West Virginia
math4life

Educators' Guide for Mathematics

Algebra 1

West Virginia DEPARTMENT OF
EDUCATION

Algebra 1

The primary goal of Algebra I is to help students understand linear, quadratic, and exponential functions. They learn to analyze relationships and apply linear models to data with a linear trend. Algebra I focuses on Data & Statistics, Linear Functions, Exponential Functions, and Quadratic Functions. The standards are organized into subcategories including Sequences, Multiple Representations, Heart of Algebra and Systems. These concepts should be integrated throughout the school year for a comprehensive learning experience.

Mathematical Representations

Contextual: contextual representations create connections between mathematics concepts and the real world.

Physical: physical representations allow students to use tangible objects or manipulatives to represent mathematics concepts or contexts.

Visual: visual representations can be drawn to help students understand a mathematical concept.

Symbolic: symbolic representations connect mathematics concepts to the mathematics language used to represent them including the variable, equation, table, graph, or any other symbols used to represent a mathematical concept.

Verbal: verbal representations are descriptions of student thinking about mathematical concepts, contexts, connections, notices, wonders, or other ideas.

It is important to note that the representations provided in this document should be used as examples of what could be used to teach these Algebra 1 standards in the classroom. When possible, teachers should use concrete objects to help students create physical representations and visualize problems. Teachers should use this guide as inspiration to be creative when teaching Algebra 1 concepts.

Connecting Mathematical Habits of Mind and Content

The Mathematical Habits of Mind (**MHM**) apply throughout each course and, together with the content standards, prescribe that students experience mathematics as a coherent, relevant, and meaningful subject. The MHM represent a picture of what it looks like for students to understand and do mathematics; therefore, to the extent possible, content instruction includes attention to appropriate practice standards. Students are afforded ample opportunities to engage in each Mathematical Habit of Mind in Algebra I. The following table offers some general examples.

Mathematical Habits of Mind Algebra I

Mathematical Habits of Mind	**Explanation and Examples**
MHM1 Make sense of problems and persevere in solving them.	Be patient and persistent when tackling math problems. Understand the problem fully, identify relevant information, and distinguish it from unnecessary details. Explore different methods to solve problems effectively.
MHM2 Reason abstractly and quantitatively.	Think about numbers beyond their values. For example, understand slope not just as a steepness measure but as the rate of change in a linear function. Apply this concept to any function and calculate average rates of change over specific intervals.
MHM3 Construct viable arguments and critique the reasoning of others.	Develop logical and well-reasoned arguments when solving equations. Avoid blindly following rules but rather explain the reasoning behind your solution steps using phrases like "If this happens, then that must be true." Critique others' approaches constructively.
MHM4 Model with mathematics.	Use math to explore real-world situations. Analyze data patterns and apply mathematical concepts like exponential, linear, and quadratic functions to solve practical problems.
MHM5 Use appropriate tools strategically.	Utilize appropriate tools like graphing calculators or software to represent equations and functions visually. Create complex graphs to interpret results effectively. Draw diagrams when solving problems to enhance understanding.
MHM6 Attend to precision.	Be precise and clear in mathematical definitions and concepts. Understand the precision necessary for the context of the problem. Determine if an equation represents a function correctly by ensuring each input has a unique output.
MHM7 Look for and make use of structure.	Discover patterns and structures in math. For example, derive formulas like $(a \pm b)^2 = a^2 \pm 2ab + b^2$ by applying the distributive property. Understand how certain expressions take specific forms and use them to draw conclusions.
MHM8 Look for and express regularity in repeated reasoning.	Recognize key features of lines in a plane, such as slopes and intercepts. Grasp how these features influence the behavior and characteristics of linear functions. This idea of finding patterns and using them to explain what's happening is an essential skill in math!

Category: Data and Statistics

In Algebra I, standards **M.A1HS.32**– **M.A1HS.34** build on students' understanding of key ideas for describing distributions (**shape, center, and spread**) and types of data displays presented in the standards for Grades 6 through 8. Students are now expected to compare data sets, using calculations or data displays, and/or determine the best measure of center/spread to describe data.

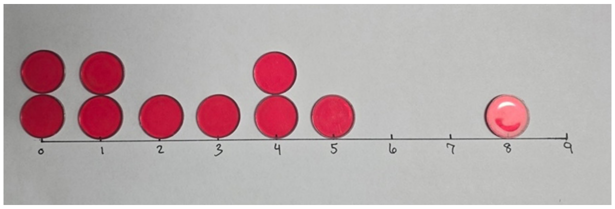
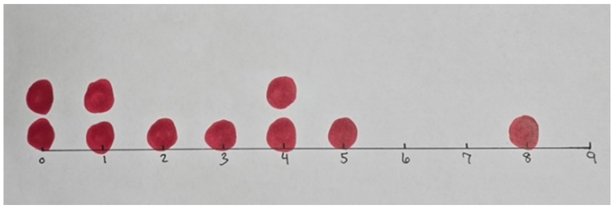
Generally, students consider variation, as indicated by **standard deviation**, **interquartile range (IQR)**, and the question(s) asked to decide on the **median** or **mean** as the more appropriate measure of center. Students should justify their choice through statistical reasoning.

Students' understanding of standards **M.A1HS.32** – **M.A1HS.34** can be called upon for deeper understanding of **M.A1HS.35** and in Algebra II/Math III when statistics is revisited.

Statistics and Probability
Summarize, represent, and interpret data on a single count or measurement variable. **M.A1HS.32** Select applicable representations to display data on the real number line (e.g., dot plots, histograms, and box plots). **M.A1HS.33** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation only as a tool to describe spread and not to explicitly find standard deviation) of two or more different data sets. **M.A1HS.34** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).

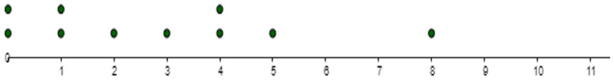
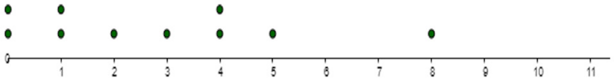
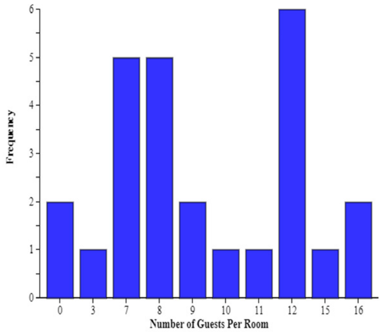
Representations:

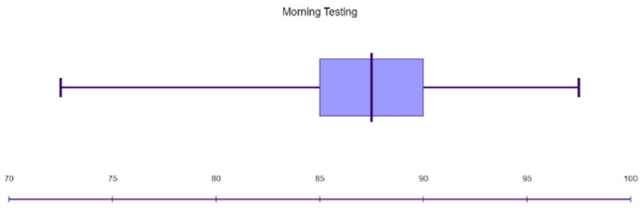
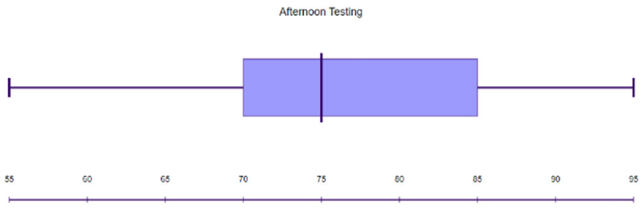
Contextual: A curious math student took a sample of 10 teachers and asked them how many siblings they have. Here are the results (**M.A1HS.32**).

Physical: Students use bingo counters to represent data in a dot plot.  	**Visual:** Students draw the dot plot they created by hand or using technology. 
Symbolic: Students analyze the list of data values collected from the sample of 10 teachers. 0, 0, 1, 1, 2, 3, 4, 4, 5, 8	**Verbal:** Students describe the method used to create the dot plot using appropriate vocabulary for the center and spread. "I put two dots above 4 because there were two teachers with 4 siblings." "The median number of siblings for the teachers asked is 2.5 because there are 5 dots above 2.5 and 5 dots below 2.5".

Examples:

As students complete these examples, special attention must be given to reasoning, both abstractly and quantitatively as the students sort through different data displays (**MHM2**). While answering the questions, students will construct viable arguments to defend their claims (**MHM3**). To allow students to fully embrace **MHM3**, give students the opportunity to critique the reasoning of other students in the class.

Example Problems	**Answers**
1. A curious math student took a sample of 10 teachers and asked them how many siblings they have. Here are the results.  a. What is the mean number of siblings for the sample of teachers? b. What is the median number of siblings for the sample of teachers? c. The teacher with 8 siblings actually had 16 siblings. If we were to correct this in the data set, how would that affect the mean and the median? (**M. A1HS.33**) Adapted from a lesson in *https://mathmedic.com/*	a. 2.8 b. 2.5 c. Mean would increase to 3.6 and median would stay the same
2. Leo owns multiple vacation rental properties and counted the number of guests staying in each one. He then created both a bar graph and a dot plot to display the same data: (**M.A1HS.32**) a. Which display (bar graph/dot plot) can be used to find which were without guests? b. Which display (bar graph/dot plot) makes it easier to see clusters and gaps in the data?   	a. dot plot b. dot plot

Example Problems	Answers
3. The two boxplots below are comparing scores on math tests taken in the morning and taken in the afternoon to see if there is any significant difference. (**M.A1HS.33-34**) a. Which group (Morning/Afternoon) had a larger median score? b. Which group (Morning/Afternoon) had a larger interquartile range?  	a. Morning b. Afternoon

Misconceptions:

- Students often use the word "outlier" inaccurately, failing to verify that it satisfies the necessary conditions (1.5 standard deviations from the mean). Using terms such as unusual feature or data point can help students avoid using the term outlier when it isn't appropriate.
- Students commonly believe that any data that is collected should follow a normal distribution/normal curve.
- Students need to understand that association does not necessarily provide evidence for cause and effect.
- Students often judge variability by focusing on heights of bars on histograms, implying the variability in the frequencies, rather than variation in the data values.

Applications:

- Personal Finance - Students will create a fictional personal budget based on a part-time job or allowance scenario. Over a simulated month, they'll track income and expenses in categories like food, transportation, entertainment, and savings. Students will:
  - Provide a sample income (e.g., $100 / week).
  - Allocate spending in a variety of categories and record their simulated weekly spending data.
  - Use dot plots and histograms to represent total weekly spending.
  - Calculate mean, median, mode, and range for weekly savings or total spending.
  - Write a reflection detailing whether their spending is consistent, if there are outliers and how they affect summary statistics, and any other observations they can infer from the data collected.
- Sports Statistics - Students will choose a player (basketball, soccer, baseball, etc.) or be assigned one. Using real stats from a recent season, they'll analyze the athlete's performance over time. Students will:
  - Research their player and gather data.
  - Create a dot plot or line plot of their chosen statistic over time.
  - Calculate measures of center and spread. Discuss the differences between those measures and why one might be preferable when evaluating an athlete's performance.
  - Identify outliers and how they might impact interpretations of the data.
  - Gather data on a second comparable player and write a discussion comparing the two. Which one had the better season and why?
- Health and Wellness Tracking - Students will track a health metric of their choice for 5 -7 days (e.g., number of steps, hours of sleep, screen time, water intake). Students will:
  - Choose a variable to track and collect daily data at school or at home.
  - Create a line plot or histogram of their data.
  - Calculate mean, median, and range.
  - Reflect on what the data says about their habits. Are there patterns or inconsistencies? Alternatively, the data could be collected for the whole class leading to a larger pool of data and different types of discussion.
- Environmental Data - Students will explore local temperature and/or rainfall data to gather daily high temperatures for 30 days. Students will:
  - Find a source for weather related data in their area.
  - Represent the data using a box plot and histogram.
  - Analyze range, interquartile range, and any outliers. Reflect on whether the weather was consistent. Discuss a 'typical' day for the observed period.
  - Gather the same data for a different city and explain the differences in variability and center.
- Polls and Surveys - Students will design a one-question survey with a numerical response and collect data from classmates or the school community. Some examples might be: hours of sleep on school nights, text messages sent in a day, times you eat fast food in a week, etc. Students will:
  - Create and administer the survey (easiest to use something like an online platform).
  - Compile and organize their data.
  - Represent the data using dot plots and box plots.
  - Calculate mean, median, and spread.
  - Share findings and reflect on what the data says.

Career Connections:

Understanding and interpreting data is central to many career paths. Below are a few key fields that heavily rely on skills introduced in this unit:

- Healthcare Professions (e.g., Nurse, Doctor, Public Health Analyst)
  - Nurses and doctors track patient vitals over time (heart rate, blood pressure, temperature) and must interpret the data trends to determine treatment responses.
  - Public health officials use statistical data to track disease outbreaks, monitor vaccination rates, and communicate health risks.
- Marketing and Business Analysts
  - Marketers collect consumer data and interpret graphs showing engagement, sales, and other trends. Knowing how to choose and interpret data displays helps them make data-driven business decisions.
  - Business analysts use measures of center and spread to evaluate performance metrics like customer satisfaction or delivery times.
- Urban Planning and Civil Engineering
  - Planners and engineers analyze traffic data, utility usage, or population growth to guide infrastructure design. Understanding distributions, variability, and patterns is essential.
- Sports Analytics
  - Sports analysts and scouts interpret performance data to compare athletes, make predictions, and build strategies. Data displays help identify consistent players vs those with outlier performances.
- Social Sciences (e.g., Psychologists, Sociologists)
  - Social scientists collect data from surveys or experiments and use visual tools to spot patterns in human behavior. Summary statistics are used to draw conclusions and write reports.

Category: Linear Functions

Subcategory: Sequences – Linear

Sequences are tools for understanding patterns and predicting future values based on known trends or relationships. Sequences are an example of functions and often use **function notation**, f(n), where n represents the number of the term in the sequence (**M.A1HS.19-21**).

Students have worked with patterns as early as kindergarten and have continued this work in subsequent years. In Algebra I, students will extend and formalize their earlier understanding of patterns to both arithmetic and geometric sequences (**M.A1HS.26-27**); this subcategory will focus on **arithmetic sequences** since they are a model of linear functions.

Function notation (**M.A1HS.20**) will be introduced in this subcategory and will be further explored in the next subcategory. Use of sequences and function notation will continue to be developed in future math courses, including Algebra II/Math III.

Functions

Understand the concept of a function and use function notation.

M.A1HS.19

Use multiple representations of linear and exponential functions to recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Develop function notation utilizing the definition of a function to represent situations both algebraically and graphically.

M.A1HS.20

Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context.

M.A1HS.21

Recognize arithmetic and geometric sequences are functions, sometimes defined recursively, whose domain is a subset of the integers (e.g., the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ *for* $n \geq 1$).

Build a function that models a relationship between two quantities.

M.A1HS.26

Write linear, exponential, and quadratic functions that describe a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations.

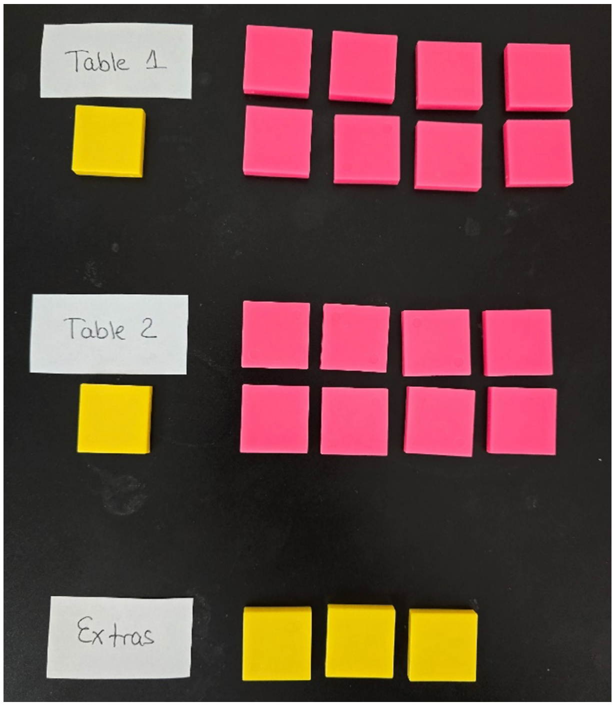
M.A1HS.27

Construct linear and exponential functions, including arithmetic and geometric sequences to model situations, given a graph, a description of a relationship or given input-output pairs (include reading these from a table).

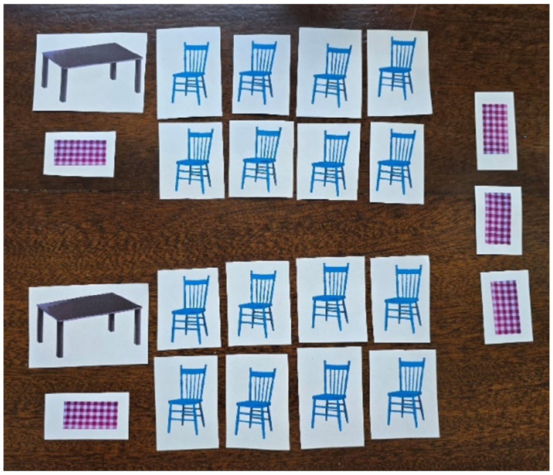
Representations:

Contextual: Callie's Catering Company prepares for an event by setting up tables and chairs. Each table is set with 8 chairs. Callie always brings 3 extra tablecloths than what she needs for the tables (**M.A1HS.20-21**).

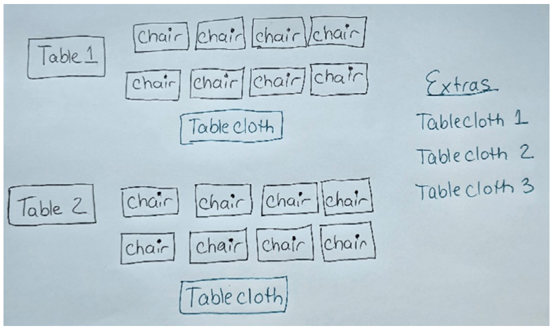
Physical: Students use manipulatives to represent the data for any given number of tables.



Visual: Students draw the data they organized for any given number of tables.



Or



Symbolic: Students create a table for the data.

Tables	Chairs	Tablecloths
1	8	4
2	16	5
3	24	6
4	32	7
5	40	8
6	48	9

Verbal: Students describe patterns they see in the data.

"For two tables, I counted 16 chairs because there are 8 chairs for each table. Callie would need 2 table cloths for 2 tables, plus 3 extra table cloths. So, she needs 5 table cloths if there are 2 tables."

Examples:

As students evaluate the given sequences, looking for and making use of the structure will prove to be beneficial in answering the questions that follow correctly (**MHM7**). After finding the correct pattern, students may look for and express regularity in repeated reasoning (**MHM8**) as they use this new knowledge to complete the remainder of the questions.

Example Problems

Callie's Catering Company prepares for an event by setting up tables and chairs. Each table is set with 8 chairs. Callie always brings 3 extra tablecloths than what she needs for the tables.

1. Callie needs to make a spreadsheet so that her crew will know what to bring to each event. Help her fill out the missing information.

