BUILDING NUMERACY
THROUGH MATHEMATICAL PROGRESSIONS

Birth to Grade 3

West Virginia Department of Education

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Mathematics Progressions for Birth to Grade Three

"Numeracy" is a term that refers to all the mathematics that young students learn including number, operations, and geometry and measurement concepts. This document was created by the West Virginia Department of Education Office of Early Learning primarily as a tool to help teachers and parents to understand the role of progressions in developing numeracy skills in children. Having knowledge of how children progress in their early development of numeracy concepts helps teachers and parents to select and use activities that will intentionally build numeracy skills in children. Building these skills is foundational for children as they progress through their study of mathematics. This document is designed to facilitate discussions related to mathematics progressions and to indicate the body of concepts that can be developed in young children.

A study within a professional learning community of the resource documents used to create these progressions would be a recommended way to build educator capacity and deepen understanding of developing numeracy in young children. An informed teacher could then use this document as a quick reference to isolate children's learning in a particular domain. Determining the next steps in learning and the associated instructional tasks to accomplish that learning would move children forward in mastering the standards within the domain. It should be noted that while the Mathematics Progressions for Birth to Grade Three is divided into mathematical domains, there is much cross-over between these domains. This linear model does not accurately represent those connections or the coherence found between domains. Reading the listed resources and the progressions found in the Progressions Documents for Math Standards, and The University of Arizona Institute for Mathematics and Education *http://ime.math.arizona.edu/progressions/* the will help teachers make those connections.

Research has shown that children are capable of learning math concepts at a much earlier age than has been previously recognized. A study has indicated that when controlling for IQ, family income, gender, temperament, type of previous educational experience, and whether children came from single or two parent families; the mastery of early math concepts upon school entry was the strongest predictor of future academic success. Children develop their cognitive knowledge at different rates and the suggested age ranges are only that.
It is the hope of the West Virginia Department of Education Office of Early Learning that this document will be the start of a discussion and then a movement to help children develop their interest and capacity to learn meaningful mathematics in their early years.

School readiness and later achievement.
Duncan, Greg J.; Dowsett, Chantelle J.; Claessens, Amy; Magnuson, Katherine; Huston, Aletha C.; Klebanov, Pamela; Pagani, Linda S.; Feinstein, Leon; Engel, Mimi; Brooks-Gunn, Jeanne; Sexton, Holly; Duckworth, Kathryn; Japel, Crista
Developmental Psychology, Vol 43(6), Nov 2007, 1428-1446. *http://dx.doi.org/10.1037/0012-1649.43.6.1428*

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Operations and Algebraic Thinking

	Operations	Algebraic Thinking
0-36 months	• Uses gestures to ask for more. • Understands getting more or having less. • Uses fingers to show how old they are, or how many objects they have.	• Active explorations in relationship to people and properties • Demonstrates an awareness that people and objects still exist when they are out of sight or sound range. • Uses a variety of search strategies to find hidden objects • Uses familiar objects to represent something else. • Understands that some symbols have meaning.
3-4 years	• Knows that a whole is bigger than its parts. • Finds the total sum of small groups of items. • Combines and separates up to five objects and describes the parts.	• Intuitively has understanding that when adding two numbers, it does not matter which number "comes first."
4-5 years	• Finds results for joining and take-away by using concrete objects and counting all.	• Intuitively has understanding that when adding three numbers it does not matter which two you add first.
5-Kindergarten	• Uses counting-on and counting-up-to strategies to find results. • Fluently adds and subtracts within 5. • Represents addition and subtraction within 10 with objects. • Makes sets of 6-10 objects and describes the parts, identifying which part has more, less, or the same.	• Makes algebraic generalizations (such as, subtracting zero from any number gives that number, or subtracting a number from itself gives zero), with guidance.
Grade 1	• Represents and solves all addition and subtraction situations (result unknown, change unknown, start unknown) within 20. • Adds and subtracts within 20 using a variety of informal and intuitive strategies and can describe the strategies used. • Demonstrates fluency within 10. • Works with addition and subtraction equations with the unknown in all positions.	• Understands and applies properties of operations and the relationship between addition and subtraction. • With guidance, makes algebraic generalizations, such as any number added to and then subtracted from x, leaves x.
Grade 2	• Solves one- and two-step word problems within 100 with unknowns in any situation and subtype. • Describes thinking when solving a word problem. • Adds and subtracts within 20, knowing facts by end of year.	• Works with equal groups of objects to gain foundations for multiplication.
Grade 3	• Represents and solves problems involving multiplication and division. • Multiplies and divides within 100. • Multiplies a single digit by a multiple of 10. • Solves problems involving the four operations.	• Understands properties of multiplication and the relationship between multiplication and division. • Identifies and explains patterns in arithmetic.

