date)

Dear Parent or Guardian,

I look forward to being your child's teacher for the Calculus course! High school can be an exciting, intimidating, and overwhelming experience – all at once. I want to welcome your child to my classroom and give you a preview of what to expect in mathematics for the upcoming school year. As always, if you have any questions or concerns, please feel free to contact me using the information below.

Providing students with the skills they need to be college- and career-ready is the ultimate goal of the educational standards in West Virginia. Students in this course will continue enhancing skills through a developmentally-appropriate progression of standards, building on what they learned in prior courses and preparing them for what they will need in future courses. The following summary highlights some of the mathematical understandings students will develop this school year.

Algebra

- A utility company burns coal to generate electricity. The cost C in dollars of removing p% of the air pollutants emissions is $C = \frac{90000p}{100 - p}$, $0 \le p < 100$. Find the cost of removing (a) 10%, (b) 25%, and (c) 75% of the pollutants. Find the limit of C as $p \to 100^-$
- A management company is planning to build a new apartment complex. Knowing the maximum number of apartments the lot can hold and given a function for the maintenance costs, determine the number of apartments that will minimize the maintenance costs.
- The velocity v of the flow of blood at a distance r from the central axis of an artery of radius R is $v = k(R^2 - r^2)$ where k is the constant of proportionality. Find the average rate of flow of blood along a radius of the artery. (Use 0 and R as the limits of integration.)

Data Analysis and Probability

- The average data entry speeds S (words per minute) of a business student after t weeks of lessons are recorded in the following table.

t	5	10	15	20	25	30
S	28	56	79	90	93	94

  A model for the data is $S = \frac{100t^2}{65 + t}$, $t > 0$. Do you think that there is a limiting speed? If so, what is the limiting speed? If not, why?
- Identify a real life situation that involves quantities that change over time and develop a method to collect and analyze related data. Develop a continuous function to model the data and generalize the results to make a conclusion.
- A sheet of typing paper is ruled with parallel lines that are 2 inches apart. A two-inch needle is tossed randomly onto the sheet of paper. The probability that the needle will touch a line is $P = \frac{2}{\pi}\int_0^{\pi/2} \sin\theta \, d\theta$ where θ is the acute angle between the needle and any one of the parallel lines. Find the probability.

Geometry

- The radius of a right circular cylindrical balloon is given by $\sqrt{t + 2}$ and its height is $\frac{1}{2}\sqrt{t}$, where t is time in seconds and the dimensions are in inches. Find the rate of change of the volume with respect to time.
- Given 50 meters of framing material, construct a window that will let in the most light if the middle of the window is a rectangle and the top and bottom of the window are semi-circles.
- The graph of f consists of the three line segments joining the point (0,0), (2–2), (6,2), and (8,3). The function F is defined as follows $F(x) = \int_0^x f(t)\,dt$. Find the total enclosed areas generated by f and the x-axis. Determine the points of inflection of F on the interval (0,8).

With your assistance and continued support, your student will have a successful school year. I am available by phone and email if you have any questions or concerns or if you would like to set up a conference.

Sincerely,

Email:
Phone:

Sample Parent Letter
High School Mathematics IV – Trigonometry/Pre-calculus

(Insert School Address)

(Insert Date)

Dear Parent or Guardian,

I look forward to being your child's teacher for the Math IV – Trigonometry/Pre-calculus course! High school can be an exciting, intimidating, and overwhelming experience – all at once. I want to welcome your child to my classroom and give you a preview of what to expect in mathematics for the upcoming school year. As always, if you have any questions or concerns, please feel free to contact me using the information below.

Providing students with the skills they need to be college- and career-ready is the ultimate goal of the educational standards in West Virginia. Students in this course will continue enhancing skills through a developmentally-appropriate progression of standards, building on what they learned in prior courses and preparing them for what they will need in future courses. The following summary highlights some of the mathematical understandings students will develop this school year.