Tables	Chairs	Tablecloths
1		
2		
3		
4		
5		
6		

2. What patterns do you notice in the table?
3. If Callie brings 11 tables, how many chairs will she bring? How many tablecloths will she bring?
4. If Callie brings 120 chairs, how many tables will she bring? How many tablecloths?
5. If Callie brings 16 tablecloths, how many tables will she bring? How many chairs?
6. Write an equation that shows the relationship between the number of tables, *t*, and the number of chairs, *c*.
7. Write an equation that shows the relationship between the number of tables, *t*, and the number of tablecloths, *b*.
8. Write an equation that shows the relationship between the number of tablecloths, *b*, and the number of chairs, *c*. (**M.A1HS.20-21, 26**)

Source: Medic, M. (n.d.). Math Medic Teacher Portal. *https://portal.mathmedic.com/lesson-plans/course/Algebra-1/unit/1/day/2*

Answers

1.

Tables	Chairs	Tablecloths
1	8	4
2	16	5
3	24	6
4	32	7
5	40	8
6	48	9

2. Answers may vary; but students should notice at minimum that the number of chairs increase by 8 each time and the number of tablecloths increase by 1. They may also notice that the number of tables times 8 is the number of chairs and the number of tables plus 3 is the number of tablecloths.
3. 88 chairs; 14 tablecloths
4. 15 tables; 18 tablecloths
5. 13 tables; 104 chairs
6. $c = 8t$
7. $b = t + 3$
8. $\frac{c}{8} = t + 3$

<table>
<tr><th>Example Problems</th><th>Answers</th></tr>
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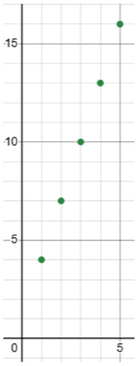
<td>
Consider the sequence 4, 7, 10, 13, 16
1. What is the pattern of this sequence?
Now, assume 4 is the first term of the sequence.
2. Make a table illustrating the sequence.
3. Make a graph illustrating the sequence.
4. What is a recursive rule for this sequence?
5. What is the explicit formula for this sequence?
(M.A1HS.27)
</td>
<td>
1. “Plus 3”; common difference of 3
2.
<table>
<tr><td>1</td><td>4</td></tr>
<tr><td>2</td><td>7</td></tr>
<tr><td>3</td><td>10</td></tr>
<tr><td>4</td><td>13</td></tr>
<tr><td>5</td><td>16</td></tr>
</table>
3.


4. $f(1) = 4, f(n + 1) = f(n) + 3.$
5. $f(n) = 3n + 1$
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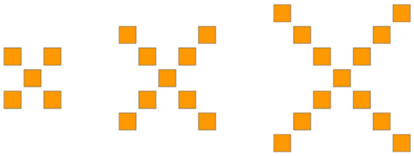
<td>

Potential questions to ask:
1. Will any of the figures have 75 squares?
2. Is the number of squares in each figure growing at a constant rate? How could you describe what the 10th figure would look like so someone else could draw it?
3. Can the number of squares ever be exactly 15 times greater than the figure number?
4. Will any figures have a prime number of squares?
5. What is the function for the relationship between the figure number and the number of squares?
(M.A1HS.27)
Source: Patterns 1 through 50 — Visual Patterns. (n.d.). Visual Patterns. <i>https://www.visualpatterns.org/patterns/1-through-50</i>
</td>
<td>
1. No, because the figure is adding 4 each time after starting with figure 1 having 5 squares and 70 (75 – 5 original) is not a multiple of 4.
2. Yes; Potential Answer for part 2: 4 squares are being added each time, one to each end of the diagonals. The 10th figure would have a total of 41 squares. There would be one square in the center and 10 squares in a diagonal off each of the four corners.
3. No, because using the function in #5, and setting $f(n)=15n$ gives
$$15n = 4(n - 1) + 5$$
$$15n = 4n - 4 + 5$$
$$11n = 1$$
$$n = 1/11$$
and n must be a whole number.
4. Yes, figures 1, 2, and 3 all have prime numbers of squares. In addition, figure 4 would also have a prime number of squares. A better question may be: “<i>Will any figures have a composite number of squares?</i>”
5. $f(n) = 4(n - 1) + 5$ where n is the figure number and f(n) is the number of squares.
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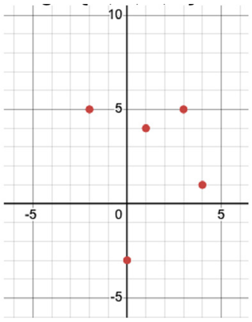
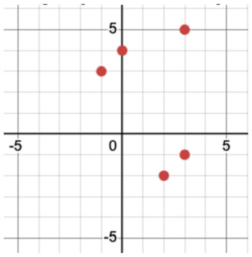
<table>
<tr><th>Example Problems</th><th>Answers</th></tr>
<tr>
<td>
Determine if the given tables represent a function. Justify your answer by graphing and stating the domain and range. (M.A1HS.19)
1.
<table>
<tr><td>x</td><td>-2</td><td>0</td><td>1</td><td>3</td><td>4</td></tr>
<tr><td>y</td><td>5</td><td>-3</td><td>4</td><td>5</td><td>1</td></tr>
</table>
2.
<table>
<tr><td>x</td><td>-1</td><td>0</td><td>2</td><td>3</td><td>3</td></tr>
<tr><td>y</td><td>3</td><td>4</td><td>-2</td><td>-1</td><td>5</td></tr>
</table>
</td>
<td>
1. Function
Domain: {-2, 0, 1, 3, 4}
Range: {-3, 1, 4, 5}


2. Not a function
Domain: {-1, 0, 2, 3}
Range: {-2, -1, 3, 4, 5}


</td>
</tr>
</table>

Misconceptions:

- Misinterpreting the meaning of "*f(x)*": Some students may believe that "*f(x)*" means multiplying *f* and *x* instead of realizing that it denotes the output of the function f when the input is *x*.
- Students often believe that all functions must use the symbols *f(x)* or *y*.
- Students may mistakenly treat the function name (e.g., *f*(*x*)) as a variable rather than understanding that it represents an entire rule or process that relates inputs (*x*-values) to outputs (*y*-values).

Applications:

The following are some sample application scenarios that could be used with students to reinforce sequences and function notation:

- Weekly Savings Plan - Students will model a simple savings plan over time using an arithmetic sequence. Provide the students with the scenario where they will start saving $10 in Week 1 and increase their deposit by $5 each week.
- Social Media Followers - Students will analyze and extend a pattern representing a steady weekly gain in followers. Provide the students with the scenario where a small business gains 120 followers each week starting from 0.
- Stadium Steps - Students will examine a pattern based on stair height and seating levels Provide the students with the information that each step in a stadium rises 7 inches above the one below it. The first step is 14 inches above ground level.

Once a scenario is chosen, the students will then:

- Write out the first ten terms in the sequence.
- Identify the first term and the common difference.
- Create a table comparing the two quantities from the scenario.
- Write an explicit formula using function notation.
- Use the new formula to predict the values beyond the table.

Career Connections:

Understanding sequences prepares students for careers that require identifying patterns, projecting outcomes, and making decisions based on steady rates of change. Here are several career paths where these skills are directly applied:

- Financial and Investment Advisors
    - Financial professionals help clients build savings plans that grow steadily over time through regular deposits. These plans often model arithmetic sequences, where the same amount is added each month.
    - Advisors use explicit formulas to calculate how long it will take to reach savings goals and help clients understand predictable growth over time.
- Construction and Civil Engineering
    - Engineers and construction professionals use arithmetic sequences to plan spacing, height, and material requirements — such as steps in a staircase, layers in a wall, or rows of support beams.
    - They also use function notation and arithmetic modeling to estimate material needs and costs when structures increase uniformly.
- Social Media Managers and Content Creators
    - Digital content professionals track followers, views, or likes that often grow steadily over time. Arithmetic sequences help them model growth, predict trends, and set content release goals.
    - Function notation is used to represent patterns in audience engagement and analyze when major growth milestones will be reached.
- Event Planners and Venue Designers
    - Event planners and stage designers organize spaces (e.g., chairs, tables, seating rows) that often follow linear patterns. For example, adding 10 seats per row in a venue layout forms an arithmetic sequence.
    - They use these patterns to predict space requirements and ensure capacity goals are met for audiences or attendees.
- Personal Trainers and Physical Therapists
    - Fitness professionals use arithmetic sequences when designing progressive workout plans. Clients might increase weights or reps by the same amount each session.
    - Therapists use these sequences to gradually build strength or range of motion and track progress over time using function-based models.
- Small Business Owners and Freelancers
    - Entrepreneurs often charge flat rates per item or service, forming a linear pricing structure. Arithmetic sequences help them project profits, manage inventory, and understand growth.
    - Function notation allows for modeling income over time and setting realistic sales targets based on repeatable patterns.

Subcategory: Multiple Representations – Linear (Graphing)

At the heart of graphing equations and inequalities lies a fundamental big idea in mathematics: the graphical representation of mathematical relationships and solution sets. The standards, **M.A1HS.16** and **M.A1HS.18**, encompass both representing and solving equations and inequalities graphically. When dealing with linear equations in two variables, the graph visually represents all the solutions on the coordinate plane (**M.A1HS.16**), connecting algebraic concepts to geometric representations. Additionally, graphing linear inequalities involves shading the region that satisfies the inequality, illustrating it as a half-plane, and graphing the solution set to a system of linear inequalities as the intersection of the corresponding half-planes (**M.A1HS.18**). Understanding this big idea enables students to interpret mathematical relationships visually, enhancing problem-solving skills, and fostering a deeper appreciation of the interplay between algebra and geometry.

Students in 6th grade begin representing and analyzing quantitative relationships between dependent and independent variables. In Algebra II/Math III, students will interpret functions that arise in applications in terms of a context, analyze functions using different representations, and build new functions from existing functions.

Expressions and Equations

Represent and solve equations and inequalities graphically.

M.A1HS.16

Recognize that the graph of a linear or exponential equation in two variables is the set of all its solutions plotted in the coordinate plane.

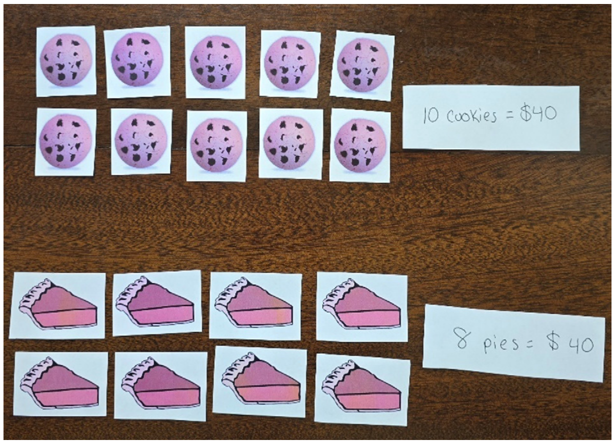
M.A1HS.18

Graph the solutions of a linear inequality in two variables as a half-plane and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

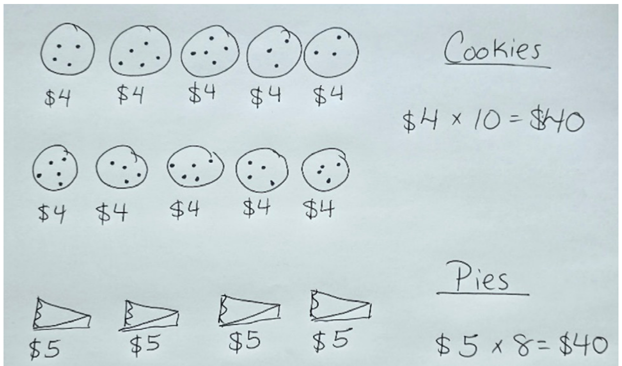
Representations:

Contextual: The track team is having a bake sale. They sell cookies for $4 and pies for $5. Their goal is to make at least $40 in the first hour. What is the minimum number of each baked good needed to be sold to reach their goal (**M.A.1HS.18**)?

Physical: Students use manipulatives to represent the minimum number of cookies or pies that would earn $40. This is one example – students may create variations of cookie and pie combinations.



Visual: Students draw the minimum number of cookies or pies that earn $40.



Symbolic: Students create a table and/or equation by hand or using technology for the data they calculated for the minimum number of cookies or pies and all possible combinations to earn a minimum of $40.

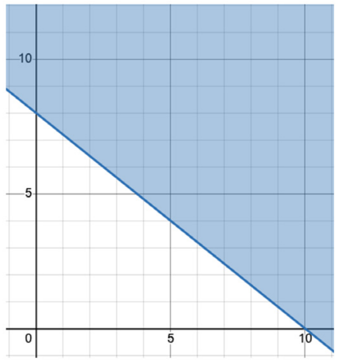
Equation: $4x + 5y \geq 40$, where x is the number of cookies sold and y is the number of pies sold.

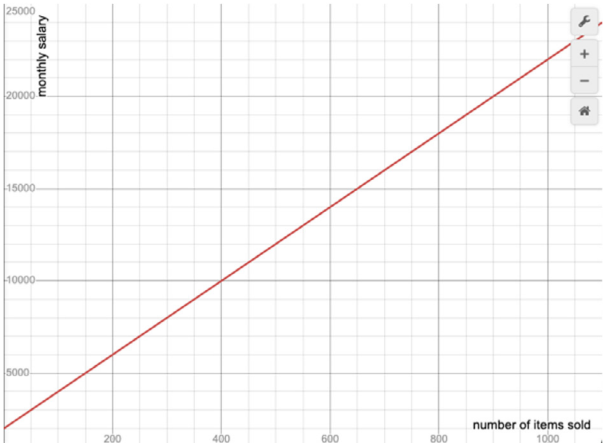
Verbal: Students describe patterns they see in the data.

"I noticed that the *x* intercept is *10* and the *y* intercept is *8* which are the minimum number of cookies or pies, respectively, needed to make $40."

Examples:

When students consider the following example, patience and persistence will allow students to correctly make sense of the problem and persevere in solving it (**MHM1**). Students are given the opportunity to model real-world mathematics (**MHM4**) to bring the content to life with connections to their own lives. To elevate this example even further, allow students the opportunity to choose tools strategically, such as a graphing calculator, to evaluate the problem (**MHM5**).

Example Problems	**Answers**
1. The track team is having a bake sale. They are selling cookies for $4 and pies for $5. Their goal is to make at least $40 in the first hour. How many of each baked good would they need to sell to reach their goal? (**M.A1HS.18**)	1. $4x+5y\geq40$ where x = number of cookies sold and y = number of pies sold Solving for y gives: $5y \geq 40 - 4x$ $y \geq 8 - \frac{4}{5}x$ Since the inequality is greater than or equal to, we will graph a solid line with y-intercept of 8, slope of $-\frac{4}{5}$ and shading above the line. Thinking of the context of the question, we can ignore all but quadrant I since we cannot sell negative numbers of cookies or pies. 

Example Problems	**Answers**
2. Sarah is considering a job that offers her a base salary of $2,000 per month plus $20 for each item she sells. Write and graph the equation that represents Sarah's monthly income (y), where x is the number of items she sells. (**M.A1HS.16**)	
3. You can work a total of no more than 20 hours each week at your two jobs. Housecleaning pays $20 per hour and your sales job pays $10 per hour. You need to earn at least $200 each week to pay your bills. Write and graph a system of inequalities that shows the various number of hours you can work at each job. (**M.A1HS.18**)	3. Let x be the number of hours housecleaning and y be the number of hours working the sales job. Then, $x + y \le 20$ $20x + 10y \ge 200$ Solving each inequality for y gives, $y \le 20 - x$ $y \ge 20 - 2x$ When graphing, we shade below the first (red) line and above the second (blue) line. The solution set is where the shading overlaps. Since the context involves numbers of hours worked at various locations, we can ignore any solutions outside of quadrant 1. 

Misconceptions:

- Ignoring extraneous solutions: When graphing equations that relate to real world contexts, students might forget to consider restrictions or domain limitations, leading to solutions that do not make sense in the given context.
- Not recognizing all solutions: Students might believe that only the points on the graph are solutions, overlooking the possibility of solutions that fall between grid points (for example, fractional solutions or irrational solutions).

Applications:

- Road Trip Budgeting – Students will use graphs of linear equations and inequalities to model travel expenses and budget constraints. Scenario: A student is planning a road trip and wants to track how far they can travel based on gas money. They have a budget of $120 for gas and their car gets 30 miles per gallon with gas at $4/gallon. Students will:
  - Define variables: x = number of gallons, y = miles driven.
  - Write a linear equation: $y = 30x$ and a linear inequality: $4x \leq 120$.
  - Graph the equation and shade the region that represents possible mileage under budget.
  - Interpret the graph:
    - "What's the maximum distance they can travel on $120?"
    - "Could they go 1,000 miles?"
- Cell Phone Plan Constraints - Students will model a cell phone plan that charges for data usage, then graph allowable usage under a fixed monthly budget. Scenario: A student has a $60 budget for data. Their plan charges $10 for a base fee and $5 per GB of data used. Students will:
  - Write a linear inequality: $10 + 5x \leq 60$, where $x = GB$ of data used.
  - Define the variable y, and rearrange into slope-intercept form: $y = 5x + 10$.
  - Then solve and graph the inequality, $y > 5x + 10$.
  - Identify the maximum number of GB of data used the student can afford.
  - Reflect: "What does the shaded region mean in this context?"
- Job Offer Comparison - Students will graph and compare two job options using systems of inequalities. Scenario: Job A: $15/hour, up to 25 hours/week and Job B: $12/hour, up to 35 hours/week Students will:
  - Write inequalities representing both jobs.
  - Graph the inequalities and identify overlapping regions (feasible work schedules).
  - Determine which job pays more for different hour ranges.
  - Analyze: "What hours maximize earnings under each job's restrictions?"
- Club Fundraiser - Students will model item sales for a fundraiser and explore constraints on inventory and revenue. Scenario: A club sells two types of snacks — brownies ($2 each) and cookies ($1.50 each). They have a total of 100 items to sell and want to raise at least $160. Students will:
  - Define variables.
  - Write inequalities.
  - Graph both inequalities and shade the feasible region.
  - Use the graph to identify sales combinations that meet the goal.
  - Reflect: "What combinations maximize sales and meet the target?"

Career Connections:

- Urban Planners and Civil Engineers
    - Urban planners use linear graphs to model population growth, traffic flow, and infrastructure needs over time.
    - Civil engineers use inequalities to model physical constraints (e.g., maximum load, land usage boundaries) and identify feasible solutions when designing roads, bridges, or buildings.
- Financial Analysts and Accountants
    - Financial professionals graph earnings, spending, and budget projections over time.
    - They use inequalities to represent budget limits or target earnings and analyze when revenues meet or exceed goals.
- Marketing and Sales Professionals
    - Marketers graph sales data and advertising impact to analyze trends and predict outcomes.
    - They use linear models to determine when investments in advertising or pricing strategies pay off and use inequalities to stay within advertising budgets or sales thresholds.
- Environmental Scientists
    - Environmental researchers use graphs to monitor data like CO_2 levels, pollution output, or deforestation rates over time.
    - They use inequalities to define safe vs. dangerous levels and systems of constraints to model multiple environmental factors at once.
- Logistics and Supply Chain Managers
    - Logistics professionals graph fuel costs, delivery time, and distance relationships to optimize routing.
    - Systems of inequalities help them plan efficient shipping schedules while staying within budget, time, and vehicle capacity constraints.
- Software Developers/Game Designers
    - Developers use linear functions to model animations, player stats, or progression curves in games.
    - They apply inequality logic to limit user input ranges, determine success/failure conditions, or calculate in-game currency thresholds.
- Architects and Designers
    - Architects use linear functions to scale designs and model elements such as lighting angles or elevation changes.
    - Inequalities represent restrictions such as height codes, space limitations, and budget caps when working on project proposals.