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Measurement and Data

	Measurement	Geometric Measurement
0-36 months	• Attends to overall appearance of size, labeling as big/little. • Explores measuring tools, such as measuring cups, or a ruler. • Understands some effects of size or weight when picking up or moving objects.	• Manipulates shapes individually, but does not combine to compose a larger shape. • Explores or examines objects and watches objects when they move. • Explores how things fit together. • Predicts how things move or fit together, or inside other things.
3-4 years	• Compares size by sight, feel, and comparing to hands, feet, etc. • Compares and orders a small set of objects as appropriate according to size, length, weight, area, volume. • Knows usual sequence of basic daily events. • Knows a few ordinal numbers.	• Fills simple pattern block puzzles using trial and error.
4-5 years	• Physically aligns two objects to determine which is longer or if same length. • May be able to measure with ruler, but often lacks understanding or skill.	• Puts several shapes together to make one part of a picture.
5-Kindergarten	• Describes measureable attributes of objects. • Directly compares two objects. • Uses ordinal numbers from first to tenth. • Uses measurement words and some standard measurement tools accurately.	• Analyzes and compares two-dimensional and three-dimensional shapes in different sizes and orientations. • Models shapes by building them from component parts. • Composes simple shapes to make larger shapes.
Grade 1	• Measures lengths indirectly and by iterating length units without gaps or overlaps. • Tells and writes time in hours and half-hours.	• Composes two-dimensional and three-dimensional shapes to create a composite shape.
Grade 2	• Measures and estimates lengths in standard units. • Relates addition and subtraction to length and the number line. • Tells time to the nearest 5 minutes. • Solves problems involving money. • Recognizes that comparisons are valid only when referring to the same unit.	• Partitions a rectangle into rows and columns of same size squares and count to find total.
Grade 3	• Tells and writes time to the nearest minute and measures time intervals. • Solves addition and subtraction time interval problems on a number line. • Measures and estimates liquid volumes and masses of objects using grams, kilograms, and liters. • Solves problems of all types that involve measurement.	• Understands concepts of area and relates area to multiplication and to addition. • Recognizes perimeter as an attribute of plane figures and solves area and perimeter problems.

Mathematics Progressions for Birth to Grade Three

Measurement and Data

	Categorical Data	Measurement Data
0-36 months	• Sorts toys or other objects by color shape or size. • Matches objects by similar or related characteristics. • Explores or examines differences between different types of objects. • Sorts objects into two groups based on a single characteristic.	
3-4 years	• Sorts and/or describes objects by a non-geometric attribute (size, color) or by shape.	
4-5 years	• Sorts objects by one attribute	
5-Kindergarten	• Sorts and classifies objects into more than one category and counts the number in each. • Resorts objects into new categories.	
Grade 1	• Organizes, represents and interprets data with up to three categories.	
Grade 2	• Draws a bar and/or picture graph and answers questions related to them.	• Generates measurement data by measuring length to the nearest whole unit. • Shows length measurements on a line plot marked with whole number units, connecting this to the representation of whole numbers on a number line.
Grade 3	• Draws scaled picture and bar graphs to represent a data set and solves problems.	• Generates data by measuring lengths marked with whole numbers, halves and fourths and shows on a line plot, connecting this to the representations of fractions on a number line.