Building Relationships among Complex Numbers, Vectors, and Matrices	**Analysis and Synthesis of Functions**
• Represent abstract situations involving vectors symbolically.	• Write a function that describes a relationship between two quantities. (e.g., if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.)
Trigonometric and Inverse Trigonometric Functions of Real Numbers	**Derivations in Analytic Geometry**
• Make sense of the symmetry, periodicity, and special values of trigonometric functions using the unit circle. • Prove trigonometric identities and apply them in problem solving situations.	• Make sense of the derivations of the equations of an ellipse and a hyperbola.
Modeling with Probability	**Series and Informal Limits**
• Develop a probability distribution. (e.g., Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.)	• Apply mathematical induction to prove summation formulas.

With your assistance and continued support, your student will have a successful school year. I am available by phone and email if you have any questions or concerns or if you would like to set up a conference.

Sincerely,

Email:
Phone:

Sample Parent Letter
STEM Readiness

(Insert School Address)

(Insert Date)

Dear Parent or Guardian,

I look forward to being your child's teacher for the STEM Readiness course! High school can be an exciting, intimidating, and overwhelming experience – all at once. I want to welcome your child to my classroom and give you a preview of what to expect in mathematics for the upcoming school year. As always, if you have any questions or concerns, please feel free to contact me using the information below.

Providing students with the skills they need to be college- and career-ready is the ultimate goal of the educational standards in West Virginia. Students in this course will continue enhancing skills through a developmentally-appropriate progression of standards, building on what they learned in prior courses and preparing them for what they will need in future courses. The following summary highlights some of the mathematical understandings students will develop this school year.

Arithmetic and Algebra of Complex Numbers	**Polynomial, Rational, and Radical Relationships**
• Understand that the arithmetic and algebra of expressions involving rational numbers is governed by the same rules as the arithmetic and algebra of real numbers.	• Derive the formula for the sum of a geometric series, and use the formula to solve problems. (e.g., Calculate mortgage payments.)
Probability for Decisions	**Trigonometry of General Triangles**
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	• Apply knowledge of the Law of Sines and the Law of Cosines to determine distances in realistic situations. (e.g., Determine heights of inaccessible objects.)
Functions and Modeling	
• Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision. (e.g., Estimate water and food needs in a disaster area, or use volume formulas and graphs to find an optimal size for an industrial package.)	

With your assistance and continued support, your student will have a successful school year. I am available by phone and email if you have any questions or concerns or if you would like to set up a conference.

Sincerely,

Email:
Phone:

Sample Parent Letter
Transition Mathematics for Seniors

(Insert School Address)

(Insert Date)

Dear Parent or Guardian,

I look forward to being your child's teacher for the Transition Mathematics for Seniors course! High school can be an exciting, intimidating, and overwhelming experience – all at once. I want to welcome your child to my classroom and give you a preview of what to expect in mathematics for the upcoming school year. As always, if you have any questions or concerns, please feel free to contact me using the information below.

Providing students with the skills they need to be college- and career-ready is the ultimate goal of the educational standards in West Virginia. Students in this course will continue enhancing skills through a developmentally-appropriate progression of standards, building on what they learned in prior courses and preparing them for what they will need in future courses. The following summary highlights some of the mathematical understandings students will develop this school year.

Number and Quantity: The Real Number System The Complex Number System	**Algebra:** Seeing Structure in Expressions Arithmetic with Polynomials and Rational Expressions Creating Equations Reasoning with Equations and Inequalities
• Develop an understanding of basic operations, equivalent representations, and properties of the real and complex number systems.	• Create equations or inequalities that model physical situations. • Solve systems of equations, with an emphasis on efficiency of solution as well as reasonableness of answers, given physical limitations.
Functions: Interpreting Functions Building Functions	**Geometry:** Geometric Measuring and Dimension Expressing Geometric Properties with Equations Modeling with Geometry
• Develop knowledge and understanding of the concept of functions as they use, analyze, represent and interpret functions and their applications.	• Use coordinates to prove geometric properties algebraically.
Statistics and Probability: Interpreting Categorical and Quantitative Data Making Inferences and Justifying Conclusions	
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	

With your assistance and continued support, your student will have a successful school year. I am available by phone and email if you have any questions or concerns or if you would like to set up a conference.