Subcategory: Multiple Representations – Linear Interpretation

These standards form comprehensive understanding and analysis of linear functions through various representations. Students are guided to interpret functions arising in real-world applications by exploring key features of their graphs and table, including **intercepts, intervals of increase and decrease, positivity or negativity, and end behavior (M.A1HS.22-23)**. They learn to relate the **domain** of a linear function to its graph and the quantitative relationship it describes. Additionally, students are encouraged to graph linear functions, paying particular attention to intercepts and other crucial features. They analyze linear functions expressed symbolically and recognize the effect of shifts and scaling on their graphs (**M.A1HS.28**). Furthermore, students compare properties of two linear functions represented differently, whether algebraically, graphically, numerically, or verbally (**M.A1HS.24**). By developing a deeper appreciation for linear functions and their versatile representations, students enhance their problem-solving skills and apply their knowledge in diverse mathematical contexts. Students' ability to apply their knowledge should be demonstrated as they determine situations that would have a linear relationship based on a scenario's constant rate of change (**M.A1HS.29**).

Foundational skills built in this subcategory will be revisited with exponential and quadratic functions later in this course and will be extended to any function in future math courses.

Functions

Interpret functions that arise in applications in terms of a context.

M.A1HS.22

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of quantities, and sketch graphs showing key features given a verbal description of the relationship. Relate the domain of a function to its linear, exponential, and quadratic graphs and, where applicable, to the quantitative relationship it describes.

a. Key features of linear and exponential graphs include: intercepts; and intervals where the function is increasing, decreasing, positive, or negative.
b. Key features of quadratic graphs include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximum or minimum; symmetry; and end behavior.

Analyze functions using different representations.

M.A1HS.23

Graph linear, exponential, and quadratic functions expressed symbolically and show key features of the graph.

a. For linear functions, focus on intercepts.
b. For exponential functions, focus on intercepts and end behavior.
c. For quadratic functions, focus on intercepts, maxima, minima, end behavior, and the relationship between coefficients and roots to represent in factored form.

Instructional Note: Provide opportunities for students to graph and show key features by hand and using technology.

Functions (Continued)

M.A1HS.24

Compare properties of two linear, exponential, or quadratic functions each represented in a different way, such as algebraically, graphically, numerically in tables, or from verbal descriptions.

M.A1HS.25

Write a function defined by a linear, exponential, or quadratic expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square for $a = 1$ only in a quadratic function to show zeros, extreme values, symmetry of the graph, the relationship between coefficients and roots represented in factored form and interpret these in terms of a context.
b. Use the properties of exponents to interpret expressions in exponential functions.

Build new functions from existing functions.

M.A1HS.28

Identify the effect on the graphs of linear and exponential functions, $f(x)$, with $f(x) + k$, and the graphs of quadratic functions, $g(x)$, with $g(x) + k$, $k\,g(x)$, $g(kx)$, and $g(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Construct and compare linear, quadratic and exponential models and solve problems.

M.A1HS.29

Distinguish between situations that can be modeled with linear functions, with exponential functions, and with quadratic functions.

a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
d. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. Extend the comparison of linear and exponential growth to quadratic growth.

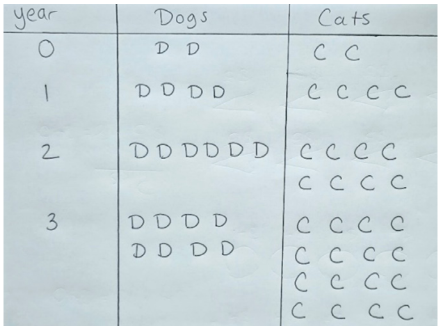
Representations:

Contextual: Tristan has two dogs and two cats. Each year, one of his dogs has two puppies. His cat population doubles each year. How many dogs and cats will he have in 3 years (**M.A.1HS.29**)?

Physical: Students use manipulatives to represent the data pattern for each population over 3 years.

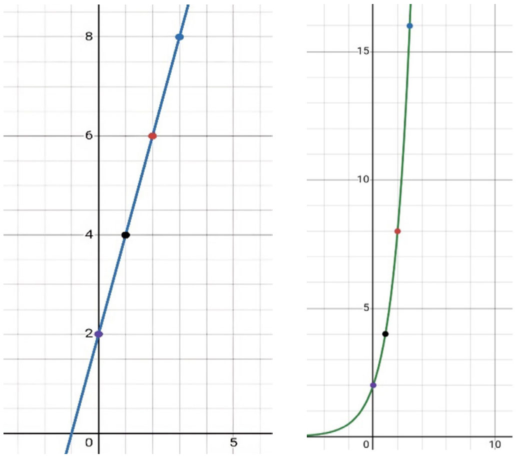
Visual: Students draw or graph the data pattern for each population over 3 years

Or



year	Dogs	Cats
0	D D	C C
1	D D D D	C C C C
2	D D D D D D	C C C C C C C C
3	D D D D D D D D	C C C C C C C C C C C C C C C C

Or



Representations:

Symbolic: Students create an equation and/or table by hand or using technology for the data they calculated for each population over 3 years.

Dogs

Equation: $y = 2x + 2$

Year	Dogs
0	2
1	4
2	6
3	8

Cats

Equation: $y = (2)2^x$

Year	Dogs
0	2
1	4
2	8
3	16

Verbal: Students describe patterns they see in the data.

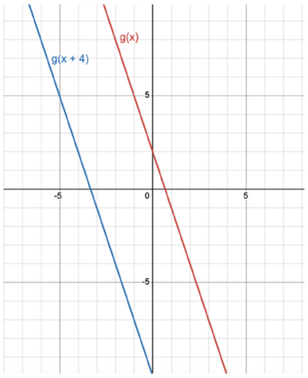
"I drew a line to represent the linear pattern of the dog population increasing by 2 dogs each year."

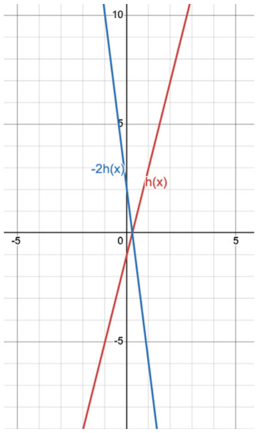
"The cat population increases more rapidly than the dog population."

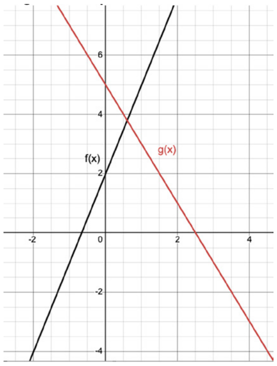
"I drew a curve to represent the exponential pattern of the cat population doubling each year."

Examples:

As students work with contextual values, it is important to attend to precision (**MHM6**). Students will need to see numbers beyond their values in order to reason abstractly and quantitatively (**MHM2**) while completing the example. Identification of key features and expressing regularity in repeated reasoning (**MHM8**) will allow students the best opportunity to correctly identify the correct response.

<table>
<tr><th>Example Problems</th><th>Answers</th></tr>
<tr><td>1. Tristan has two dogs and two cats. Each year, one of his dogs has two puppies. His cat population doubles each year. How many dogs and cats will he have in 4 years? (M.A1HS.29)</td><td>1. Let $d(x)$ = number of dogs, $c(x)$ = number of cats, and x = number of years. Then,
<table>
<tr><th>x</th><th>$d(x) = x+2$</th><th>$c(x) = 2 \cdot 2^x$</th></tr>
<tr><td>0</td><td>2</td><td>2</td></tr>
<tr><td>1</td><td>4</td><td>4</td></tr>
<tr><td>2</td><td>6</td><td>8</td></tr>
<tr><td>3</td><td>8</td><td>16</td></tr>
<tr><td>4</td><td>10</td><td>32</td></tr>
</table>
Tristan will have 10 dogs and 32 cats in 4 years.</td></tr>
<tr><td>2. A taxi company charges a base fare of $5 plus $0.75 for each mile traveled. Write the linear function representing the total cost of a taxi ride in terms of the distance traveled, in miles. Interpret the slope, <i>y</i>-intercept, and the meaning of the coefficients. (M.A1HS.22)</td><td>2. The total cost, $C(m)$, in terms of m miles, can be modeled as
$$C(m) = 0.75m + 5$$
The slope of 0.75 means that for every mile traveled, the cost increases by $0.75. This is also the coefficient of the independent variable, m.
The y-intercept of 5 is the initial cost for getting into the taxi without driving any miles. This is also the constant coefficient in the function.</td></tr>
<tr><td>3. Consider the linear function $g(x) = -3x + 2$. Describe the effect on the graph when $g(x)$ is transformed to $g(x + 4)$. Illustrate and explain the effect using technology. (M.A1HS.28)</td><td>3. The graph of $g(x)$ is translated to the left 4 units with the transformation of $g(x + 4)$.

</td></tr>
</table>

Example Problems	Answers
4. Investigate the impact on the graph of the linear function $h(x) = 4x - 1$ when $h(x)$ is transformed to $-2h(x)$. Use technology to illustrate and explain the effects. (**M.A1HS.28**)	4. The graph of $h(x)$ is reflected across the y-axis and has a vertical stretch of a factor of 2 with the transformations $-2h(x)$. 

<table>
<tr><th>Example Problems</th><th>Answers</th></tr>
<tr>
<td>5. Compare the linear functions <i>f(x) = 3x + 2</i> and <i>g(x) = -2x + 5</i> algebraically, graphically, and numerically. Discuss how different representations highlight different aspects of their contexts. (M.A1HS.24)</td>
<td>5. Algebraically, we see that <i>f(x) = 3x + 2</i> has a smaller <i>y</i>-intercept than <i>g(x) = -2x + 5</i> since the constant of 2 in <i>f(x)</i> is smaller than the constant of 5 in <i>g(x)</i>.
Graphically, we see that <i>f(x)</i> is increasing since it has a positive slope, but <i>g(x)</i> is decreasing with a negative slope.


Numerically, when looking at the table of values for each function, we see that <i>f(x)</i> is getting more positive faster than <i>g(x)</i> is getting more negative. This is because the absolute value of the average rate of change for <i>f(x)</i> is larger than the absolute value of the average rate of change for <i>g(x)</i>.
<table>
<tr><th>x</th><th>f(x)</th></tr>
<tr><td>-2</td><td>-4</td></tr>
<tr><td>-1</td><td>-1</td></tr>
<tr><td>0</td><td>2</td></tr>
<tr><td>1</td><td>5</td></tr>
<tr><td>2</td><td>8</td></tr>
</table>
<table>
<tr><th>x</th><th>g(x)</th></tr>
<tr><td>-2</td><td>9</td></tr>
<tr><td>-1</td><td>7</td></tr>
<tr><td>0</td><td>5</td></tr>
<tr><td>1</td><td>3</td></tr>
<tr><td>2</td><td>1</td></tr>
</table>
</td>
</tr>
</table>

<table>
<tr><th>Example Problems</th><th>Answers</th></tr>
<tr><td>6. The distance <i>D(t)</i>, in miles, traveled by a car in <i>t</i> hours is given in the table below.
<table>
<tr><th>t</th><th>D(t)</th></tr>
<tr><td>1</td><td>60</td></tr>
<tr><td>2</td><td>120</td></tr>
<tr><td>3</td><td>180</td></tr>
<tr><td>4</td><td>240</td></tr>
</table>
Determine whether this situation can be modeled by a linear, exponential, or quadratic function. Justify your choice and discuss growth characteristics. (M.A1HS.29)</td><td>6. This situation is best modeled by a linear function because as the input values (hours) are increasing by 1, the output values are increasing by a constant rate of 60 miles.</td></tr>
</table>

Misconceptions:

- Overlooking domain and range: Students may overlook the relationship between the domain of a function and its graph, especially for exponential and quadratic functions where the domain might have restrictions.
- Confusing equivalent forms: Students may struggle to write functions in different but equivalent forms to reveal various properties of the function, such as converting a quadratic function from standard form to factored form.
- Inappropriate function selection: Students may have difficulty distinguishing between real-world situations that can be modeled with linear, exponential, or quadratic functions, leading to incorrect choices of functions to represent these scenarios.

Applications:

- Part-Time Job Earnings - Students will model and interpret a part-time job's weekly earnings, represented through multiple formats (table, graph, and equation). Scenario: A student earns $14/hour and works up to 20 hours per week. Students will:
    - Create a table of values for weekly earnings based on hours worked.
    - Write the equation: $E(x) = 14x$, where $E(x)$ represents earnings and x is the number of hours worked.
    - Graph the equation and identify the intercept (0,0) and slope (14).
    - Interpret key features: Slope = rate of pay, Positive interval = entire domain, Meaning of intercept (0 [hours worked], [$]0 [earned])
    - Answer: “How much will you earn if you work 12.5 hours?” and “What does a point like (10, 140) represent?”
- Water Tank Fill-Up - Students will interpret linear models describing how quickly a tank fills over time. Scenario: A tank fills at a rate of 6 gallons per minute. It starts empty. Students will:
    - Create a table for the first 10 minutes.
    - Write a function describing the situation (e.g., $W(t) = 6t$), and graph it.
    - Interpret the slope and y-intercept.
    - Sketch the graph and identify increasing intervals.
    - Compare it to a second tank that starts with 12 gallons and fills at 4 gallons/min:
    - Compare the functions, graphs, intercepts, and rates.
    - Answer: “When will both tanks contain the same amount of water?”

- Amusement Park Ride Fees - Students will interpret and compare linear cost plans. Scenario: Amusement park offers two ride-pass options: Plan A: $15 entry + $3 per ride, Plan B: $5 entry + $5 per ride. Students will:
    - Create tables and equations for both plans.
    - Graph both functions on the same coordinate plane.
    - Identify: Intercepts (fixed cost), Slope (cost per ride), Break-even point (intersection).
    - Answer: “Which plan is better for 4 rides?” and “At what number of rides is Plan A cheaper?”
- Temperature Conversion - Students interpret a linear function relating Celsius and Fahrenheit. Given: $F = (9/5)\,C + 32$ Students will:
    - Create a table and graph for -20°C to 40°C.
    - Identify key features: y-intercept (freezing point), slope (rate of change), intervals of increase.
    - Answer: “What’s the Fahrenheit equivalent of 20°C?” and “At what point is Fahrenheit equal to Celsius?”
- Bus Route Schedule - Students model and interpret the travel time of a city bus route. Scenario: A bus departs at 7:00 a.m. and stops every 10 minutes at a new location along its route. Students will:
    - Generate a function to model the situation (e.g., $T(x) = 7 + 10x$, where $T(x)$ is the time and x = number stops).
    - Create a schedule table showing arrival times at each stop.
    - Graph the function (using time on the y-axis).
    - Answer: “What time will the bus arrive at the 5th stop?” and “How many stops before 8:00 a.m.?”

Career Connections:

- Financial Planners and Budget Analysts
    - Financial professionals create and interpret graphs and tables showing income, expenses, and savings over time.
    - They use key features of linear graphs (slope = savings rate, intercepts = initial balance or breakeven point) to advise clients and model future financial scenarios.
- Data Analysts and Business Intelligence Specialists
    - Analysts regularly compare data from spreadsheets, graphs, and equations to identify trends and patterns.
    - They interpret slope and intercepts to explain growth or decline in customer engagement, sales, or website traffic, and determine when key benchmarks will be met.
- Engineers and Technicians
    - Engineers graph data from experiments or performance tests and analyze trends to assess efficiency, speed, or strength.
    - Linear models are used to describe constant rates of change, such as temperature, pressure, or voltage over time.
- Transportation and Logistics Coordinators
    - Professionals in logistics use linear models to track delivery times, route lengths, and fuel usage.
    - They compare different transportation options by analyzing graphs and tables to optimize schedules and minimize costs.

- Meteorologists and Environmental Scientists
    - Meteorologists graph weather data (temperature changes, rainfall, wind speed) and interpret key features to describe trends and make predictions.
    - Environmental scientists use graphs to model pollution levels or population growth and determine whether values are increasing, decreasing, or approaching dangerous thresholds.
- Educators and Education Researchers
    - Educators use data from assessments and surveys to track student progress.
    - They compare scores over time using tables and graphs, interpreting slope as growth rate and intercepts as baseline performance.

Subcategory - Linear Regression

One application of linear functions is **linear regression** which is explored through this subset of standards (**M.A1HS.35-37**) that summarize, represent, and interpret **bivariate data** or data that relates two variables. Students learn to create **scatterplots** to display the relationship between two **quantitative variables** and describe the nature of their **correlation** (**M.A1HS.35**). Students determine a function that best fits the data with the support of technology, with particular emphasis on linear and exponential functions. Students then utilize these fitted functions to solve problems relevant to the data's context including interpreting its **rate of change** and **constant term** to support understanding (M.A1HS.36). Students also gain insight into assessing the fit of a function by analyzing **residuals**, with a focus on situations where linear models are appropriate (**M.A1HS.35**).

Students gain a vital understanding of the distinction between **correlation** and **causation** (**M.A1HS.37**), emphasizing the importance of interpreting data carefully and avoiding making erroneous assumptions about cause-and-effect relationships. By mastering these skills, students develop a deeper appreciation for data literacy, allowing them to make informed decisions and draw meaningful conclusions from real-world data.

Statistics and Probability

Summarize, represent, and interpret data on two categorical and quantitative variables.

M.A1HS.35

Represent data on two quantitative variables on a scatter plot and describe how the variables are related.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models.
b. Informally assess the fit of a function by plotting and analyzing residuals. Focus should be on situations for which linear models are appropriate.
c. Fit a linear function for scatter plots that suggest a linear association.

Interpret linear models.

M.A1HS.36

Interpret the rate of change and the constant term of a linear model in the context of the data. Use technology to compute and interpret the correlation coefficient of a linear fit.

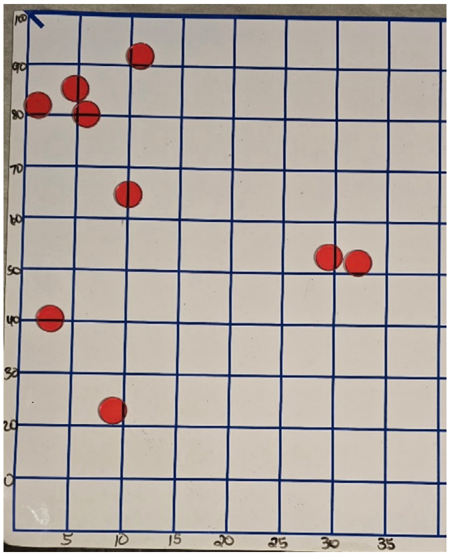
M.A1HS.37

Distinguish between correlation and causation.