Mathematics Progressions for Birth to Grade Three

Geometry

	Shapes	Spatial Relationships/Structuring
0-36 months	• Puts things together, such as simple matching puzzles, nesting cups. • Matches familiar shapes (circle, square, typical triangle) with the same size and orientation. • Recognizes and names circles and squares, maybe triangles. • Matches shapes by rotating to prototype. • Judges shapes the same if they are more visually similar than different.	• Follows simple directions related to position (in, on, up, down).
3-4 years	• Recognizes more shapes in real-world, less typical triangle shapes, and some rectangles with same size and orientation. • Recognizes some shapes with different sizes and orientation.	• Follows simple directions related to proximity (behind, under, beside, next to, between).
4-5 years	• Recognizes and compares most familiar shapes and typical examples of other shapes, such as cubes and sphere. • Composes simple shapes to form larger shapes. • Names at least some three-dimensional shapes. • Recognizes sides and angles as distinct geometric objects.	• Identifies positions of objects in space by using words like, beside, inside, next to, above, below, under. • Uses and responds to positional words.
5-Kindergarten	• Names shapes regardless of orientation. • Identifies shapes as two-dimensional and three-dimensional • Recognizes shapes in the environment.	• Uses and makes simple sketches to locate objects. • Correctly uses position words to describe objects.
Grade 1	• Distinguishes between defining and non-defining attributes.	
Grade 2	• Recognizes and draws shapes having specified attributes.	• Mentally constructs an array that is tiled with squares that line up in row and columns.
Grade 3	• Understands that shapes in different categories may share attributes. • Recognizes rhombuses, rectangles, and squares as quadrilaterals.	• Distinguishes between linear and area measures.

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Mathematical Habits of Mind	Ways to incorporate into the classroom	Questions to ask
1. 1. Make sense of problems and persevere in solving them.	• Give students problems worthy of being solved. • Let students develop their own plan to solve a problem.	• What do you notice about_____? • Tell me what you have already done. What would be a good next step? • Is there another way to do this?
2. 2. Reason abstractly and quantitatively.	• Expect students to model, interpret, and connect multiple representations.	• How did you decide what to do? • What do the numbers (or objects) represent? • Explain your model to me.
3. 3. Construct viable arguments and critique the reasoning of others.	• Create a safe environment so that all students feel free to explain their thinking and to critique the reasoning of others. • Have students use objects or pictures to justify reasoning. • Ask students to justify their own reasoning.	• How do you know? • What is the same? • What is different? • Will it still work if _____?
4. 4. Model with mathematics.	• Expect students to be able to model a situation. • Have a number of students show and explain their models.	• How can you represent this? • Could you use these (manipulatives, blocks, counters) to represent the situation?
5. 5. Use appropriate tools strategically.	• Use technology and math tools (manipulatives, measurement tools, calculators) when appropriate.	• What is your estimate? • Is there any tool that might help you solve this?
6. 6. Attend to precision.	• Use precise mathematical language in the classroom and expect the same of students. • Ask students to identify unites used.	• Is your answer reasonable? • How do you know? • Explain how your solution answers the question.
7. 7. Look for and make use of structure.	• Encourage students to find numerical patterns in their work. • Ask students to look for the component parts to a problem and explain each.	• What do you notice? • Do you see any patterns? • Where have you seen this before? • What do you already know that would help you solve this?
8. 8. Look for and express regularity in repeated reasoning.	• Encourage students to generalize numerical patterns. • Ask students to predict answers.	• Is this always true? • Can you find a rule for this? • What would you predict to be the solution? Why?

Mathematics Progressions for Birth to Grade Three

Resources Adapted from:

West Virginia Early Learning Standards Framework: Infant/Toddler
http://www.wvearlychildhood.org/resources/wv_early_learning_standards_framework_infant_toddler.pdf , www.wvchildcare.org

Early Learning Standards Framework for West Virginia Pre-K, 2014 West Virginia Department of Education, Office of Early Learning
https://wvde.state.wv.us/oel/docs/earlylearning_standardsframework_brochureWEB.pdf

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Washington State Early Learning and Development Guidelines: Birth through 3rd Grade 2012, Washington State Department of Early Learning, Olympia, WA

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West Virginia College and Career Ready Standards for Mathematics, Grades K-3 2016, West Virginia Department of Education, Charleston, WV
http://wvde.state.wv.us/policies/

Implementing Standards for Mathematical Practices, Institute for Advanced Study/ Park City Mathematics Institute/Created by Learning Services, Modified by Melisa Hancock 2013.

Progressions Documents for Math Standards, the Brookhill Foundation, and The University of Arizona Institute for Mathematics and Education 2013. http://ime.math.arizona.edu/progressions/

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