Sincerely,

Email:
Phone:

Appendix D
Mathematics Standards Progressions

Skill Progressions in West Virginia College- and Career-Readiness Standards for Mathematics

The following pages outline the skill progressions found in the West Virginia College- and Career Readiness Standards for Mathematics. In Mathematics, the sequence of topics follow a programmatic progression that are reflected in the domains. These domains have been organized into programmatic levels where grade-level clusters provide detail about the skill progressions. The language of the clusters illustrates the advancing rigor and complexity of the expectations for what students should know, understand, and be able to do. Because the diversity of the mathematics in the Fourth Course Options does not support a similar skills progression alignment for these course, the document ends with a listing of the Fourth Course Options in Mathematics.

This document is intended to be a resource to foster and support discussion among teachers or how to best personalize and differentiate instruction for their students. The progression of skills toward college- and career-readiness that are outlined here can be used to scaffold instruction, assist with remediation, and develop instructional plans that meet the specific needs of each student.

Mathematics Progressions – High School Traditional Pathway

Domain: The Real Number System

Course	Clusters
High School Algebra I for 8th Grade	• Extend the properties of exponents to rational exponents. • Use properties of rational and irrational numbers.
High School Algebra I	• Extend the properties of exponents to rational exponents. • Use properties of rational and irrational numbers.
High School Geometry	Not a primary focus of High School Geometry
High School Algebra II	Not a primary focus of High School Algebra II

Domain: Quantities

Course	Clusters
High School Algebra I for 8th Grade	• Reason quantitatively and use units to solve problems. (Foundation for work with expressions, equations, and functions.)
High School Algebra I	• Reason quantitatively and use units to solve problems. (Foundation for work with expressions, equations, and functions.)
High School Geometry	Not a primary focus of High School Geometry
High School Algebra II	Not a primary focus of High School Algebra II

Domain: The Complex Number System

Course	Clusters
High School Algebra I for 8th Grade	Initial focus begins in High School Algebra II
High School Algebra I	Initial focus begins in High School Algebra II
High School Geometry	Initial focus begins in High School Algebra II
High School Algebra II	• Perform arithmetic operations with complex numbers. • Use complex numbers in polynomial identities and equations. (Polynomials with real coefficients)

Domain: Seeing Structure in Expressions

Course	Clusters
High School Algebra I for 8th Grade	• Interpret the structure of expressions. (Linear, exponential, quadratic) • Write expressions in equivalent forms to solve problems. (Quadratic and exponential)
High School Algebra I	• Interpret the structure of expressions. (Linear, exponential, quadratic) • Write expressions in equivalent forms to solve problems. (Quadratic and exponential)
High School Geometry	Not a primary focus of High School Geometry
High School Algebra II	• Interpret the structure of expressions. (Polynomial and rational) • Write expressions in equivalent forms to solve problems.

Domain: Arithmetic with Polynomials and Rational Expressions

Course	Clusters
High School Algebra I for 8th Grade	• Perform arithmetic operations on polynomials. (Linear and quadratic.)
High School Algebra I	• Perform arithmetic operations on polynomials. (Polynomials that simplify to quadratics.)
High School Geometry	Not a primary focus of High School Geometry
High School Algebra II	• Perform arithmetic operations on polynomials. (Beyond quadratics) • Understand the relationship between zeros and factors of polynomials. • Use polynomial identities to solve problems. • Rewrite rational expressions. (Linear and quadratic denominators)

Domain: Creating Equations

Course	Clusters
High School Algebra I for 8th Grade	• Create equations that describe numbers or relationships. (Linear, quadratic, and exponential (integer inputs only))
High School Algebra I	• Create equations that describe numbers or relationships. (Linear, quadratic, and exponential (integer inputs only))
High School Geometry	Not a primary focus of High School Geometry
High School Algebra II	• Create equations that describe numbers or relationships. (Equations using all available types of expressions, including simple root functions.)