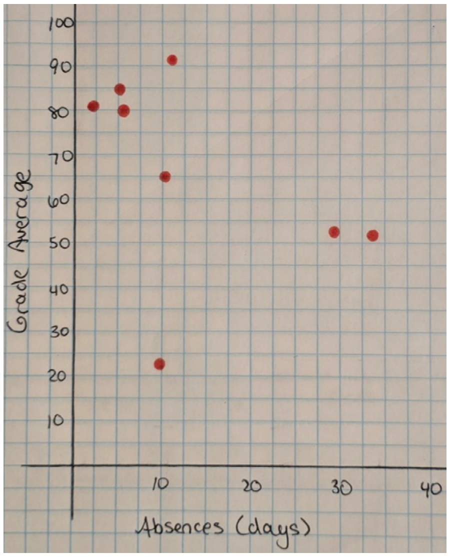
Representations:

Contextual: Ms. Thomas's math class collected data to see if there was a relationship between student attendance and final grades for the course (**M.A1HS.35**). Students missed the following numbers of days: 2, 33, 5, 11, 29, 6, 10, 9. The following student averages corresponded to attendance respectively: 81, 52, 85, 92, 53, 80, 65, 23).

Physical: Students use manipulatives on a large graph to plot the data points using the x variable (attendance) and corresponding y variable (student average).



Visual: Students draw or graph the data.



Symbolic: Students create a table and/or equation by hand or using technology for the data they gathered.

Days Absent	Student Average
2	81
33	52
5	85
11	92
29	53
6	80
10	65
9	23

Verbal: Students describe patterns they see in the data.

"The student averages decrease as the number of absences increase."

"The data pattern is negative and linear."

"I see one outlier."

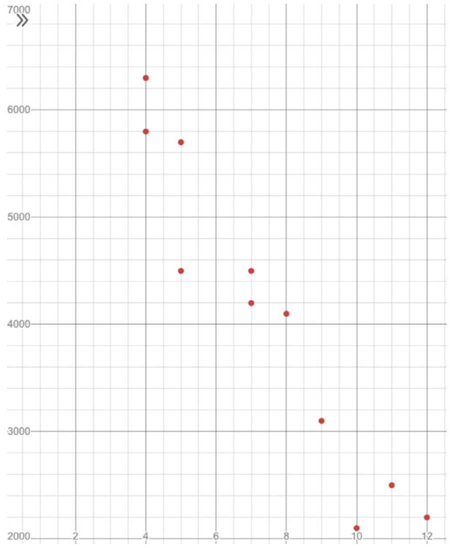
Examples:

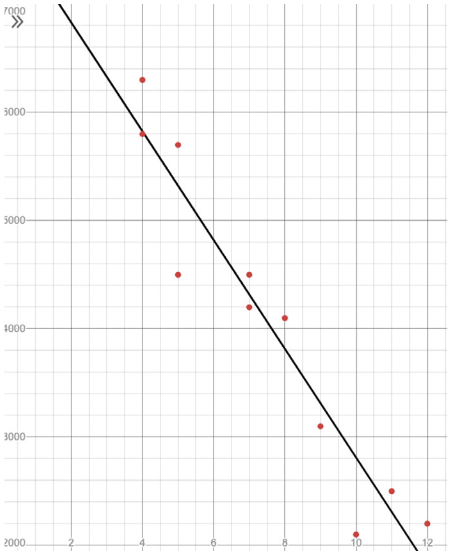
Students are given the opportunity to model real-life situations in mathematics (**MHM4**) when completing the following examples. Context outside the classroom allows students the opportunity to have a more meaningful learning experience. When asked to use technology, students are able to practice use with appropriate tools (**MHM5**), providing skills necessary for life after high school. Students are also asked to create viable arguments and justify their reasoning (**MHM3**) to solidify their knowledge of these concepts.

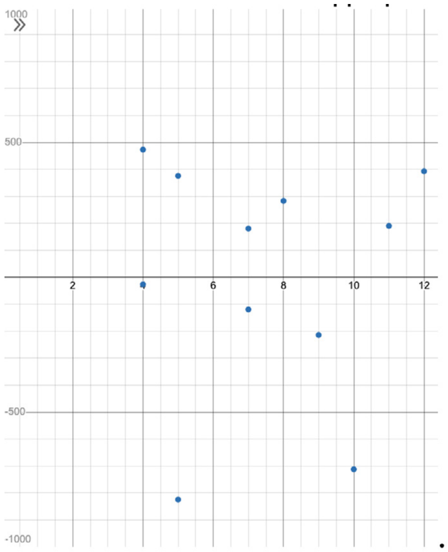
Example Problems	**Answers**

Is there a relationship between the age of cars and their sale prices? The data for used cars sold at a car dealership for the last year are included in the following table.

Car Age (in years)	4	4	5	5	7	7	8	9	10	11	12
Selling Price (in dollars)	6300	5800	5700	4500	4500	4200	4100	3100	2100	2500	2200

1. Create a scatter plot of the data.	
2. Describe the correlation in the data if there is any. (**M.A1HS.37**)	2. There is a strong, negative correlation between the car age in years and the selling price of the car in dollars. This is because the points are decreasing in a linear fashion.

Example Problems	**Answers**
3. Find by hand or using technology the line of best fit for the data. (**M.A1HS.35**)	3. Line of best fit added using the table and linear regression features of technology tools, such as the Desmos graphing calculator or handheld graphing calculator. 
4. Find the equation for the line of best fit by hand or using technology. (**M.A1HS.35**)	4. $y = -502.42x + 7836.26$
5. Interpret the slope of the line of best fit equation in the context of the problem. (**M.A1HS.36**)	5. The slope of -502.42 means that for every one year the car age increases, the sales price of the car decreases by $502.42.
6. Can it be determined that the age of a car affects its sales price? Justify your response. (**M.A1HS.37**)	6. No, we cannot be certain that the age of the car affects the sales price even though the data appears to have a strong correlation. We do not have enough information regarding the cars that were sold or other conditions that might have influenced their sales price.

Example Problems	Answers
7. Is a linear model appropriate for the data? Justify your response. (**M.A1HS.35**)	7. Yes, by looking at the residual plot shown below, we see there are no apparent patterns within the residual plot. Therefore, we can conclude that the linear model is appropriate. 

Misconceptions:

- Confusing correlation with causation: Students may mistakenly think that correlation implies causation. Just because two variables are correlated (show a statistical relationship) does not mean that one causes the other.
- Overlooking data patterns: When representing data on scatter plots, students may overlook patterns or trends, leading to inaccurate interpretations of the relationship between the two quantitative variables.
- Misinterpreting rate of change: When interpreting linear models, students may misinterpret the rate of change (slope) or constant term (y-intercept) in the context of the data failing to relate these two elements to the context of the problem.

Applications:

See the Applications in the Data Statistics section on *page 7* of this document.

Career Connections:

See the Career Connections in the Data Statistics section on *page 8* of this document.

Subcategory: Heart of Algebra - Linear

The standards in this section emphasize the importance of interpreting and analyzing mathematical **expressions, equations, and inequalities** with real-world contexts (**M.A1HS.1**). Students look at equivalent forms of expressions and their components like **terms, factors, and coefficients** (**M.A1HS.5**). Students will create **equations and inequalities** to represent real-world situations and use them to solve problems (**M.A1HS.7-9**). Students will also simplify polynomial expressions using operations (**M.A1HS.6**). Lastly, students will rearrange equations or inequalities with more than one variable, often referred to as “literal equations” (**M.A1HS.10**). These literal equations serve as a real-world application for equations and inequalities when common formulas are introduced.

Students have prior math experience in grades 6-8 with creating, evaluating, and comparing equivalent expressions. Algebra I will build on this prior learning and these standards will help to build a foundation for understanding in future math courses.

Equations and Expressions

Interpret the structure of expressions and equations in terms of the context they model.

M.A1HS.1

Interpret linear, exponential, and quadratic expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
c. Interpret the parameters in a linear function or exponential function of the form f(x) = a*bx in terms of a context.

Write expressions in equivalent forms to solve problems.

M.A1HS.5

Choose and produce an equivalent form of linear, exponential, and quadratic expressions to reveal and explain properties of the quantity represented by the expression through connections to a graphical representation of the function.

a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression, when a = 1 only, to reveal the maximum or minimum value of the function it defines.
c. Use the properties of exponents to transform expressions in exponential functions. For example, the expression 1.15t can be rewritten as *(1.151/12)12t ≈ 1.01212t* to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Instructional Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.

Perform arithmetic operations on polynomials.

M.A1HS.6

Recognize that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Focus on linear or quadratic terms.

Equations and Expressions (Continued)

Create equations that describe numbers or relationships to create equations.

M.A1HS.7

Create equations and inequalities in one variable, representing linear and exponential relationships, and use them to solve problems. In the case of exponential equations, limit to situations with integer inputs.

M.A1HS.8

Create equations in two or more variables, representing linear and exponential relationships between quantities. In the case of exponential equations, limit to situations with integer inputs.

M.A1HS.9

Represent constraints by linear equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

Solve equations and inequalities in one variable.

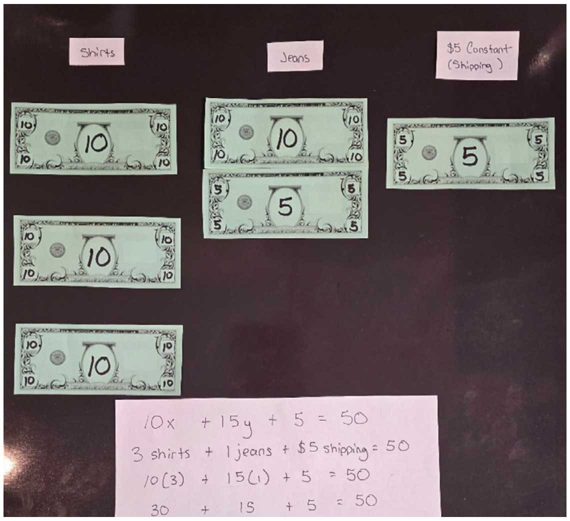
M.A1HS.10

Solve linear equations including equations with coefficients represented by letters, simple exponential equations that rely on application of the laws of exponents, and compound linear inequalities in one variable.

Representations:

Contextual: Online Clothes Inc. charges $10 per shirt, $15 per pair of jeans, and a $5 shipping fee per order. Kiya has a gift card for $50. Show one example of how Kiya might spend the gift card (**M.A1HS.9**).

Physical: Students use manipulatives to represent the data pattern.



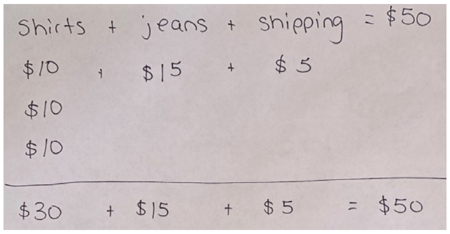
$10x + 15y + 5 = 50$

3 shirts + 1 jeans + $5 shipping = 50

$10(3) + 15(1) + 5 = 50$

$30 + 15 + 5 = 50$

Visual: Students draw or graph the data.



Shirts + jeans + shipping = $50

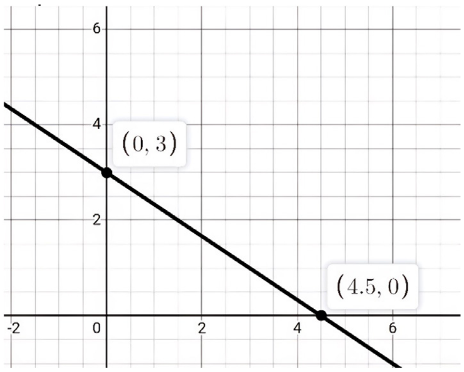
$10 + $15 + $5

$10

$10

$30 + $15 + $5 = $50

Graph



Symbolic: Students create a table and/or equation by hand or using technology for the data they gathered.

Equation: $10x + 15y = 45$ *or* $10x + 15y + 5 = 50$

Number of Shirts	Pairs of Jeans
0	3
1	2.33
2	1.67
3	1
4.5	0

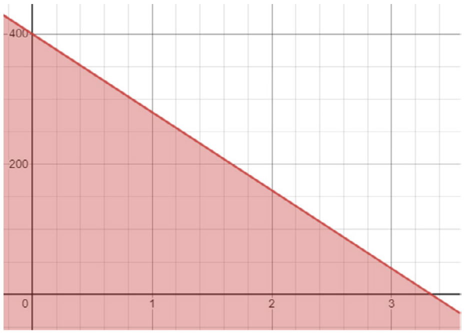
Verbal: Students describe patterns they see in the data.

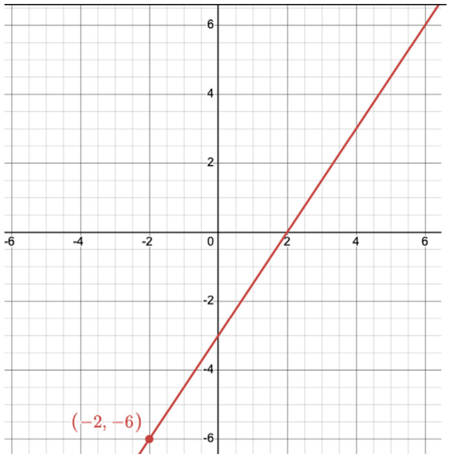
"As the number of shirts purchased increases, the number of pairs of jeans that Kiya can purchase decreases."

"The value of 4.5 for the number of shirts when zero jeans are bought is a non-viable option."

Examples:

Students are again given the opportunity to model with mathematics (**MHM4**) as they progress through the following examples. Appropriate tools (**MHM5**) pertaining to this problem might include graphing utilities, graphing software, or a hand-drawn graph. Additionally, students are given the opportunity to construct their own reasoning as it pertains to the calculations just completed in the problem (**MHM3**).

Example Problems	**Answers**
1. Consider the linear expression $h(t) = 5t + 10$, where h represents the height (in meters) of a ball thrown upward at time t (in seconds). Interpret the coefficient and constant term in the context of the problem. (**M.A1HS.1**)	1. The coefficient in the function, $h(t) = 5t + 10$, is 5, which represents the increase in height per second. The constant term is 10, which represents the starting height of the ball when $t = 0$ (the y intercept).
2. A car rental company charges a flat fee of \$30 per day plus an additional \$0.25 for every mile driven. A customer wants to rent a car for a few days but cannot spend more than \$100 on the rental. a. Write an inequality to represent the number of days "d" a car is rented and "m" miles driven. b. Can the customer afford to rent the car for 3 days and drive 150 miles? (**M.A1HS.8-9**)	2. a. $30d + .25m \leq 100$  b. $30(3) + .25(150) = 90 + 37.50 = \$127.50.$ \$127.50 is not less than \$100, so the customer cannot afford both a 3-day rental and driving 150 miles.

Example Problems	**Answers**
3. Find the slope, x-intercept, and y-intercept of the equation $y + 6 = \frac{3}{2}(x+2)$. Then, graph the equation. (**M.A1HS.5**)	3. Slope: given in point slope form equation as the coefficient of parentheses, or $3/2$. Intercepts: the y-intercept can be found by rewriting the equation of the line in slope-intercept form, $y = \frac{3}{2}x-3$, which shows the y-intercept to be -3 or $(0, -3)$. The x-intercept can be found by rewriting the equation of the line in standard form, $3x - 2y = 6$; then substituting 0 for y gives $3x = 6$ or $x = 2$, showing that the x-intercept is $(2, 0)$. 
4. Emma is saving money. She starts with \$80 and saves \$10 per week. How long will it take her to save \$180? (**M.A1HS.7**)	5. Let x = the number of weeks and E = Emma's total savings. Then, $E = 80 + 10x$ $180 = 80 + 10x$ $100 = 10x$ $10 = x$ Emma will have \$180 in 10 weeks.
6. Josie wants to buy a new phone for \$1,000. To save her money, she is investing \$800 into a savings account with a simple interest rate of 4%. How many years will it take for the interest to reach \$200 so Josie can pay for her phone? (**M.A1HS.10**)	6. The formula for simple interest is $I = prt$. Rearranging to solve for time gives $t = \frac{I}{pr}$. Substituting the values from the question gives $t = \frac{200}{(800)(0.04)} = 6.25$ It will take Josie 6.25 years to earn \$200 in interest (\$1,000 total) to pay for her new phone.
7. Rewrite the expression in standard form: $(4x - 3)(2x + 5)$. (**M.A1HS.6**)	$(4x - 3)(2x + 5) = 8x^2 + 14x - 15$

Misconceptions:

- One common misconception is treating linear inequalities as if they were linear equations. Students may forget that inequalities involve a range of solutions, not just a single value.
- Students may have difficulty understanding when to use ">" (greater than) or "<" (less than) symbols in an inequality. They may reverse the symbols when comparing values or notations in real-world problems.
- Students may have difficulty understanding that letters used as coefficients behave the same as if the coefficients were numbers. When rearranging formulas to solve for a variable of interest, it may help to practice rearranging equations with only one variable without simplifying to promote the skills needed when the coefficients are also letters.

Applications:

See the Sequences – Linear section on *page 13* in this document.

Career Connections:

See the Sequences – Linear section on *page 14* in this document.

Subcategory: Systems - Linear

This subcategory of standards guides students to graph more than one equation or inequality on a coordinate plane to solve problems or algebraically prove basic geometric theorems. Students will solve **systems of linear equations** using the graphing method with support of technology and should have an understanding that the point of intersection of two or more lines is the solution to the system (**M.A1HS.12,14,17**). This process can lead into proving the slope criteria for parallel and perpendicular lines since systems of equations with parallel lines will have no solution (**M.A1HS.30**). Graphing simultaneous linear equations can also connect to computing the **perimeters** of polygons and areas of triangles and rectangles, employing the **distance formula** to practice its connection with the Pythagorean theorem (**M.A1HS.31**).

Students will solve systems of linear equations through multiple methods- including the elimination, or manipulation, method (**M.A1HS.13**) and should be exposed to different solutions sets- one solution, no solution, or infinite solutions (**M.A1HS.14**). Using real world scenarios will require students to consider constraints and solutions that fit the given context (**M.A1HS.9**).

In grade 7 and 8, students use similar triangles to compare slopes which will be foundational for the algebraic and geometric ties explored in this subcategory of standards. These standards will also deepen students' work from grade 8 where they were introduced to graphing simultaneous linear equations. In future courses, students will use parallel and perpendicular lines to characterize polygons on a coordinate plane. They will also expand solving systems of equations to scenarios with more than two unknowns.

Expressions and Equations

Create equations that describe numbers or relationships.

M.A1HS.9

Represent constraints by linear equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

Solve systems of equations.

M.A1HS.12

Analyze and solve pairs of simultaneous linear equations.

a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.
b. Solve simple cases by inspection (e.g., $3x + 2y = 5$ *and* $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6).
c. Solve real-world and mathematical problems leading to two linear equations in two variables (e.g., given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair).

M.A1HS.13

Understand and demonstrate ways to manipulate a system of two equations and two variables while preserving its solution set.

M.A1HS.14

Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Include examples of solution sets with no solutions, an infinite number of solutions, and one solution.

Represent and solve equations and inequalities graphically.

M.A1HS.17

Explain why the x-coordinates of the points where the graphs of the linear and/or exponential equations $y = f(x)$ *and* $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations).

Geometry

Use coordinates to prove simple geometric theorems algebraically.

