Unit Name: Unit 1: Extending the Number System

Lesson Plan Number & Title: Lesson 3: Comparing and Contrasting Equivalency (Exact vs. Approximate Solutions)

Grade Level: High School Math II

Lesson Overview:
Students will be able to explain orally or in written format a working definition of equivalent values using radicals and exponents with precision and fluency. Although not directly referenced in this standard, lesson goals are to establish readiness for the unit on the topics of exponential laws, radical simplification, equivalency and numeracy. The standards do not require answers to be stated as simplified radicals. However, a student must be able to recognize equivalency such that the square root of 50 is equivalent to 5 times the square root of 2.

Focus/Driving Question:
Can you think of multiple representations for the same number? Do you know if a number is considered exact or approximate? When is it appropriate to use an exact answer? When is it appropriate to approximate?

West Virginia College- and Career-Readiness Standards:
M.2HS.1
Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. (e.g., We define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.)

Manage the Lesson:
Step 1: What is the relationship between an exact vs. approximate number? Develop the relationship with your students by watching the video Radical Numbers: How to Find Radicals From Decimals – http://www.youtube.com/watch?v=kNqtCFVQdqk on how to convert a decimal into a radical. Students should follow the examples with their calculators to investigate the relationship between a radical and decimal. In a partner activity, have each student create a decimal number less than three. Ask them to exchange their number with a partner to investigate converting the number into a radical. What did they observe during the process? Is there a connection to exponential numbers? How might they use their knowledge of exponents to explain the process to someone else? Have the students share with one another in a whole class discussion.

Step 2: Develop vocabularies and create a word wall with your students and develop the following definitions-relate the vocabulary to their investigation/explore activity (Step 3) on the accompanying vocabulary listed in Vocabulary Development. Ask them to create examples of the vocabulary on the word wall.

Step 3: Build upon student knowledge- Investigate/Explore Can you think of multiple representations for the same number? Using the Equivalency of Square Roots Sorting Activity individually or in groups, students sort the cards into equivalent values. Discuss with students the reasons for their choices. Are there any patterns they observe?

Step 4: Students will demonstrate their knowledge through the incorporation of student practice utilizing a variety of materials. The combination of materials listed can be adapted to your students learning styles and abilities. For example, breaking the assignment into shorter tasks can guide your instruction and

provide informal assessment on student mastery. Depending upon access to computers, the use of computer games as student practice can create higher student interest for the reluctant learner or student with attention span issues. All of the materials listed below may not be needed for student mastery of the lesson objective. When planning lesson implementation, select the materials most appropriate for your student's needs.

Instructional Activities
[Equivalency of Square Roots Contest](#) challenge your students to develop multiple representations for the given radical
[Exact vs. Approximate](#) use the prompt to stimulate discussion on a comparison of numbers
[Find and Fix – Square Roots](#) individually or in groups, students determine where the errors are and correct them
Illuminations: Stacking Squares – http://illuminations.nctm.org/LessonDetail.aspx?id=L622 illuminations activity to investigate squares vs. cubes equivalency

Computer Practice
http://www.sascurriculumpathways.com/portal/ Activity # 86 complete the related handout and conduct online research to investigate the connection between radicals and powers

Puzzles and Games
Estimation Games – http://www.mathsisfun.com/numbers/estimation-game.php

Step 5: Ask students to create one example each using exponents and radicals. This activity can be adapted to student instructional levels and needs. Each example should include an "answer" key shown on the second page of the printable [My Quiz](#), where they need to provide 3 alternative forms their problems can be written as equivalent expressions. Each student trades with a partner or group the first page of the activity *My Quiz*, keeping the answer key hidden. Once the quiz is completed, it is returned to the designer who grades it using their key. In whole group, have students discuss the results. Did everyone develop similar solutions? Were there any equivalent expressions not thought of by the key developed that were correct? Students can hand in the results of the activity to develop an individual assessment using the student created problems.

Step 6: Reflect with your students regarding the lesson. Ask them to consider the following: Can you think of multiple representations for the same number? Do you know if a number is considered exact or approximate? When is it appropriate to use an exact answer? When is it appropriate to approximate? Keeping these questions in mind or write them on the board as focus questions, ask the students to write 3 response statements on their paper to hand in as exit slips before leaving the class. Read their responses carefully to gain insight into student understanding. Do you need to further develop the concept of equivalency or have they established mastery to continue further in the unit?

Academic Vocabulary Development:
Develop vocabularies and create a word wall with your students and develop the following definitions-relate the vocabulary to their investigation/explore activity (Step 3) on the accompanying vocabulary listed in Vocabulary Development. Ask them to create examples of the vocabulary on the word wall.

equivalent expression - Two algebraic expressions are said to be equivalent if their values obtained by substituting the values of the variables are same
exact solution - a solution in which no rounding has been utilized to derive a solution
approximate solution - a solution in which rounding was given to simplify the solution

Launch/Introduction:
What is the relationship between an exact vs. approximate number? Develop the relationship with your students by watching the video Radical Numbers: How to Find Radicals From Decimals – http://www.youtube.com/watch?v=kNqtCFVQdqk on how to convert a decimal into a radical.