Domain: Reasoning with Equations and Inequalities

Course	Clusters
High School Algebra I for 8th Grade	• Understand solving equations as a process of reasoning and explain the reasoning. (Master linear, learn as a general principle) • Solve equations and inequalities in one variable. (Linear inequalities; literal equations that are linear in the variables being solved for; quadratics with real solutions) • Analyze and solve linear equations and pairs of simultaneous linear equations. • Solve systems of equations (Linear-linear and linear-quadratic) • Represent and solve equations and inequalities graphically. (Linear and exponential; learn as a general principle)
High School Algebra I	• Understand solving equations as a process of reasoning and explain the reasoning. (Master linear, learn as a general principle) • Solve equations and inequalities in one variable. (Linear inequalities; literal equations that are linear in the variables being solved for; quadratics with real solutions) • Solve systems of equations (Linear-linear and linear-quadratic) • Represent and solve equations and inequalities graphically. (Linear and exponential; learn as a general principle)
High School Geometry	Not a primary focus of High School Geometry
High School Algebra II	• Understand solving equations as a process of reasoning and explain the reasoning. (Simple radical and rational) • Represent and solve equations and inequalities graphically. (Combine polynomial, rational, radical, absolute value, and exponential functions)

Domain: Interpreting Functions

Course	Clusters
High School Algebra I for 8th Grade	• Define, evaluate, and compare functions. • Understand the concept of a function and use function notation. (Learn as a general principle. Focus on linear and exponential (integer domains) and on arithmetic and geometric sequences.) • Use functions to model relationships between quantities. • Interpret functions that arise in applications in terms of a context. (Linear, exponential, and quadratic) • Analyze functions using different representations. (Linear, exponential, quadratic, absolute value, step, piecewise-defined)
High School Algebra I	• Understand the concept of a function and use function notation. (Learn as a general principle. Focus on linear and exponential (integer domains) and on arithmetic and geometric sequences.) • Interpret functions that arise in applications in terms of a context. (Linear, exponential, and quadratic) • Analyze functions using different representations. (Linear, exponential, quadratic, absolute value, step, piecewise-defined)
High School Geometry	Not a primary focus of High School Geometry
High School Algebra II	• Interpret functions that arise in applications in terms of a context. (Emphasize selection of appropriate models) • Analyze functions using different representations. (Focus on using key features to guide selection of appropriate types of model function)

Domain: Building Functions

Course	Clusters
High School Algebra I for 8th Grade	• Build a function that models a relationship between two quantities. (Linear, exponential, and quadratic) • Build new functions from existing functions. (Linear, exponential, quadratic, and absolute value)
High School Algebra I	• Build a function that models a relationship between two quantities. (Linear, exponential, and quadratic) • Build new functions from existing functions. (Linear, exponential, quadratic, and absolute value)
High School Geometry	Not a primary focus of High School Geometry
High School Algebra II	• Build a function that models a relationship between two quantities. (Include all types of functions studied) • Build new functions from existing functions. (Include simple radical, rational, and exponential functions; emphasize common effect of each transformation across function types)

Domain: Linear, Quadratic, and Exponential Models

Course	Clusters
High School Algebra I for 8th Grade	• Construct and compare linear, quadratic, and exponential models and solve problems. • Interpret expressions for functions in terms of the situation they model. (Linear and exponential of form $f(x) = b^x + k$)
High School Algebra I	• Construct and compare linear, quadratic, and exponential models and solve problems. • Interpret expressions for functions in terms of the situation they model. (Linear and exponential of form $f(x) = b^x + k$)
High School Geometry	Not a primary focus of High School Geometry
High School Algebra II	• Construct and compare linear, quadratic, and exponential models and solve problems. (Logarithms as solutions for exponentials)

Domain: Trigonometric Functions

Course	Clusters
High School Algebra I for 8th Grade	Initial focus begins in High School Algebra II
High School Algebra I	Initial focus begins in High School Algebra II
High School Geometry	Initial focus begins in High School Algebra II
High School Algebra II	• Extend the domain of trigonometric functions using the unit circle. • Model periodic phenomena with trigonometric functions. • Prove and apply trigonometric identities.