M.A1HS.30

Prove the slope criteria for parallel and perpendicular lines and use the slope criteria to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

M.A1HS.31

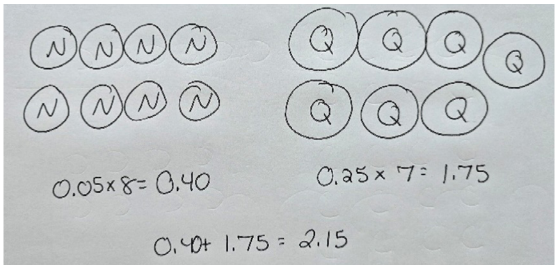
Use coordinates to compute perimeters of polygons and areas of triangles and rectangles. Instructional Note: Using the distance formula provides practice with the distance formula and its connection with the Pythagorean theorem.

Representations:

Contextual: There is a combined total of 15 nickels and quarters in a jar. There is \$2.15 in the jar. How many nickels and quarters are in the jar (**M.A1HS.14**)?

Physical: Students use manipulatives to represent the situation.

Visual: Students draw or graph the data.



Symbolic: Students create a table and/or equation by hand or using technology for their solution.

$N + Q = 15$

$0.05N + 0.25Q = 2.15$

Verbal: Students describe how they created the equations to represent each situation of the system or how they interpreted their answer on a graph.

"I knew that the number of nickels plus the number of quarters equaled a total of 15 coins."

"Each nickel is worth 5 cents and each quarter is worth 25 cents. Their combined total is \$2.15. So, I knew these monetary values needed to be in the same equation."

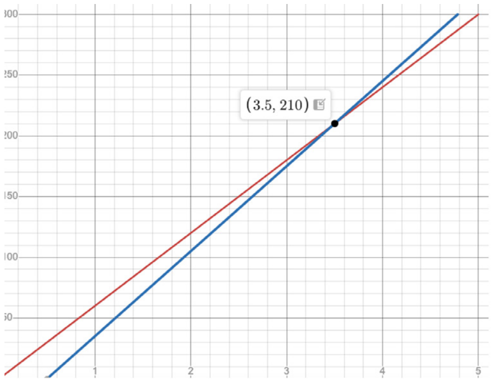
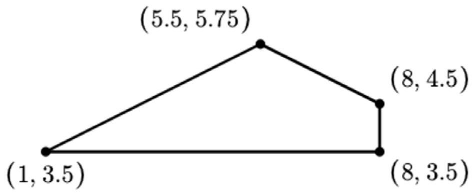
"I knew that the answer to the system was the coordinate (8,7) where the two lines intersected. So, the *x* value 8 meant there were 8 nickels and the *y* value 7 meant there were 7 quarters."

Examples:

Students are given the opportunity to apply the concepts of linear functions to solve a practical problem (**MHM4**) in the following examples. As students begin to recognize the structure necessary to correctly complete the problem, conclusions can be drawn from specific parts of each function (**MHM7**). Students are also given real world contexts in which they must make sense of (**MHM1**), convert the abstract thoughts and information into quantitative values (**MHM2**) and give answers to a level of precision necessary for the context (**MHM6**).

Example Problems	Answers
1. Some Washington High School students want to order shirts with their Patriot logo to wear to all sporting events. Company A charges \$9.75 per shirt plus a setup fee of \$40. Company B charges \$8.50 per shirt plus a \$55 fee. For what number of shirts would the cost be the same? **(M.A1HS.9, 12**)	1. Equation for Company A: $C = 9.75s + 40$ where C is the total cost and s is the number of shirts ordered. Equation for Company B: $C = 8.50s + 55$ where C is the total cost and s is the number of shirts ordered. Since we are looking for the shirt order where the costs are equal, one method would be substitution, where we substitute for C, so we set the two equations equal to each other and solve for s (the number of shirts). $9.75s + 40 = 8.50s + 55$ $9.75s = 8.50s + 15$ $1.25s = 15$ $s = 12$ If 12 shirts were ordered from either Company A or Company B, the cost would be \$157.00.

<table>
<tr><th>Example Problems</th><th>Answers</th></tr>
<tr><td>2. Samantha is moving from Madison, Wisconsin to La Crosse, Wisconsin. She will do all the packing and unpacking herself with her brother. The moving company quoted a price of $1250 for 8 hours of loading and unloading and driving 130 miles. The company quoted the same price if the truck drives an extra 30 miles to pick up Samantha’s brother. Samantha figures that with her brother’s help she only needs to hire the movers for 6 hours. How much does the company charge per hour for loading/unloading? How much do they charge for driving? (M.A1HS.13, 14)</td><td>2. Let x represent the hourly cost for loading and unloading. Let y represent the per-mile cost for renting the truck.
<table>
<tr><th></th><th>Mileage</th><th>Load/Unload Hours</th></tr>
<tr><td>Without brother</td><td>130</td><td>8</td></tr>
<tr><td>With brother</td><td>160</td><td>6</td></tr>
</table>
Write an equation that models each situation:
<i>labor cost+mileage cost=Total cost</i>
$8x + 130y = 1250$
$6x + 160y = 1250$
This system could be solved by graphing by hand, with technology, or by an algebraic method. To use elimination to solve this system, both of the equations need to be put into the same form of a line. This is one possible method:
Multiply the first equation by -3 and the second equation by +4
$-24x - 390y =- 3750$
$24x + 640y = 5000$
If the two equations are now combined, the x variable is eliminated because $-24x + 24x = 0x$.
$250y = 1250$ <i>and can now be solved for</i> y
$y = 5$ <i>indicating that the mileage rate is $5.00 per mile</i>
Substitute that value for y into either of the original equations to find the per hour labor cost.
$8x + 130(5) = 1250$
$8x + 650 = 1250$
$8x = 600$
$x=75$ <i>indicating that the hourly rate for labor is $75 per hour.</i>
Samantha pays $75 per hour for loading and unloading and $5 per mile for driving.</td></tr>
</table>

Example Problems	**Answers**
3. Jonathan and Susie are both driving the same route to the state track meet in Charleston. Jonathan leaves at 12:00 PM and drives at 60 mph. Susie leaves at 12:30 PM and drives at 70 mph. Will Susie pass Jonathan on the 300-mile drive? Justify your answer by graphing. (**M.A1HS.12, 14, 17**)	3. Jonathan's travel can be modeled by $y = 60x$ and Susie's travel can be modeled by $y = 70(x-0.5)$ where x is the number of hours since 12:00 PM and y is the number of miles driven. Graphing these equations gives the following:  The graphs intersect when $x = 3.5$. Therefore, Jonathan and Susie will be in the same place 3.5 hours after 12:00 PM (or 3:30 PM) and Susie will pass Jonathan before reaching their destination in Charleston because they have traveled less than 300 miles when they pass on the road.
4. Find the equation of a line passing through the point (6,-2) and perpendicular to the line $y = \frac{3}{2}x+4$. (**M.A1HS.30**)	4. Perpendicular slopes are opposite reciprocals, so the slope of the new line will be $-\frac{3}{2}$. Using point-slope form of a line, the equation could be $y + 2 = -\frac{3}{2}(x-6)$. To reinforce slope-intercept form, this equation could be converted to $y = -\frac{3}{2}x + 7$.
5. The polygon below is given with the indicated coordinates. Find the perimeter. (**M.A1HS.31**)  	5. The side from (1, 3.5) to (8, 3.5) is 7 units long. The side from (8, 3.5) to (8, 4.5) is 1 unit long. Using the distance formula, the side from (8, 4.5) to (5.5, 5.75) is 2.795 units long and the side from (5.5, 5.75 to (1, 3.5) is 5.031 units long. Adding these 4 sides together give a perimeter of 15.826 units.

Misconceptions:

- Misconception about slopes: Some students may mistakenly believe that the slope of a line is solely determined by the *y*-intercept. They might think that any two lines with the same *y*-intercept are parallel, which is not true, two lines are parallel if and only if they have the same slope.
- Inaccurate use of distance formula: While using the distance formula to calculate the distance between two points, students might forget to take the square root of the sum of squared differences which is grounded in an understanding of the Pythagorean Theorem.
- Misinterpretation of area and perimeter: Students may mix up the formulas for calculating the area and perimeter of various shapes without a firm mastery of the concepts.
- Misunderstanding of algebraic proofs: Students might struggle to understand the logic and steps involved in algebraic proofs for geometric theorems.

Applications:

See the Multiple Representations – Linear (Graphing) section on *page 19* in this document.

Career Connections:

See the Multiple Representations – Linear (Graphing) section on *page 20* in this document.

Category: Exponential Functions

Subcategory: Sequences - Exponential

Sequences are tools for understanding patterns and predicting future values based on known trends or relationships. Sequences are an example of functions and often use **function notation**, *f(n)*, where *n* represents the number of the term in the sequence and is an element of the **domain** (**M.A1HS.19-21**).

In Algebra I, students will extend and formalize their earlier understanding of patterns to both arithmetic and geometric sequences; this subcategory focuses on **geometric sequences** since they are a model of **exponential functions** (**M.A1HS.26-27**).

Students have experience with properties of exponents from grade 8 math courses. Students will continue work with exponential functions in Algebra II/Math III and beyond and will derive the formula for the sum of a finite geometric sequence and use the formula to solve problems.

Functions

Understand the concept of a function and use function notation.

M.A1HS.19

Use multiple representations of linear and exponential functions to recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. Develop function notation utilizing the definition of a function to represent situations both algebraically and graphically.

M.A1HS.20

Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context.

M.A1HS.21

Recognize arithmetic and geometric sequences are functions, sometimes defined recursively, whose domain is a subset of the integers (e.g., the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n + 1) = f(n) + f(n - 1)$ *for* $n \geq 1$).

Build a function that models a relationship between two quantities.

M.A1HS.26

Write linear, exponential, and quadratic functions that describe a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations.

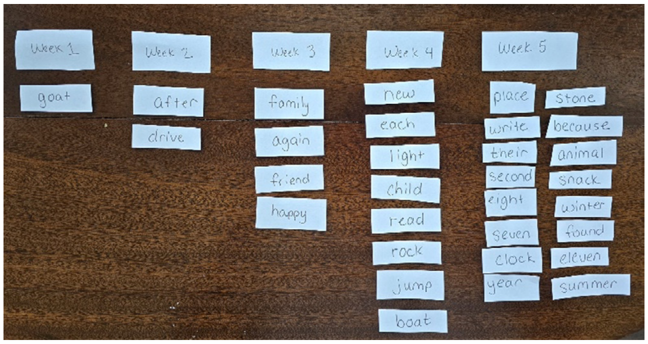
M.A1HS.27

Construct linear and exponential functions, including arithmetic and geometric sequences to model situations, given a graph, a description of a relationship or given input-output pairs (include reading these from a table).

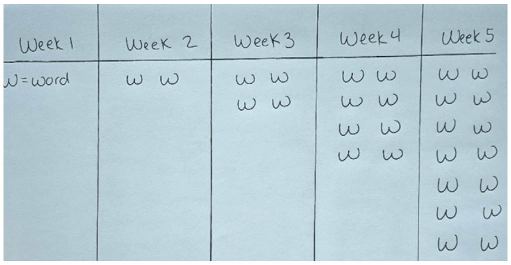
Representations:

Contextual: Studying spelling words is part of every student's homework in Cabell County. Each week, the second-grade teachers ask their students to double the number of spelling words that they study from the previous week. If in the first week of school, students are assigned 1 spelling word, how many spelling words will they be studying by week 5 (**M.A1HS.27**)?

Physical: Students use manipulatives to represent the data pattern.

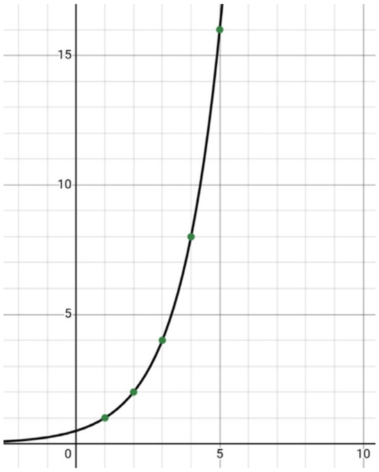


Visual: Students draw or graph the data.



Week 1	Week 2	Week 3	Week 4	Week 5
w = word	w w	w w w w	w w w w w w w w	w w w w w w w w w w w w w w

Graph



Symbolic: Students create a table and/or equation by hand or using technology for their solution.

Week	Number of Words
1	1
2	2
3	4
4	8
5	16

Verbal: Students describe patterns they see in the data.

"The number of spelling words increased rapidly by week 5."

"By week 7, there will be 64 words."

Examples:

Students should identify and use patterns in the structure of the following examples (**MHM7**) to draw conclusions about the questions being asked. Utilizing repeated reasoning (**MHM8**) will allow for quicker and more efficient work as the students progress through the examples. Being able to form viable arguments and critique the reasoning of others (**MHM3**) will be needed to be successful with these examples.

<table>
<tr><th>Example Problems</th><th>Answers</th></tr>
<tr><td>1. Studying spelling words is part of every student’s homework in Cabell County. Each week, the second-grade teachers ask their students to double the number of spelling words that they study from the previous week. If in the first week of school, students are assigned 1 spelling word, how many spelling words will they be studying by week 4? Write a function to model this situation. (M.A1HS.26-27)</td><td>1. Let S be the number of spelling words studied in w week. Then,
$S(w) = 2^{w-1}$
Substituting in w=4 (for 4 weeks) gives
$S(4) = 2^{4-1} = 2^{3} = 8$
They will be studying 8 words by week 4.</td></tr>
<tr><td>2. A city's population grows exponentially. The population $P(t)$ after "t" years is given by $P(t) = 1000 \cdot 1.05^{t}$. Evaluate $P(10)$ and interpret the result in the context of the problem. (M.A1HS.20)</td><td>2. $P(10) = 1000 \cdot 1.05^{10} = 1628.8946$
The city will have a population of approximately 1628 after 10 years.</td></tr>
<tr><td>3. Jackson says the following relation is not an exponential function because the output values are all fractions and are not multiplied by an integer to get the next value. Is Jackson correct? Why or why not? (M.A1HS.19)
<table>
<tr><th>Input</th><th>Output</th></tr>
<tr><td>1</td><td>$\frac{1}{3}$</td></tr>
<tr><td>2</td><td>$\frac{1}{9}$</td></tr>
<tr><td>3</td><td>$\frac{1}{27}$</td></tr>
<tr><td>4</td><td>$\frac{1}{81}$</td></tr>
<tr><td>5</td><td>$\frac{1}{243}$</td></tr>
</table></td><td>3. Jackson is <i>not</i> correct because the common ratio in an exponential function does not need to be an integer. Each of these output values is multiplied by 1/3 (or divided by 3), so the common ratio is 1/3. A function to model this relationship would be
$f(n) = \frac{1}{3}^{n}$</td></tr>
</table>

<table>
<tr><th>Example Problems</th><th>Answers</th></tr>
<tr><td>4. Write a function that models the following geometric sequence:
3, 6, 12, 24, 48, 96

(M.A1HS.21)</td><td>4. If we call the initial term, $g_0 = 3$, then we see that
<table>
<tr><th>Input</th><th>Output</th></tr>
<tr><td>0</td><td>3</td></tr>
<tr><td>1</td><td>6</td></tr>
<tr><td>2</td><td>12</td></tr>
<tr><td>3</td><td>24</td></tr>
<tr><td>4</td><td>48</td></tr>
<tr><td>5</td><td>96</td></tr>
</table>
Since exponential functions are modeled as $f(x) = a \cdot b^x$ where a and b are constants, we see that the initial value of 3 will equal a, and our constant ratio is 2. Therefore, the function could be
$f(x) = 3 \cdot 2^x$</td></tr>
<tr><td>5. Determine if the following sequences are arithmetic or geometric. Justify your answers.
a. 2, -6, 18, -54, 162, -486
b. 17, 11, 5, -1, -7, -13
c. 1, 1, 2, 3, 5, 8

(M.A1HS.27)</td><td>5. a. Geometric; common ratio is -3
b. Arithmetic; common difference is -6
c. Neither; this is a Fibonacci sequence where the previous two numbers are added together to get the next number.</td></tr>
</table>

Misconceptions:

- All sequences are geometric sequences: Students might assume that any sequence with numbers getting larger or smaller is a geometric sequence; however, geometric sequences follow a pattern where each term is multiplied by a constant ratio to get to the next term.
- All geometric sequences start with 1 as the first term: Students might assume that the first term of a geometric sequence is always 1. While it is common to start with 1, geometric sequences can have any real number as the first term.
- All geometric sequences are increasing: Some students might mistakenly think that geometric sequences are always increasing. If the common ratio is positive and greater than 1, the sequence increases. However, if the common ratio is:
  - Negative and the absolute value of the r is greater than one, the sequence alternates signs and decreases in magnitude;
  - Negative and the absolute value of r is between 0 and 1, the sequence alternates signs and decreases in magnitude; or
  - Positive between 0 and 1, the sequence decreases.
- Applying arithmetic series formulas to geometric series: Students might mix up the formulas for arithmetic series and geometric series, leading to incorrect calculations.

Applications:

- Bacterial Growth Simulation - Students will model population growth using a geometric sequence. Scenario: A single bacterium doubles every hour in a controlled lab setting. Students will:
    - Write the first 8 terms of the sequence starting at 1: *1, 2, 4, 8, 16, 32, 64, 128*.
    - Identify the common ratio $r = 2$.
    - Write an explicit rule using function notation: $f(n) = 1 * 2^n$.
    - Use the formula to answer: "How many bacteria after 12 hours?" and "When will the population exceed 1,000?"
    - Graph the sequence to observe exponential growth.
- Cell Phone Depreciation - Students will model exponential decay using a real-world depreciation scenario. Scenario: A new phone is purchased for $800. Its value decreases by 20% each year. Students will:
    - Identify the common ratio $r = 0.8$ (retains 80% each year)
    - Write the explicit formula: $f(n) = 800 * 0.8^n$.
    - Create a table and graph for the first 6 years.
    - Answer "What is the phone's value after 3 years?" and "When is the phone worth less than $200?"
- Social Media Sharing - Students model viral growth as a geometric sequence. Scenario: A student posts a video that is shared by 3 people. Each person shares it with 3 new people the next day. Students will:
    - Model the sequence: $f(n) = 3^n$, where $f(n)$ represents the number of new shares on day n.
    - Graph and compare cumulative vs. daily shares.
    - Determine "How many new people will see the post on Day 7?" and "How many total people have seen it after 7 days?"
- Investment with Compound Interest - Students will explore compound interest as a real-world geometric sequence. Scenario: A student invests $1,000 in an account that earns 5% interest annually, compounded once per year. Students will:
    - Identify: $r = 1.05$, initial term = $1000.
    - Write: $f(n) = 1000 * 1.05^n$.
    - Create a table for 0–10 years and graph the growth.
    - Answer "When does the account reach $1,500?" and "How much is earned in the 5th year alone?"
- Radioactive Decay Modeling - Students will model exponential decay of a substance with a fixed half-life. Scenario: A chemical substance has a half-life of 3 hours. Start with 80 g. Students will:
    - Recognize: $r = 0.5$.
    - Write: $f(n) = 80 * 0.5^n$, where n is the number of 3–hour intervals.
    - Create a table and graph for up to 24 hours.
    - Answer "When will only 5 g remain?"

Career Connections:

- Medical Researchers and Epidemiologists
    - Epidemiologists study the spread of infectious diseases, which often follow exponential growth patterns (e.g., each infected person spreads the disease to multiple others).
    - They use exponential models to predict outbreaks, track infection rates, and plan responses to the spread.
- Financial Advisors and Investment Analysts
    - Financial professionals use exponential functions to calculate compound interest on savings, investments, and loans.
    - Understanding geometric sequences allows them to model long-term growth, compare investment options, and communicate how small changes in interest rates or time affect returns.
- Software Engineers and Data Scientists
    - Software engineers and data scientists work with algorithms and databases where exponential growth occurs in areas like search time, data storage needs, and network reach.
    - They use geometric sequences to model how quickly systems scale and to optimize performance.
- Environmental Scientists
    - Environmental researchers track exponential processes like deforestation, population growth, carbon emissions, and species extinction.
    - They use recursive and explicit exponential models to forecast trends and guide sustainability efforts.
- Marketing Analysts and Social Media Strategists
    - Marketers and strategists analyze viral content patterns, where views or shares increase exponentially over short periods.
    - They use geometric models to estimate reach, evaluate campaign success, and understand the tipping point for audience growth.
- Nuclear Technicians and Health Physicists
    - Professionals working with radioactive materials use exponential decay models to calculate safe handling times, determine half-lives, and ensure safety protocols.
    - Geometric decay is used to predict how much of a substance remains over time and when it reaches safe levels.

Subcategory: Heart of Algebra - Exponential

This subcategory will take a deeper look at previously explored algebraic concepts like interpreting **expressions and equations** through exponential models (**M.A1HS.1**). Students will use their prior knowledge of exponent properties to rewrite exponential expressions into equivalent forms which, in some cases, may help with interpreting their real-world application (**M.A1HS.2, 5, 10**). Teachers can also build on students' understanding of linear functions and expand understanding to exponential models since they are a concept new to Algebra I students. Students will create and use exponential functions to solve problems which will require them to know real-world situations that model exponential **growth or decay** (**M.A1HS.7, 8**).

Students have previously learned and applied exponential properties in Math 8; this section will extend that understanding to **rational exponents** and how they relate to radical expressions (**M.A1HS.3, 4**). Students will use radical expressions in Geometry and will consider radical functions in future math courses

Equations and Expressions

Interpret the structure of expressions and equations in terms of the context they model.

M.A1HS.1

Interpret linear, exponential, and quadratic expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
c. Interpret the parameters in a linear function or exponential function of the form $f(x) = a * b^x$ in terms of a context.

M.A1HS.2

Use the structure of quadratic and exponential expressions to identify ways to rewrite them.

Extend the properties of exponents to rational exponents.

M.A1HS.3

Explain the connections between expressions with rational exponents and expressions with radicals using properties of exponents. Extend from application of properties of exponents for expressions with integer exponents.

M.A1HS.4

Rewrite expressions involving radicals, including simplifying, and rational exponents using the properties of exponents.

Equations and Expressions (Continued)

Write expressions in equivalent forms to solve problems.

M.A1HS.5

Choose and produce an equivalent form of linear, exponential, and quadratic expressions to reveal and explain properties of the quantity represented by the expression through connections to a graphical representation of the function.

a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression, when $a = 1$ only, to reveal the maximum or minimum value of the function it defines.
c. Use the properties of exponents to transform expressions in exponential functions. For example, the expression 1.15^t can be rewritten as $(1.151/12)^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Instructional Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.

Create equations that describe numbers or relationships to create equations.

M.A1HS.7

Create equations and inequalities in one variable, representing linear and exponential relationships, and use them to solve problems. In the case of exponential equations, limit to situations with integer inputs.

M.A1HS.8

Create equations in two or more variables, representing linear and exponential relationships between quantities. In the case of exponential equations, limit to situations with integer inputs.

M.A1HS.9

Represent constraints by linear equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context.

Solve equations and inequalities in one variable.

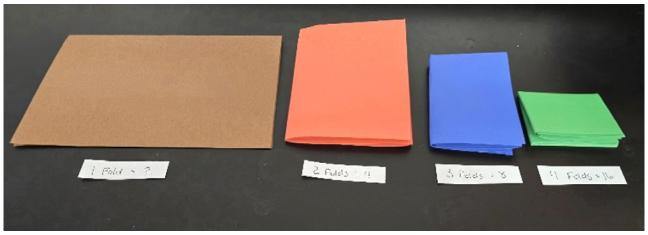
M.A1HS.10

Solve linear equations including equations with coefficients represented by letters, simple exponential equations that rely on application of the laws of exponents, and compound linear inequalities in one variable.

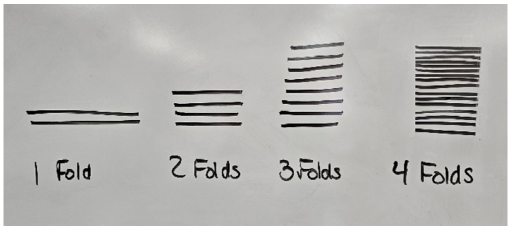
Representations:

Contextual: As the folds on a paper increase, the thickness of the paper increases exponentially (**M.A1HS.7**).

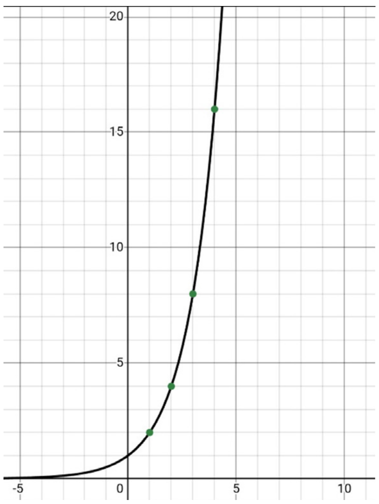
Physical: Students use manipulatives to represent the data pattern.



Visual: Students draw or graph the data.



Graph



Symbolic: Students create an equation and/or table by hand or using technology for their solution.

Fold	Thickness
1	2
2	4
3	8
4	16

Verbal: Students describe patterns they see in the data.

"The thickness of the paper quickly increases at 4 folds."

"I predict that 5 folds would yield a thickness of 32."

Examples:

As students complete the following examples, students are given the opportunity to make sense of real-life problems and persevere through solving them (**MHM1**). As students' reason, seeing numbers beyond their value (**MHM2**) will give important context into the connections made to their lives. Finding and making use of the structure (**MHM7**) allows students to draw conclusions about the example.

Example Problems	Answers
1. You put $500 in a savings account. Each month, the value of the account increases by 3%. How much money will be in the account after 3 years? What is the annual interest rate for the account? (**M.A1HS.1, 2, 5, 7**)	1. Let A be the total amount in the account after t months. Then, $A(t) = 500(1.03)^t$ Since 3 years is 36 months, substituting in t=36 gives $A(36) = 500(1.03)^{36} = 1449.139$ There will be approximately $1449.14 in the account after 3 years. To find the annual interest rate, we know that $\frac{t}{12} = s$ where s is the number of years the account has been open. Therefore, $t = 12s$. Substituting this into the functions gives, $A(t) = 500(1.03)^{12s}$ Using exponent properties, $A(t) = 500(1.03^{12})^s$ $A(t) = 500(1.4258)^s$ This reveals that 0.4258, or 42.58% is the approximate annual interest rate on the account. This is approximate due to rounding of 1.03^{12}.

Example Problems	**Answers**
2. A bacteria population triples every 4 hours. Initially, there were 10 bacteria. Write an exponential function $B(t)$ to model the bacterial population after "t" hours and determine the population after 12 hours. (**M.A1HS.3-4, 7, 9**)	2. This exponential function will have the general form $B(t) = b_0 \cdot r^t$ where b_0 is the initial value and r is the common ratio. Substituting in the initial value from the question and the information regarding the rate gives $B(4) = 10 \cdot (r)^4 = 30$ Solving for r yields $r = \sqrt[4]{3} = 3^{\frac{1}{4}}$ Substituting b_0 and r into the initial equation gives $B(t) = 10 \cdot 3^{\frac{t}{4}}$ To find the size of the bacterial population after 12 hours, let $t = 12$. Then, $B(12) = 10 \cdot 3^{\frac{12}{4}}$ $= 10 \cdot 3^3$ $= 10 \cdot 27$ $= 270$ After 12 hours, there will be 270 bacteria.
3. Simplify: $\frac{x^4y^3}{\sqrt{xy^5}}$ (**M.A1HS.2-4**)	3. $\frac{x^4y^3}{\sqrt{xy^5}} = \frac{x^4y^3}{x^{\frac{1}{2}}y^{\frac{5}{2}}} = x^{\frac{7}{2}}y^{\frac{1}{2}} = \sqrt{x^7y} = x^3\sqrt{xy}$
4. Solve: $2^{x+1} = 8$ (**M.A1HS.10**)	4. $2^{x+1} = 8$ $2^{x+1} = 2^3$ Since the base on each side is 2, we can conclude that $x + 1 = 3$, so $x = 2$.
5. Theo is interested in investing in a savings account with an annual interest rate of 4%, compounded monthly. Write an equation to model this situation. (**M.A1HS.8**)	5. The compound interest formula is $A = P\left(1 + \frac{r}{n}\right)^{nt}$ where A = total amount, P = principal (starting amount), r = annual rate, n = number of times compounded per year, and t = time in years. Using the values from the given question, $A = P\left(1 + \frac{0.04}{12}\right)^{12t}$ models the situation.

Misconceptions:

- Simplification of radical expressions and equations: Students often have difficulty differentiating between finding the solution to an equation such as $k^2 = 25$ and taking the square root of a number. The equation $k^2 = 25$ has two solutions, 5 and -5. Expressions such as $\sqrt{3^2}$ or $\sqrt{(-7)^2}$ ask for the “principal” square root. As a result, $\sqrt{3^2} = 3$ and $\sqrt{(-7)^2} = 7$.
- Squares and square roots of negative numbers: Student confusion in differentiating between expressions involving the square of a negative number and the negative of a square, often extends to square roots. Students may need help in appreciating the Order of Operations in differentiating between the expressions.

$(-5)^2 \rightarrow (-5)(-5) \rightarrow 25$
$-(5)^2 \rightarrow -(5)(5) \rightarrow -25$
$\sqrt{(-5)^2} \rightarrow \sqrt{(25)} \rightarrow 5$
$-\sqrt{5^2} \rightarrow -\sqrt{25} \rightarrow -5$

Applications:

See the Sequences – Exponential section on *page 53* in this document.

Career Connections:

See the Sequences – Exponential section on *page 54* in this document.

Subcategory: Multiple Representations – Exponentials

This section will expand on the prior learning of linear functions to include exponential functions and the many ways they can be represented- graphically, algebraically, or numerically in tables (**M.A1HS.16, 24**). Students should be able to identify and interpret key features of exponential functions (**M.A1HS.22**), including if the function represents **growth or decay** (**M.A1HS.29**), and x or y intercepts (**M.A1HS.23**). Students will determine the solution of a system of equations involving an exponential function with the use of technology to view the graph or using a table of values (**M.A1HS.17**). Using prior knowledge of exponent properties, students will interpret and write equivalent exponential expressions (**M.A1HS.25**). Students will be able to write equations given an exponential parent graph that is transformed or should be able to determine the impact of operations on an exponential function's graph (**M.A1HS.28**).

Exponential functions relate to many other fields of study and will continue to be used in future math courses. In Algebra II/Math III, students will use exponential functions to explore their inverses, which are logarithmic functions.

Expressions, Equations, and Functions

Represent and solve equations and inequalities graphically.

M.A1HS.16

Recognize that the graph of a linear or exponential equation in two variables is the set of all its solutions plotted in the coordinate plane.

M.A1HS.17

Explain why the *x*-coordinates of the points where the graphs of the linear and/or exponential equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations).

Interpret functions that arise in applications in terms of a context.

M.A1HS.22

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of quantities, and sketch graphs showing key features given a verbal description of the relationship. Relate the domain of a function to its linear, exponential, and quadratic graphs and, where applicable, to the quantitative relationship it describes.

a. Key features of linear and exponential graphs include: intercepts; and intervals where the function is increasing, decreasing, positive, or negative.
b. Key features of quadratic graphs include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximum or minimum; symmetry; and end behavior.

Expressions, Equations, and Functions (Continued)

Analyze functions using different representations.

M.A1HS.23

Graph linear, exponential, and quadratic functions expressed symbolically and show key features of the graph.

a. For linear functions, focus on intercepts. For exponential functions, focus on intercepts and end behavior.
b. For quadratic functions, focus on intercepts, maxima, minima, end behavior, and the relationship between coefficients and roots to represent in factored form.

Instructional Note: Provide opportunities for students to graph and show key features by hand and using technology.

M.A1HS.24

Compare properties of two linear, exponential, or quadratic functions each represented in a different way, such as algebraically, graphically, numerically in tables, or from verbal descriptions.

M.A1HS.25

Write a function defined by a linear, exponential, or quadratic expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square for $a = 1$ only in a quadratic function to show zeros, extreme values, symmetry of the graph, the relationship between coefficients and roots represented in factored form and interpret these in terms of a context.
b. Use the properties of exponents to interpret expressions in exponential functions.

Build new functions from existing functions.

M.A1HS.28

Identify the effect on the graphs of linear and exponential functions, $f(x)$, with $f(x) + k$, and the graphs of quadratic functions, $g(x)$, with $g(x) + k$, $k\,g(x)$, $g(kx)$, and $g(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Construct and compare linear, quadratic, and exponential models and solve problems.

M.A1HS.29

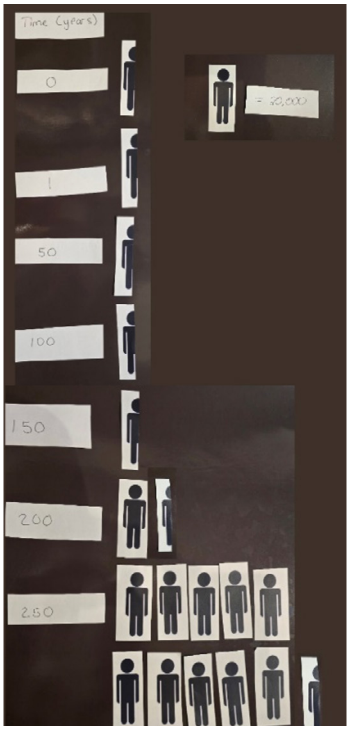
Distinguish between situations that can be modeled with linear functions, with exponential functions, and with quadratic functions.

a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
d. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. Extend the comparison of linear and exponential growth to quadratic growth.

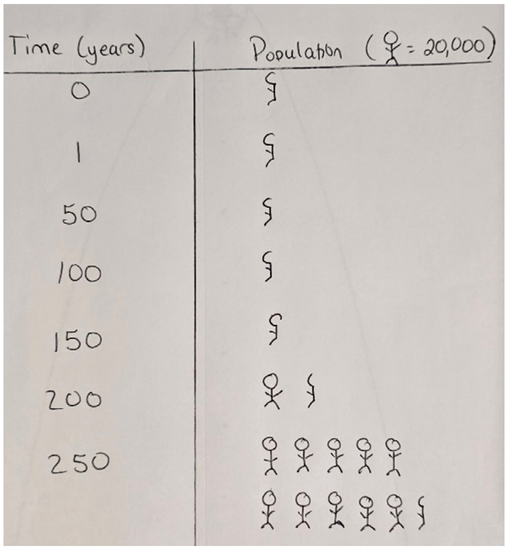
Representations:

Contextual: Suppose we have a town with an initial population of 10,000 people, and the population is growing exponentially. The population is modeled by the function $P(t) = 10{,}000 + 1.05^t$, where t represents time in years. What will the population be in 250 years (**M.A1HS.23**)?

Physical: Students use manipulatives to represent the situation.



Visual: Students draw or graph the data.



Symbolic: Students create a table and/or equation by hand or using technology for their solution.

Time (Years)	Population
0	10,000
1	10,000.05
50	10,011.47
100	10,131.50
150	11,507.98
200	27,292.58
250	208,300.94

Verbal: Students describe patterns they see in the data.

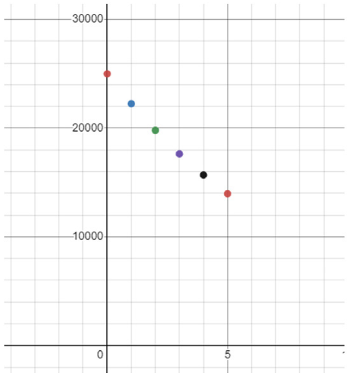
"There was very little growth until after 150 years."

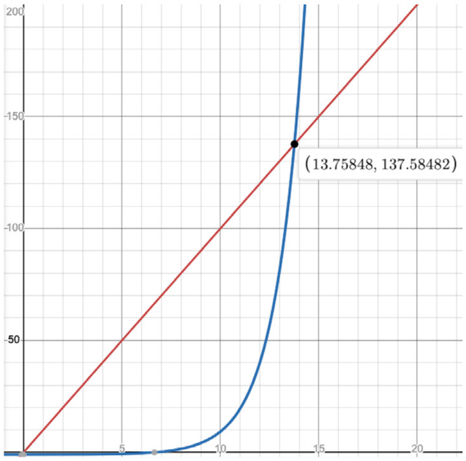
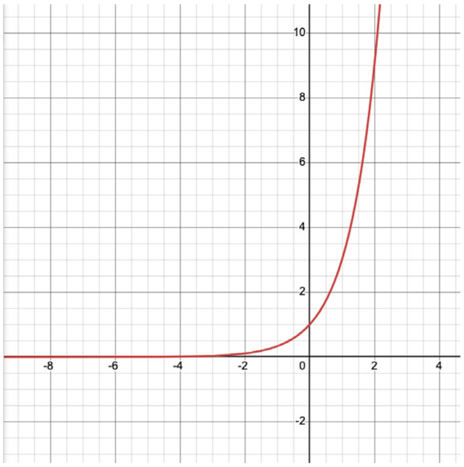
"In one year, the growth was less than 1."

"The y-intercept was the initial population of 10,000."

Examples:

Repeated reasoning (**MHM8**) will give students an upper hand when completing the following examples by allowing the opportunity to apply key features to explain what is happening as the problem changes. Attending to precision (**MHM6**) will ensure students arrive at accurate answers that make sense in the context of the example.

<table>
<tr><th>Example Problems</th><th>Answers</th></tr>
<tr><td>1. You buy a new car that costs $25,000. The car depreciates at a rate of 11% per year.</td><td>1.</td></tr>
<tr><td>a. Write an equation to represent the exponential function
(M.A1HS.24)</td><td>a. $f(n)=25000(.89)^n$, where n is the number of years, $25000 is the starting value of the car and 1 - .11 = .89 is the retention of the cars value rate.</td></tr>
<tr><td>b. Create a table to represent the exponential function, then identify the growth/decay rate on your table and explain how the growth/decay rate represents an exponential function.
(M.A1HS.24)</td><td>b.
<table>
<tr><th>Years (n)</th><th>Value of car</th></tr>
<tr><td>0</td><td>25000</td></tr>
<tr><td>1</td><td>22250 = 25000 * $(.89)^1$</td></tr>
<tr><td>2</td><td>19802.50 = 25000 * $(.89)^2$</td></tr>
<tr><td>3</td><td>17624.23 = 25000 * $(.89)^3$</td></tr>
<tr><td>4</td><td>15685.56 = 25000 * $(.89)^4$</td></tr>
</table></td></tr>
<tr><td>c. Create a graph to represent the exponential function.
(M.A1HS.23)</td><td>c.