Domain: Congruence

Course	Clusters
High School Algebra I for 8th Grade	Initial focus begins in High School Geometry

High School Algebra I	Initial focus begins in High School Geometry
High School Geometry	• Experiment with transformations in the plane. • Understand congruence in terms of rigid motions. (Build on rigid motions as a familiar starting point for development of concept of geometric proof) • Prove geometric theorems. (Focus on validity of underlying reasoning while using variety of ways of writing proofs) • Make geometric constructions. (Formalize and explain processes)
High School Algebra II	Not a primary focus of High School Algebra II

Domain: Similarity, Right Triangles, and Trigonometry

Course	Clusters
High School Algebra I for 8th Grade	Initial focus begins in High School Geometry
High School Algebra I	Initial focus begins in High School Geometry
High School Geometry	• Understand similarity in terms of similarity transformations. • Prove theorems involving similarity. (Focus on validity of underlying reasoning while using variety of formats) • Define trigonometric ratios and solve problems involving right triangles. • Apply trigonometry to general triangles.
High School Algebra II	Not a primary focus of High School Algebra II

Domain: Circles

Course	Clusters
High School Algebra I for 8th Grade	Initial focus begins in High School Geometry
High School Algebra I	Initial focus begins in High School Geometry
High School Geometry	• Understand and apply theorems about circles. • Find arc lengths and area of sectors of circles. (Radian introduced only as a unit of measure)
High School Algebra II	Not a primary focus of High School Algebra II

Domain: Expressing Geometric Properties with Equations

Course	Clusters
High School Algebra I for 8th Grade	Initial focus begins in High School Geometry
High School Algebra I	Initial focus begins in High School Geometry
High School Geometry	• Translate between the geometric description and the equation for a conic section. • Use coordinates to prove simple geometric theorems algebraically. (Include distance formula; relate to Pythagorean Theorem)
High School Algebra II	Not a primary focus of High School Algebra II

Domain: Geometric Measurement and Dimension

Course	Clusters
High School Algebra I for 8th Grade	• Understand and apply the Pythagorean Theorem. (Connect to radicals, rational exponents, and irrational numbers)
High School Algebra I	Not a primary focus of High School Algebra I
High School Geometry	• Explain volume formulas and use them to solve problems. • Visualize the relation between two-dimensional and three-dimensional objects. • Apply geometric concepts in modeling situations.
High School Algebra II	Not a primary focus of High School Algebra II

Domain: Interpreting Categorical and Quantitative Data

Course	Clusters
High School Algebra I for 8th Grade	• Summarize, represent, and interpret data on a single count or measurement variable. • Investigate patterns of association in bivariate data. • Summarize, represent, and interpret data on two categorical and quantitative variables. (Linear focus; discuss general principle) • Interpret linear models.
High School Algebra I	• Summarize, represent, and interpret data on a single count or measurement variable. • Investigate patterns of association in bivariate data. • Summarize, represent, and interpret data on two categorical and quantitative variables. (Linear focus; discuss general principle) • Interpret linear models.
High School Geometry	Not a primary focus of High School Geometry
High School Algebra II	• Summarize, represent, and interpret data on two categorical and quantitative variables.

Domain: Making Inferences and Justifying Conclusions

Course	Clusters
High School Algebra I for 8th Grade	Initial focus begins in High School Algebra II
High School Algebra I	Initial focus begins in High School Algebra II
High School Geometry	Initial focus begins in High School Algebra II
High School Algebra II	• Understand and evaluate random processes underlying statistical experiments. • Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Domain: Conditional Probability and the Rules of Probability

Course	Clusters
High School Algebra I for 8th Grade	Initial focus begins in High School Geometry

High School Algebra I	Initial focus begins in High School Geometry
High School Geometry	• Understand independence and conditional probability and use them to interpret data. (Link to data simulations or experiments) • Use the rules of probability to compute probabilities of compound events in a uniform probability model.
High School Algebra II	Not a primary focus of High School Algebra II

Domain: Using Probability to Make Decisions

Course	Clusters
High School Algebra I for 8th Grade	Initial focus begins in High School Geometry
High School Algebra I	Initial focus begins in High School Geometry
High School Geometry	• Use probability to evaluate outcomes of decisions. (Introductory; apply counting rules)
High School Algebra II	• Use probability to evaluate outcomes of decisions. (Include more complex situations)

FOURTH COURSE OPTIONS

Fourth course options available to students in either pathway:

- Advanced Mathematical Modeling
- Calculus
- High School Mathematics IV - Trigonometry/Pre-calculus
- STEM Readiness
- Transition Mathematics for Seniors
- AP® Calculus
- AP® Computer Science
- AP® Statistics
- Dual credit mathematics courses and advanced mathematics courses offered through WV Virtual School.

Appendix E
Mathematics: Grade-level and Course Overview

Grade/Course	Number of Standards
Mathematics – Kindergarten	22
Mathematics – Grade 1	21
Mathematics – Grade 2	26
Mathematics – Grade 3	25
Mathematics – Grade 4	28
Mathematics – Grade 5	26
Mathematics – Grade 6	29
Mathematics – Grade 7	26
Mathematics – Grade 8	28

Integrated Pathway		Traditional Pathway	
Grade/Course	**Number of Standards**	**Grade/Course**	**Number of Standards**
8th Grade High School Mathematics I	64	High School Algebra I for 8th Grade	73
High School Mathematics I	51	High School Algebra I	60
High School Mathematics II	61	High School Geometry	55
High School Mathematics III	48	High School Algebra II	45

Fourth Course Options	
Grade/Course	**Number of Standards**
Advanced Mathematical Modeling	38
Calculus	23
High School Mathematics IV - Trigonometry/Pre-calculus	40
STEM Readiness	31
Transition Mathematics for Seniors	52

AP® Calculus
AP® Computer Science
AP® Statistics

Additional course options include dual credit mathematics courses and advanced mathematics courses offered through WV Virtual School. School teams, including counselors, teachers and administrators, should confer with the student and his/her parents to decide what fourth year mathematics course best meets the needs of the student.

Appendix F
West Virginia's Comprehensive Assessment System

West Virginia's Comprehensive Assessment System

A Comprehensive Assessment System

When some people hear the phrase "state assessment," they think of the test students have to take at the end of the school year. While summative tests are important and provide valuable information about student performance, a comprehensive assessment system goes well beyond the end-of-the-year test. In addition to the summative assessment given to students in the spring of the school year, West Virginia's Comprehensive Assessment System also includes access to interim and diagnostic assessments, as well as formative assessment resources in the Digital Library. All of these are critical to support meaningful instruction and help students in the successful demonstration of knowledge and skills based upon rigorous college and career readiness standards.

Summative Assessment

West Virginia's summative test is known as the West Virginia General Summative Assessment (WVGSA). The English language arts (ELA) and mathematics assessments are computer adaptive, which means each student's individual test adjusts based on how he or she responds. For example, if a student answers questions correctly, the next set of questions the student receives will be more difficult; if a student answers questions incorrectly, the next set of questions the student receives will be easier. A computer adaptive test allows a more precise, valid and reliable score for each student. A more detailed look at the WVGSA is provided on the next page. The state also administers an alternate summative assessments to students with significant cognitive disabilities. For more information about the alternate assessment, contact the Office of Assessment.

Interim/Diagnostic Assessments

The interim and diagnostic assessments are optional tests that allow educators to measure student progress throughout the year and support instruction of the standards. There are two types of interim assessments—the Interim Comprehensive Assessment (ICA) and the Interim Assessment Block (IAB). The ICAs mirror the end-of-year summative test. The item types and formats, including performance tasks, are similar to those students will encounter on the summative test. The IABs are shorter and focus on sets of targets. The diagnostic assessments were created by West Virginia teachers to focus on specific skills aligned to various targets.