</td></tr>
<tr><td>d. Identify the following:
• Independent Variable
• Dependent Variable
• Domain
• Range
• Starting Point
(M.A1HS.22)</td><td>d. Independent Variable = number of years
Dependent Variable = value of car
Domain = $x \geq 0$
Range = $0 \leq y \leq 25000$
Starting Point: value of the car on the day purchased = $25,000</td></tr>
</table>

Example Problems	Answers
2. You won a contest with a monetary prize. They provide you with the following two options to receive your winnings. Which option gives more money? How many days would you need to collect money for your option to be worth more than the other option? When would both options give the same amount of money? Option 1: Receiving \$10 every day Option 2: Receiving a penny on Day 1, then doubling the amount received every day. (**M.A1HS.16, 17, 29**)	2. Option 1: $y = 10x$ Option 2: $y = 0.01 \cdot 2^{(x-1)}$ where x = number of days receiving money Graphing these equations shows that Option 1 gives the most money for days 1 – 13, but then Option 2 exceeds Option 1 and will continue to be greater than Option 1 since exponential functions grow faster than linear functions. Both accounts give the same amount of money on Day 13.76 since this is the intersection of the two graphs. 
3. The graph below shows the function $f(x) = 3^x$. What will the graph of $g(x) = 3^x - 4$ look like? (**M.A1HS.28**) 	3. The function $g(x)$ will translated down 4 units, meaning that 4 will be subtracted from every output value of $f(x)$.

Example Problems	Answers
4. Andy bought a new car for $35,000. The value of his car can be modeled by the equation $y = 35000(0.9875)^m$, where m is the number of months since purchase. What is the yearly depreciation rate? (**M.A1HS.25**)	4. Since there are 12 months in a year, let $m = 12t$ where t is the number of years since purchase. Then, $y = 35000(0.9875)^{12t}$ Using properties of exponents, we can rewrite as $y = 35000(0.9875^{12})^t$ $y = 35000(0.86)^t$ The car is depreciating at a rate of 1 – 0.86 = 14% per year.

Misconceptions:

- Difference in effect of linear versus exponential growth: Students fail to grasp the effect of the growth factor in an exponential function and its impact on the growth rate.
- Exponential change can be positive or negative: Students assume that exponential functions always represent rapid and unbounded growth, neglecting the possibility of exponential decay.

Applications:

- Account Comparison - Students compare two savings plans using graphs and tables. Scenario: Plan A: Deposit $50 every month → linear growth, Plan B: Invest $100 with 10% monthly interest → exponential growth. Students will:
  - Write equations for the scenarios.
    - Plan A: $f(x) = 50x$
    - Plan B: $g(x) = 100(1.10)^x$
  - Create a table of values for each function over 12 months.
  - Graph both functions on the same coordinate plane.
  - Identify the following:
    - When Plan B surpasses Plan A;
    - The meaning of the intersection point (solution to $f(x) = g(x)$); and
    - How growth differs over time.

Career Connections:

- Financial Analysts and Investment Planners
    - Financial professionals use linear models to project regular savings plans and exponential models to analyze investments with compound interest or inflation.
    - They graph both types of growth to help clients compare future values and determine which investment strategies meet long-term goals.
    - Intersections of models indicate when one plan becomes more profitable than another.
- Epidemiologists and Public Health Experts
    - Epidemiologists use exponential models to predict how quickly diseases can spread within a population.
    - They compare growth rates across regions or populations and use graphs to identify tipping points for outbreaks.
    - Interpreting graphs helps determine when interventions and responses to the spread are most effective.
- Technology and App Growth Analysts
    - Tech companies track app usage and user growth over time using exponential models.
    - Product analysts compare track app usage and user growth data to advertising spend or linear user acquisition goals to predict when growth will plateau or accelerate.
    - Understanding model transformations helps them analyze the impact of promotions or feature rollouts.
- Environmental Scientists and Sustainability Experts
    - Environmental professionals compare linear resource usage (e.g., water consumption) to exponential population growth or emissions increases.
    - They use these models to predict when resource demands will outpace availability or when environmental limits will be exceeded.
    - Interpreting graph intersections helps guide policy decisions.
- Business and Operations Managers
    - Managers use graphs to compare fixed operating costs (linear) with exponential customer demand or production output.
    - Understanding function behavior allows them to optimize staffing, supply ordering, and marketing schedules.
    - They rely on graphical tools to explain projections to stakeholders and respond to changing trends.
- Software Engineers and Systems Architects
    - Engineers model linear processing times and exponential data growth to plan system infrastructure.
    - They use comparisons to determine when performance upgrades are needed or when a current system will be overwhelmed.
    - Graphing transformations of function behavior (e.g., load vs. response time) is critical for capacity planning.

Category: Quadratic Functions

Subcategory: Heart of Algebra – Quadratic

This subcategory of standards focuses on the quadratic functions and students' understanding of their equations. Students will apply the vocabulary of terms, factors, and coefficients to this new context of quadratics (**M.A1HS.1**). Students should explore multiple ways to represent quadratic expressions (**standard form, vertex form**) and to solve quadratic functions (factoring, quadratic formula) to connect the equation and solutions to their graphs (**M.A1HS.2, 5, 11**). Students should also form an understanding of the equivalence between different forms of quadratic functions (**M.A1HS.6**).

In Algebra II/Math III, students will continue this learning as they create equations and inequalities in one variable, representing linear, quadratic, simple rational, and exponential relationships, and use them to solve problems.

Equations and Expressions

Interpret the structure of expressions and equations in terms of the context they model.

M.A1HS.1

Interpret linear, exponential, and quadratic expressions that represent a quantity in terms of its context.

a. Interpret parts of an expression, such as terms, factors, and coefficients.
b. Interpret complicated expressions by viewing one or more of their parts as a single entity.
c. Interpret the parameters in a linear function or exponential function of the form $f(x) = a * b^x$ in terms of a context.

M.A1HS.2

Use the structure of quadratic and exponential expressions to identify ways to rewrite them.

Write expressions in equivalent forms to solve problems.

M.A1HS.5

Choose and produce an equivalent form of linear, exponential, and quadratic expressions to reveal and explain properties of the quantity represented by the expression through connections to a graphical representation of the function.

a. Factor a quadratic expression to reveal the zeros of the function it defines.
b. Complete the square in a quadratic expression, when $a = 1$ only, to reveal the maximum or minimum value of the function it defines.
c. Use the properties of exponents to transform expressions in exponential functions. For example, the expression 1.15t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%.

Instructional Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.

Equations and Expressions (Continued)

Perform arithmetic operations on polynomials.

M.A1HS.6

Recognize that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Focus on linear or quadratic terms.

Solve equations and inequalities in one variable.

M.A1HS.11

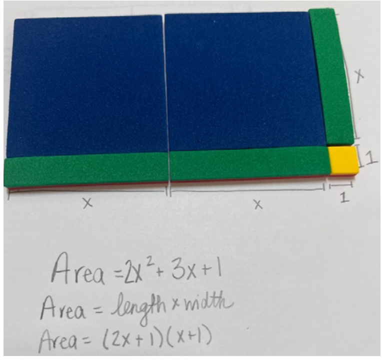
Solve quadratic equations in one variable by inspection (e.g., for $x^2 = 49$), taking square roots, factoring, completing the square when a = 1 only, and the quadratic formula, as appropriate for the initial form of the equation.

a. Recognize the concept of complex solutions when the quadratic formula gives complex solutions.
b. Use the method of completing the square to transform any quadratic equation in *x* into an equation of the form $(x - p)2 = q$. Derive the quadratic formula from this method of completing the square.

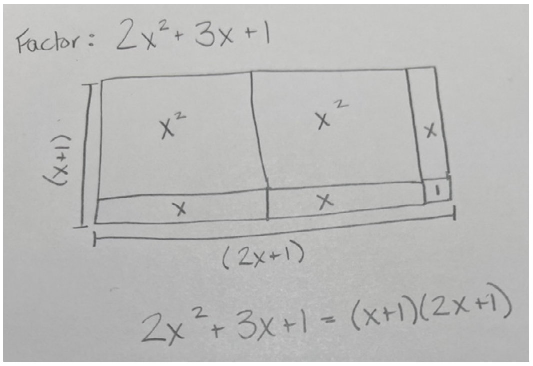
Representations:

Contextual: Factor the expression $2x^2 + 3x + 1$.

Physical: Students use algebra tiles to represent the situation.



Visual: Students draw or graph the data.



Representations:

Symbolic: Students use algebraic methods of factoring, such as the key number method, to factor the expression. $2x^2 + 3x + 1$ $2x^2 + 2x + 1x + 1$ $(2x^2 + 2x) + (+ 1x + 1)$ $2x(x + 1) + 1(x + 1)$ $(x + 1)(2x + 1)$	**Verbal:** Students describe their thought process using structure of expressions to describe factors with variables, coefficients, and constants. "I know there will be two factors since it is a quadratic expression. I notice the each term in the expression is positive, so I will start working with positive factors. Then, I know the only way to multiply to get $2x^2$ is found by multiplying $2x$ and x, so those must be the x terms in the factors. I also know the only way to multiply two constants to get 1 is $1 \cdot 1$, so those must be the constant terms. Then, since the constant term is positive, both operations must be the same as the sign of the b term, so they must both be positive. That means, the factored form is the quantity of $2x + 1$ times the quantity of $x + 1$."

Examples:

Students must make sense of the given pieces of the following problems (**MHM1**) in order to understand them fully and identify relevant information. As students analyze the examples, the concept of quadratic functions can be applied to solve the practical problem (**MHM4**). Students will also need to make use of the structure of the function (**MHM7**) to be able to justify their reasoning (**MHM3**).

Example Problems	Answers
1. A cannon ball is launched into the air with an upward velocity of 233 feet per second. The height h of the cannon ball after t seconds can be found using the equation $h = -16t^2 + 233t + 2$. How tall is the cannon? Justify. Approximately how long will it take for the cannon ball to hit the ground? Round answers to the nearest tenth if necessary. (**M.A1HS.1, 11**)	1. The cannon is 2 feet tall because the constant term in the equation is 2. The cannon ball will hit the ground when $h = 0$. Therefore, $0 = -16t^2 + 233t + 2$ Using the quadratic formula gives $t = \frac{-233 \pm \sqrt{(233)^2 - 4(-16)(2)}}{2(16)}$ $t = -0.00858, \quad t = 14.57108$ Since negative time does not make sense in this context, the cannon ball will hit the ground at approximately $t = 14.6$ seconds.

Example Problems	**Answers**
2. Find the vertex of the quadratic equation: $y = x^2 - 6x - 14$ (**M.A1HS.2, 5**)	2. While the vertex formula is an option, it is also possible to complete the square to reveal the vertex by converting the quadratic to vertex form. $y + 14 = x^2 - 6x$ $y + 14 + 9 = x^2 - 6x + 9$ $y + 23 = (x - 3)^2$ $y = (x - 3)^2 - 23$ The vertex is (3,-23).
3. Multiply: $(2x - 3)(x^2 - 2x + 4)$ (**M.A1HS.6**)	3. $(2x - 3)(x^2 - 2x + 4) = 2x^3 - 7x^2 + 14x - 12$
4. The function $h = -4.9t^2 + 87.21t + 9144$ describes the altitude (h) in meters (m) of the plane in relation to the time (t) in seconds (s) after it started the parabolic maneuver. You will use this function to analyze the parabolic flight of the NASA C-9 aircraft. Round all answers to the nearest tenth. a. At what altitude did the astronaut first start to feel microgravity? (**M.A1HS.11**) b. When did the microgravity begin and end in this maneuver? (**M.A1HS.11**) c. What does 9144 meters represent in this situation? (**M.A1HS.1**)	4. a. Let $t = 0$. Then, $h = -4.9(0)^2 + 87.21(0) + 9144 = 9144$ The altitude that the astronaut will start feeling microgravity is 9,144 m. b. Since microgravity begins at 9,144 m, set $9144 = -4.9t^2 + 87.21t + 9144$ $0 = -4.9t^2 + 87.21t$ $0 = t(-4.9t + 87.21)$ $t = 0 \quad t=17.8$ Microgravity begins at 0s and ends at 17.8s. c. The 9144 meters represents the altitude where the plane started the parabolic maneuver.

Misconceptions:

- Confusion with quadratic terms: Students may have difficulty recognizing quadratic terms in more complex expressions. They might mistake linear or constant terms for quadratic terms or overlook the presence of quadratic expressions entirely.
- Overlooking equivalent forms: Students may not recognize various equivalent forms of quadratic expressions, such as standard form, factored form, or vertex form and the information that each form provides.
- Difficulty solving quadratic equations: When solving quadratic equations, students may rely solely on one specific method, such as factoring, and struggle to identify the most appropriate method, such as quadratic formula, for a given equation.

Applications:

- Parabolic Arch Design - Scenario: Civil engineers design a parabolic arch for a pedestrian bridge. The arch follows a symmetric curve that peaks at 30 feet in the middle and touches the ground 20 feet from either side of the center. Students will:
    - Model the arch with a quadratic equation using vertex form: $h(x) = a(x-0)2 + 30$, where the arch height is 0 *at* $x = \pm 20$.
    - Use given points to solve for a.
    - Convert the vertex form to standard and factored forms.
    - Use solving methods to determine width at different heights (e.g., how wide the arch is when it’s 10 ft tall?).
    - Interpret what the vertex, intercepts, and symmetry mean in the context of the structure.
- Headlight Beam Angle - Scenario: Automotive engineers test a car’s headlight beam pattern. The beam follows a parabolic path modeled by $h(x) = -0.02x2 + x$, where x is distance (in meters) from the car and $h(x)$ is the beam’s height. Students will:
    - Identify terms, coefficients, and factors in the quadratic model.
    - Graph the function and interpret the vertex (maximum height) and x-intercepts (where the beam hits the ground).
    - Use factoring or the quadratic formula to solve when $h(x) = 0.5$ (e.g., “At what distance is the beam 0.5m above the ground?”).

Career Connections:

- Civil Engineers and Architects
    - Use quadratic functions to design parabolic arches, bridges, tunnels, and suspension cables.
    - Vertex form is used to control height and symmetry; factored form helps determine structural supports and base width.
    - Understanding how to rewrite and solve quadratic equations is critical for evaluating loads and optimizing safety and material use.
- Automotive and Aerospace Engineers
    - Model the motion of vehicles, rockets, and components using quadratic equations, particularly when analyzing acceleration, deceleration, or launch arcs.
    - Use the quadratic formula and vertex interpretation to optimize travel paths, reduce impact force, or design crash-absorbent systems.
    - Complete the square is used when adjusting design to meet specific max/min performance targets.
- Financial Analysts and Business Managers
    - Analyze profit functions that follow a quadratic relationship (revenue minus quadratic cost).
    - Determine optimal pricing or production quantities using the vertex of a profit function.
    - Factor or use the quadratic formula to find break-even points (where revenue = cost).
- Video Game Developers and Animators
    - Create realistic jump or projectile mechanics using quadratic motion formulas.
    - Adjust coefficients in the quadratic equation to fine-tune how a character moves or an object behaves in gameplay.
    - Use different forms of quadratic functions (factored for timing, vertex for peak animation) in game physics engines.
- Sports Scientists and Biomechanics Experts
    - Model the path of a ball, runner, or jumper using quadratic functions to improve athletic performance.
    - Analyze data from sensors to calculate peak heights, travel times, or distances.
    - Apply completing the square or factoring to back-calculate initial velocities or jump angles.
- Environmental Engineers
    - Use quadratic models to simulate natural phenomena like erosion rates, water spray arcs in irrigation systems, or pollutant dispersion from a point source.
    - Solve quadratic systems to find optimal equipment placement or containment zones.
    - Interpret terms and coefficients to make sense of real-world limits and behaviors in environmental systems.

Subcategory: Systems – Quadratic

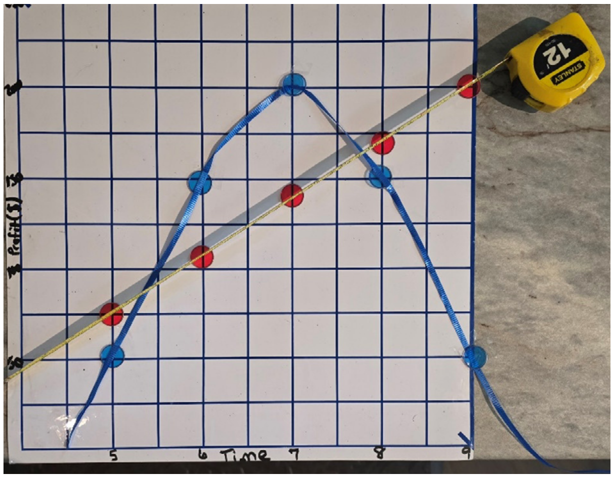
Exposing students to solving systems of equations between different types of functions is an additional topic in Algebra I. The focus is on simple systems that consist of one linear equation and one quadratic equation with two variables (**M.A1HS.15**). Teachers should focus on having students graph the equations, which may include the use of technology, to determine the solution. It is important for students to understand that the solution to a system of equations is where the functions intersect. In grade 8, students solve systems of linear equations graphically. In Algebra II/ Math III, students continue work with systems of equations and how they relate to contexts that model the real world. This requires interpretation of solutions to determine if they are viable.

Expressions and Equations
Solve systems of equations. **M.A1HS.15** Solve a simple system consisting of a linear equation and a quadratic equation in two variables graphically.

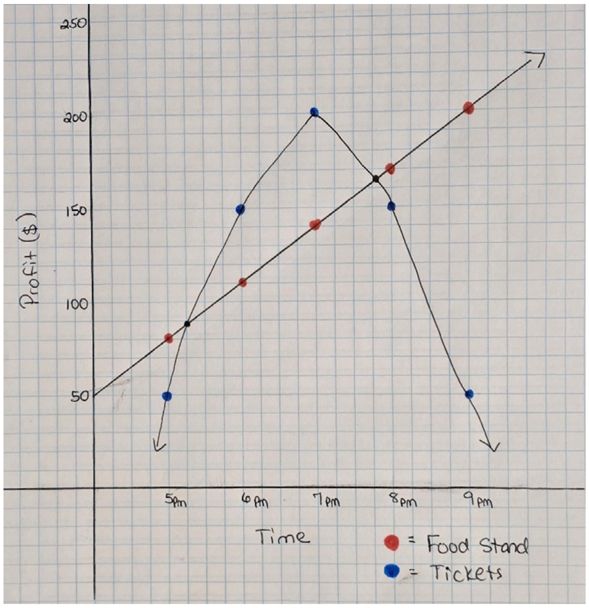
Representations:

Contextual: The following data was collected for the ticket profits recorded each hour for entrance to carnival which opens at 4 P.M. : $50 in tickets sold at 5 P.M., $150 in tickets sold at 6 P.M., $200 in tickets sold at 7 P.M., $150 in tickets sold at 8 P.M., and $50 in tickets sold at 9 P.M.. At the same carnival, the following data was collected for the profit of a food stand: $80 sold at 5 P.M., $110 sold at 6 P.M., $140 sold at 7 P.M., and $170 sold at 8 P.M. At what time(s) are the ticket and food stand profits the same (**M.A1HS.15**)?