Formative Assessment

Formative assessment is a deliberate, ongoing process used during instruction to ensure students are making progress toward specific learning goals by providing actionable feedback. Formative assessment resources are available in the Digital Library. These resources are designed to help teachers integrate formative assessment processes instructionally.

Reports

A variety of useful reports are available for the WVGSA, as well as the interim and diagnostic assessments. These reports provide valuable information to districts, schools, teachers, students and parents. The summative assessment reports include roster performance on each target and student performance on each claim and in each proficiency level. Students and parents receive individual student reports that indicate students' levels of performance. In the future, we expect to add a separate writing score report that will show how students scored on the following writing traits—purpose/organization, elaboration/evidence, and conventions. A variety reports also are available for the interim and diagnostic assessments, including item-level analysis reporting. West Virginia will continue to work toward improving reports so teachers and parents will have the information they need.

Measuring Critical Thinking

West Virginia's balanced assessment system measures critical thinking and higher-level problem solving skills through a variety of innovative test questions. Students are required to write persuasively and use evidence to support their answers. The state's assessments also measure research, listening, and communicating reasoning skills, which are not assessed by other tests.

Supports for All Students

West Virginia provides accommodations, designated supports and universal tools to ensure all students have access to the assessments. The state offers assessments in a variety of forms, including braille, large print and Spanish. For more options and information, see the West Virginia *Guidelines for Participation in Statewide Assessments* at the link at the bottom of this page.

For more information on West Virginia's statewide assessment system, call 304.558.2546 or visit the following website: http://wvde.state.wv.us/assessment/.

West Virginia General Summative Assessment (WVGSA)

Overview:
The West Virginia General Summative Assessment (WVGSA) is the state test administered at the end of each school year. Results from the test provide information about a student's academic strengths, as well as any areas that need improvement, in each assessed content area.

Test Length
The length of time a student spends taking the WVGSA depends on what grade the student is in and could be affected by the student's ability and effort. One advantage of the WVGSA is that it is an untimed test. As long as students are actively engaged in taking the test, students can take as much time as they need. An analysis of last year's test scores shows that the more time students spend working on the test, the better they perform. On average, each student will spend no more than a total of 6 to 8 hours taking the test with sessions spread over 4 to 5 days, depending on the school's testing schedule and the student's grade level. Each district sets its own overall testing window and school testing schedule.

Grades 3-8:
Students in Grades 3-8 take the English language arts (ELA) and mathematics tests. Students in Grades 4 and 6 also take the science test. The tests are aligned to the state-approved grade-level standards for each content area and provide teachers, students and parents with information on how well students are progressing toward being college and career ready when they graduate.

High School
Students in Grades 9-11 take the ELA and mathematics tests. Students in Grade 10 also take the science test. The tests are aligned to the state-approved grade-level high school standards for each content area. West Virginia also offers a Grade 12 College and Career Readiness Assessment, a retest of the Grade 11 WVGSA.

West Virginia students deserve to graduate prepared for the world that awaits them. High school is a critical time to ensure students are on track to graduate with the skills and knowledge to be college and career ready. To ensure students are making progress and meeting college and career readiness benchmarks, West Virginia administers ELA and math tests for Grades 9, 10 and 11 using a comprehensive high school item bank that includes thousands of possible questions written to various assessment targets and at various levels of difficulty.

Achievement levels have been established at each grade level so students and parents can see where students are performing based on grade-level expectations.

- **Grade 9:**
  Students are tested in ELA and math. Students get information about their progress based on West Virginia's 9th grade ELA and math achievement levels. The ELA test is aligned to specific ELA content and targets that span the breadth of high school ELA standards. The math test is aligned to the specific math content students should know by the end of their 9th grade year.

- **Grade 10:**
  Students are tested in ELA, math and science. Students get information about their progress based on West Virginia's 10th grade ELA, math and science achievement levels. The ELA test is aligned to specific ELA content and targets that span the breadth of high school ELA standards. The math test is aligned to the specific math content students should know by the end of their 10th grade year.

- **Grade 11:**
  Students are tested in ELA and math on the high school standards and get information about their progress based on 11th grade achievement levels. If students score at a 3 or higher in ELA or math, West Virginia colleges and universities recognize the student is prepared for credit-bearing courses; thus, they can skip remedial courses and enroll in credit bearing classes. This saves students time and money. Additionally, the student's scores help schools determine whether students would benefit from placement in Grade 12 transition courses to help them achieve college and career readiness.

- **Grade 12 College and Career Readiness Assessment:**
  Seniors enrolled in a Grade 12 transition course will take the Grade 12 College and Career Readiness Assessment (CCRA) unless they can provide evidence they have earned an acceptable benchmark on the Grade 11 WVGSA, ACT or SAT. Students only have to take the content area test for which they do not have a college and career ready indicator. Seniors not enrolled in a Grade 12 transition course have the option to take the Grade 12 CCRA. They can use their ACT or SAT scores to show college readiness.

Appendix G

A SNAPSHOT of Assessments and Assessment Processes for West Virginia Schools

	FORMATIVE ASSESSMENT PROCESS *(occurs daily in grades Pre-K-12; is a fundamental component of high-quality teaching and learning)*	**INTERIM/ DIAGNOSTIC ASSESSMENTS** *(occur periodically in grades Pre-K-12; are optional)*	**STATE SUMMATIVE ASSESSMENT** *(occurs yearly in grades 3-8 and grade 11 in English language arts and mathematics, and in science in grades 5,8,and 10)*
What is it?	A daily process teachers and students use that links evidence of learning to standards in order to personalize learning for all students. (Evidence of learning can include work samples, observations, anecdotal information, graded work, etc…)	Non-secure assessments used to obtain data educators can use to help identify: » strengths and weaknesses of their classes and individual students » necessary adjustments to instruction	A standardized test designed to provide a snapshot of student progress toward college and career readiness in the tested content areas
Who selects the assessment?	Is a teacher-driven process; not an isolated event	Educators	State
Who participates in it?	All educators and students in grades Pre-K-12	Students in grades Pre-K-12	All students in grades 3-8 and grade 11
When does it occur?	Daily, during high-quality instruction; the formative assessment process is NOT an event	Periodically, throughout the school year as applicable	At the end of the year or at the end of a course of study
What is done with the results?	Evidence of learning is collected and discussed by teachers and students; evidence is organized in a way that helps teachers tailor their instruction and articulate learning to families	Districts, schools, and educators use results to evaluate student achievement and learning	Long-range planning based on results can occur at the district or state levels; used in state accountability system
How much time does the assessment take?	Is an ongoing, daily process teachers use to personalize learning for all students	1 hour average	4.5 hour average for the WV General Summative Assessment (average across all grades levels and includes ELA, math, and science)

West Virginia Department of EDUCATION

Appendix H
Overview of the West Virginia TREE (Teacher Resources for Educational Excellence)

WEST VIRGINIA **TREE**
one voice. one focus. one stop.

https://wvde.state.wv.us/apps/tree/

West Virginia's online platform for educators is a one stop, grade- and/or content-specific site highlighting WV content standards, resources, and links that are essential to ensure high-quality educational programming. The resources include grade specific lessons, professional learning, and guidance documents crafted to help enhance teaching practice and guide the classroom teacher in the art of teaching. The links connect teachers with information regarding:

- Grade- and/or content-specific content standards, linked to resources to support use
- College and career readiness in West Virginia
- The formative assessment process
- Summative assessment login and resources (grades 3-12)
- Opportunities for professional learning
- Working with children with special needs
- Educator effectiveness and licensure (certification and evaluation)
- Guidance documents
- Programmatic level foundations for learning
- Additional resources

The WV TREE is designed with the teacher's busy schedule in mind, one stop, one focus, and tailored for the professional educator. This 'one stop' ensures teachers will not have to scour the WVDE website to find needed resources.

The WV TREE is a fluid website, with resources and content added on a regular basis. Additionally, future plans for the TREE include a site specific to principals, county chief instructional leaders, as well as counselors.

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