Physical: Students use algebra tiles to represent the situation.



Visual: Students draw or graph the data.



Representations:

Symbolic: Students create an equation and/or table by hand or using technology for their solution.

Time (P.m.)	Tickets ($)	Food Stand ($)
5	50	80
6	150	110
7	200	140
8	150	170
9	50	200

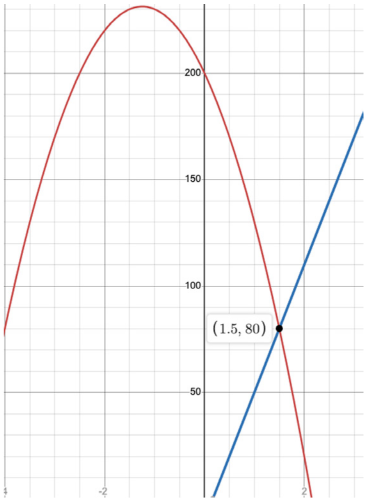
Verbal: Students describe patterns they see in the data.

“The line and parabola intersect for the first time at (5:15, 90), so at 5:15 p.m. both tickets and the food stand profit about 90 dollars.”

“The line and parabola intersect for the second time at about (7:45,165), so at 7:45 p.m. both tickets and the food stand profit about 165 dollars.”

Examples:

Students may utilize appropriate tools (**MHM5**) to create graphs or draw diagrams to aid in solving the system of equations. Making sense of the problem is necessary to understand what portion of the intersection is needed to answer the question (**MHM1**).

Example Problems	Answers
1. The revenue for a company producing electronic components is given by $R(x) = -20x^2 - 50x + 200$, where x is the price in dollars of each component. The cost for the production is given by: $C(x) = 60x - 10$. Determine the price that will allow the company to break even. (**M.A1HS.15**)	1. When graphing the revenue and cost equations, we see they intersect at the point (1.5, 80). This means, the company will break even (revenue = cost) when they charge $1.50 for the electronic component. 

Misconceptions:

- Multiple solutions: Students may only have previous exposure to linear systems of equations which only have one solution if lines intersect, no solutions if lines are parallel or infinite solutions if lines overlap. For intersections between a linear and quadratic function, systems have two solutions in most cases but could have one solution if the line touches the curve on a tangent point. There will not be infinite solutions for a system of linear and quadratic functions.
- Viewing Full Function: If students view a portion of the graph, as opposed to the full graph, they could think the curve and lines do not intersect; ensuring students view the graph outside of an initial window will be important for finding the solutions.

Applications:

- Cost vs. Revenue Analysis - Scenario: A small business tracks costs and revenue. Revenue (quadratic) is modeled by $y = -5x^2 + 100x$ and Cost (linear) is modeled by $y = 40x + 200$. Students will:
  - Graph both equations and find their points of intersection.
  - Interpret the following:
    - First intersection: break-even point (start of profitability); and
    - Second intersection: point where costs exceed revenue again.
  - Determine when and where the company is making a profit.
- Environmental Design – Scenario: An environmental designer models water sprayed by a sprinkler as: $y = -0.25x^2 + 4x$. A row of plants is located at height $y = 5$ feet (tall planters). Students will:
  - Graph both equations to determine where the water spray intersects the top of the planter row.
  - Decide whether the plants will be watered properly.
  - Alter the spray function to increase reach and repeat the analysis.

Career Connections:

- Civil and Structural Engineers
  - Engineers graph quadratic cable curves and intersect them with straight beams to model bridges, suspension systems, or roof supports.
  - They determine exact attachment points, tensions, and safety zones by analyzing these points of intersection.
  - Intersection solutions identify viable structural connections and load-bearing limits.
- Entrepreneurs and Business Analysts
  - In cost-revenue analysis, linear cost models and quadratic revenue models are graphed to determine break-even and maximum profit points.
  - Analysts find where the two graphs intersect to decide when a product becomes profitable and when losses begin again.
  - These systems help build sustainable pricing and production plans.
- Sports Scientists and Biomechanics Experts
  - Use parabolic motion models to simulate jumps, kicks, or throws and intersect them with opposing player positions (modeled linearly) to analyze in-game outcomes.
  - Graphing allows them to predict collisions, reach, or clearance during plays.
  - Athletes and coaches adjust performance based on where motion paths intersect with obstacles or defenders.
- Video Game Developers and Animators
  - Program character motion (e.g., jumps) as quadratic functions and model environmental elements (platforms, floors) as linear ones.
  - Solving the system identifies whether the character will reach the intended target.
  - Developers dynamically adjust function parameters to create realistic or challenging gameplay.
- Environmental Designers and Irrigation Engineers
  - Model water spray arcs with quadratic functions and intersect them with target areas (like hedges, raised beds, or greenhouses) using linear equations.
  - Graphical intersection points reveal coverage zones and inform placement of sprinklers or landscape features.
  - They use these tools to design efficient, sustainable irrigation systems.

Subcategory: Multiple Representations – Quadratic

In this collection of standards, the central focus lies on quadratic functions, an important type of mathematical function represented both algebraically and graphically. Students will delve into the concept of **function notation**, learning how to evaluate functions for specific inputs within their defined domains and grasp the interpretation of function notation in real-world contexts (**M.A1HS.20**). Furthermore, they will explore quadratic functions and gain the ability to interpret their essential features, such as **intercepts, intervals of increase or decrease, positivity or negativity, relative maximum or minimum points, symmetry, and end behavior** (**M.A1HS.22-23**). Students will analyze functions using various representations, honing their skills in graphing linear, exponential, and quadratic functions to highlight key characteristics (**M.A1HS.24**). A crucial aspect of their learning involves comparing properties of functions represented differently, solving problems with quadratic models, and distinguishing situations appropriate for this function type (**M.A1HS.24**). Students should consider real world scenarios that model quadratic functions and be able to build equations that describe those relationships (**M.A1HS.26, 29**). In order to graph and solve quadratics that model these real world situations, students will need to factor or complete the square (**M.A1HS.25**). The foundation in quadratic functions students build in this course will continue to be built upon in all future math courses. In Algebra II/Math III, students will work with quadratic functions with complex solutions and consider how that impacts its graph.

Functions

Understand the concept of a function and use function notation.

M.A1HS.20

Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context.

Interpret functions that arise in applications in terms of a context.

M.A1HS.22

For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of quantities, and sketch graphs showing key features given a verbal description of the relationship. Relate the domain of a function to its linear, exponential, and quadratic graphs and, where applicable, to the quantitative relationship it describes.

a. Key features of linear and exponential graphs include: intercepts; and intervals where the function is increasing, decreasing, positive, or negative.
b. Key features of quadratic graphs include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximum or minimum; symmetry; and end behavior.

Analyze functions using different representations.

M.A1HS.23

Graph linear, exponential, and quadratic functions expressed symbolically and show key features of the graph.

a. For linear functions, focus on intercepts.
b. For exponential functions, focus on intercepts and end behavior.
c. For quadratic functions, focus on intercepts, maxima, minima, end behavior, and the relationship between coefficients and roots to represent in factored form.

Instructional Note: Provide opportunities for students to graph and show key features by hand and using technology.

Functions (Continued)

M.A1HS.24

Compare properties of two linear, exponential, or quadratic functions each represented in a different way, such as algebraically, graphically, numerically in tables, or from verbal descriptions.

M.A1HS.25

Write a function defined by a linear, exponential, or quadratic expression in different but equivalent forms to reveal and explain different properties of the function.

a. Use the process of factoring and completing the square for $a = 1$ only in a quadratic function to show zeros, extreme values, symmetry of the graph, the relationship between coefficients and roots represented in factored form and interpret these in terms of a context.
b. Use the properties of exponents to interpret expressions in exponential functions.

Build a function that models a relationship between two quantities.

M.A1HS.26

Write linear, exponential, and quadratic functions that describe a relationship between two quantities.

a. Determine an explicit expression, a recursive process, or steps for calculation from a context.
b. Combine standard function types using arithmetic operations.

M.A1HS.28

Identify the effect on the graphs of linear and exponential functions, $f(x)$, with $f(x) + k$, and the graphs of quadratic functions, $g(x)$, with $g(x) + k$, $k\ g(x)$, $g(kx)$, and $g(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology.

Construct and compare linear, quadratic and exponential models and solve problems.

M.A1HS.29

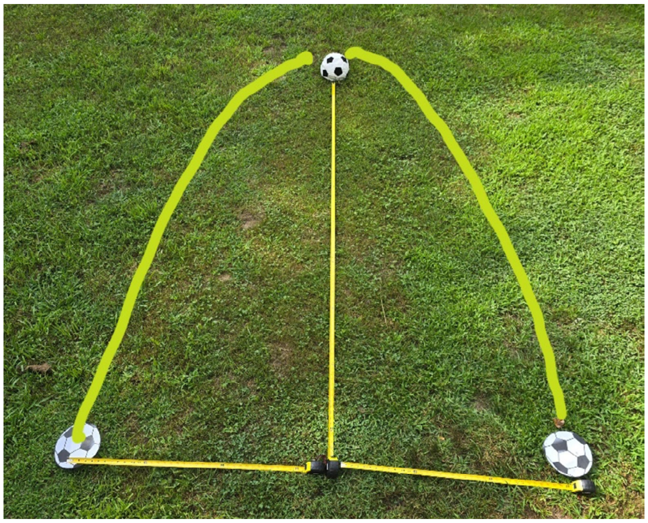
Distinguish between situations that can be modeled with linear functions, with exponential functions, and with quadratic functions.

a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals.
b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
d. Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly or quadratically. Extend the comparison of linear and exponential growth to quadratic growth.

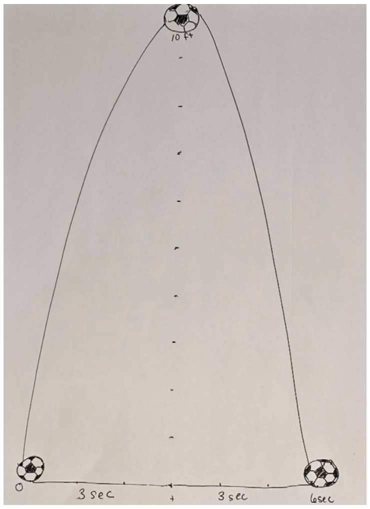
Representations:

Contextual: Casey kicked a soccer ball from the ground. After 3 seconds, it reached a maximum height of 10 feet (**M.A1HS.22**).

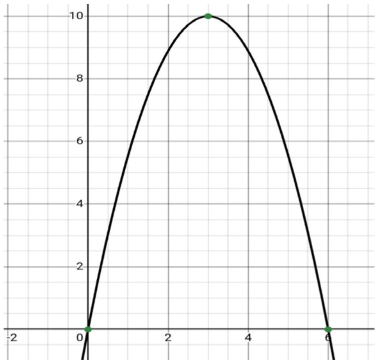
Physical: Students use manipulatives to represent the data pattern.



Visual: Students draw or graph the data.



Graph



Symbolic: Students create a table and/or equation by hand or using technology for their solution.

Time (seconds)	Height (feet)
0	0
3	10
6	0

Verbal: Students describe patterns they see in the data.

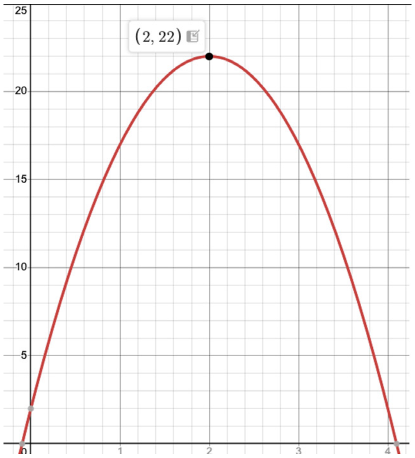
"The *y*-intercept and initial value is at the origin because the ball is on the ground at 0 seconds."

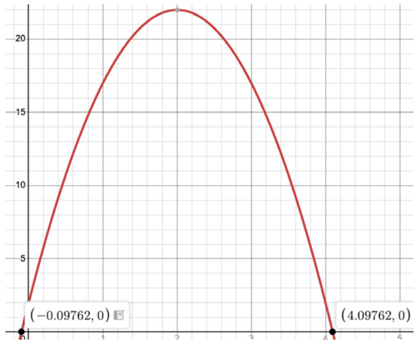
"The vertex is (3,10) because the ball reaches its maximum height of 10 feet after 3 seconds."

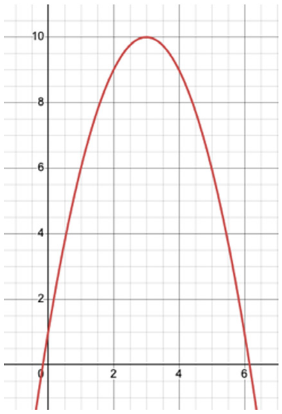
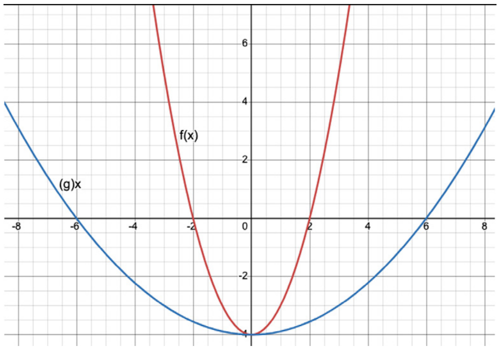
"The graph increases and then decreases because the height of the ball increases and then decreases after 3 seconds."

Examples:

Students must communicate their reasoning, both abstractly and quantitatively (**MHM2**), in order to completely defend their responses to the following examples. Students are given the opportunity to construct an argument and critique the reasoning of others (**MHM3**) as they complete the task.

<table>
<tr><th>Example Problems</th><th>Answers</th></tr>
<tr><td colspan="2">A ball is thrown upward with an initial velocity of 20 m/s from a height of 2 meters. The height "*h*" of the ball above the ground at time "*t*" seconds is given by the quadratic function $h(t) = -5t^2 + 20t + 2$.</td></tr>
<tr><td>1. When does the ball reach its maximum height and what is that maximum height? Verify your answer by graphing.
(**M.A1HS.20, 22-23**)</td><td>1. The maximum height of the ball will occur at the vertex. Using the vertex formula t = $\frac{b}{2a}$, we see the maximum height occurs when t = $-\frac{20}{2(-5)}$ = 2 seconds. Plugging this value into the function yields
$$h(2) = -5(2)^2 + 20(2) + 2 = 22$$
The maximum height of the ball is 22 meters. To verify by graphing, we look for the vertex coordinate on the graph, and see that it is located at (2, 22).
 </td></tr>
</table>

Example Problems	**Answers**
2. When does the ball hit the ground? Verify your answer by graphing. (**M.A1HS.22-23, 25**)	2. The ball will hit the ground when $h = 0$. Setting the function equal to 0 and solving gives $$0 = -5t^2 + 20t + 2$$ $$t = \frac{-20 \pm \sqrt{(20)^2 - 4(-5)(2)}}{2(-5)}$$ $$t = -0.0976, \quad t = 4.0976$$ The ball will hit the ground at approximately 4.1 seconds since we can ignore the solution where $t < 0$ in this context. The zeros of a graph are also known as the x-intercepts, so to verify by graphing we will look at the x-intercepts to find any solutions where $t > 0$. The only one occurs when $t = 4.0976$ 
3. How high is the ball off the ground at 1.5 seconds? (**M.A1HS.20**)	3. $h(1.5) = -5(1.5)^2 + 20(1.5) + 2 = 20.75$ The ball will be 20.75 meters off the ground at 1.5 seconds.

Additional examples and solutions	
4. Suppose $f(x) = -x^2 - 2x + 8$ and $g(x)$ is given by the graph below. Which function has a larger maximum value? Justify your answer.  	4. $g(x)$ has a larger maximum value of 10 because the maximum value occurs at the vertex and the vertex of $g(x)$ is seen to be (3, 10). Using the vertex formula to find the maximum value of $f(x)$ gives $x = -\frac{b}{2a}$ $x = -\frac{-2}{2(-1)} = -1$ $f(-1) = -(-1)^2 - 2(-1) + 8 = 9$ Therefore, the vertex of $f(x)$ is (- 1, 9) and the maximum value is 9. This shows the maximum value of $g(x)$ is larger than the maximum value of $f(x)$.
5. Casey kicked a soccer ball from the ground. After 3 seconds, it reached a maximum height of 10 feet. The ball hit the ground again after 6 seconds. Write a function $h(t)$ that relates the height, h, in feet to the time, t, in seconds. (**M.A1HS.22, 26**)	5. Since the zeros of the function are 0 and 6, we see that $h(t) = at(t - 6)$ where a is a constant. Using the vertex of (3, 10) and substituting into the function to solve for a gives $10 = a(3)(3 - 6)$ $10 = -9a$ $-\frac{10}{9} = a$ Replacing a in our original function gives $h(t) = -\frac{10}{9}t(t - 6)$ This could be expanded to standard form if desired.
6. The functions $f(x)$ and $g(x)$ are given on the graph and $g(x) = f(kx)$. Determine the value of k. (**M.A1HS.28**)  	6. Since the x-values of $f(x)$ have been expanded by 3, the graph of f(x) has been dilated by a factor of 1/3. Therefore, $k = 1/3$.

Additional examples and solutions

7. Determine if the given functions are linear, quadratic, exponential, or none. Justify your answers.
(**M.A1HS.29**)

a.

x	f(x)
- 2	- 4
-1	- 6
0	- 6
1	- 4

b.

x	g(x)
- 2	- 1/25
-1	- 1/5
0	- 1
1	- 5

c.

x	h(x)
- 2	- 7
-1	3
0	1
1	- 1

d.

x	j(x)
- 2	13
-1	11
0	9
1	7

7.

a. Quadratic because the second difference of the output is constant over equal length interval input values.

b. Exponential because the outputs are multiplied by a constant value (5) over equal length interval input values.

c. None. There is not a common difference, common ratio, or constant second difference over equal length interval input values.

d. Linear because the first difference of the output values is constant over equal length interval input values.

Misconceptions:

- Confusion with factored form of quadratic functions: When writing quadratic functions in factored form, students might have difficulty relating the factors to zeros, extreme values, and symmetry of the graph. This misconception can lead to errors when interpreting and graphing quadratic functions.
- Overlooking equivalent forms of functions: Students may fail to recognize the various equivalent forms of quadratic functions, such as using different methods of factoring to arrive at the same solution. This lack of understanding can limit their ability to analyze and interpret different properties of the functions.
- Difficulty interpreting graphs and tables: When analyzing graphs and tables of functions, students may struggle to identify key features accurately, such as intervals of increase, decrease, and the behavior at extreme points. This misconception can affect their ability to draw meaningful conclusions from data representations.
- Misinterpreting real-world situations: Students may misinterpret a graphical representation of a quadratic function to be a “picture” of a scenario (i.e. the path of an object in the air) instead of its graph that relates two quantities (i.e. time and height).

Applications:

See Heart of Algebra – Quadratic section on *page 72* of this document.

Career Connections:

See Heart of Algebra – Quadratic section on *page 73* of this document.

Standards Reference List by Category

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