Investigate/Explore:
Can you think of multiple representations for the same number? Using the Equivalency of Square Roots Sorting Activity individually or in groups, students sort the cards into equivalent values.
The purpose of the lesson is to scaffold necessary skills for the unit by the consideration of equivalent expressions with a focus on radicals and exponents. The launch activity encourages instructional discovery of equivalency through matching equivalent expressions and the development of vocabulary guided by the instructor through the creation of a word wall. Students develop alternative equivalent expressions for radicals and exponents in the lesson activities. The instructor can choose to do these as individual, small group or whole group instruction depending on student abilities or necessary modifications. The process of instructional practice by students can be flexible instruction by the assignment of handouts, games, or online computer practice according to instructional resources and differentiation determined by the instructor. Assessment of the lesson is a continual part of the instructional process, whether informal or formal and should guide the instructor in the directing of instruction. A short assessment in step 5 of Manage the Lesson is provided for students to create their own "quiz" with solutions and encourages students to think through the process. Both students and instructors are encouraged to reflect upon the lesson and knowledge gained in whole group discussion in terms of knowledge gained regarding the use of equivalency.

Summarize/Debrief:
Instructors can use the focus questions in Step Six of Manage the Process to create student reflections on the lesson. These can be used to gain insight into student comprehension or misunderstandings.

Materials:
Graphing Calculator, Word Wall Materials (construction paper, markers), foldable (white copy paper or notebook paper, markers or colored pencils, scissors), optional-computers
Equivalency of Square Roots Contest printable
Equivalency of Square Roots Sorting Activity printable
Exact vs. Approximate printable
Find and Fix – Square Roots printable
My Quiz printable
Websites:
http://www.sascurriculumpathways.com/portal/
http://www.mathsisfun.com/numbers/estimation-game.php
http://illuminations.nctm.org/LessonDetail.aspx?id=L622
http://www.youtube.com/watch?v=kNqtCFVQdqk
Teacher Information
https://www.teachingchannel.org/videos/common-core-state-standards-for-math?fd=1

Career Connection:
Equivalent expressions are necessary for converting units of measure, developing proportions, transformations and literal equations such as solving for a particular unknown in a formula. All of the following career clusters use the previously stated concepts: Arts and Humanities Cluster, Business and

Marketing Cluster, Engineering and Technical Cluster, Health Cluster, Human Services Cluster and Science and Natural Resources Cluster

Lesson Reflection:
In Step 6 of Manage the Lesson, students and the instructor react to statements given in a pair/share partner activity before a whole group discussion on their learning. Instructors may want to watch Common Core State Standards for Math – https://www.teachingchannel.org/videos/common-core-state-standards-for-math?fd=1 to gain a better understanding of the unit's structure and reflect in a journal entry as they begin their teaching of the unit on the day's lesson success.

Equivalency of Square Roots Sorting Activity

Each student/group will need one set of cards. Students are to sort cards into piles that represent the same quantity.

$\sqrt{100}$	$5\sqrt{4}$	10	$2\sqrt{25}$
$\sqrt{\dfrac{16}{9}}$	$\dfrac{4}{3}$	$\dfrac{\sqrt{16}}{3}$	$\dfrac{4}{\sqrt{9}}$
$\sqrt{\dfrac{1}{4}}$	$\dfrac{1}{2}$	$\dfrac{\sqrt{1}}{2}$	$\sqrt{72}$

$6\sqrt{2}$	$\sqrt{500}$	$5\sqrt{20}$	$10\sqrt{5}$
$\sqrt{169}$	13	$\sqrt{180}$	$6\sqrt{5}$
$3\sqrt{63}$	$9\sqrt{7}$	$5\sqrt{81}$	$15\sqrt{9}$

45	$\dfrac{3\sqrt{7}}{\sqrt{36}}$	$\dfrac{3\sqrt{7}}{3\sqrt{4}}$	$\dfrac{3\sqrt{7}}{6}$
$\dfrac{\sqrt{7}}{2}$	$\sqrt{64}$	$2\sqrt{16}$	8

Equivalency of Square Roots Contest

Directions for Equivalency of Square Roots Contest

Students may either be in groups or work individually.

Write the following problems (one at a time) on the board and challenge the students to rewrite the quantity in has many ways as possible. Students may either answer verbally or record their answers on paper and then respond after a given amount of time. Students earn 1 point for each correct response given. The winner will be the student/group that has the most points.

Example:

$\frac{\sqrt{25}}{\sqrt{36}}$ possible responses include but are not limited to $\frac{5}{6}, \frac{5}{\sqrt{36}}, \frac{\sqrt{25}}{6}, \frac{5}{2\sqrt{9}}, \frac{5}{3\sqrt{4}}$

1. $\sqrt{200}$
2. $\frac{\sqrt{50}}{\sqrt{2}}$
3. $\sqrt{320}$
4. $4\sqrt{72}$
5. $\frac{\sqrt{100}}{\sqrt{625}}$
6. $\sqrt{225}$
7. $\sqrt{144}$
8. $\sqrt{250}$
9. $\frac{\sqrt{180}}{\sqrt{160}}$
10. $\sqrt{120}$

~ EXACT vs. APPROXIMATE ~

Note the following statements:

$\frac{1}{3} \neq .3$

$\frac{1}{3} \neq .33$

$\frac{1}{3} \neq .333$

$\frac{1}{3} \neq .3333$

$\frac{1}{3} \neq .33333$

$\frac{1}{3} \neq .333333$

$\frac{1}{3} = .3333 \ldots$

Similarly,

$\sqrt{2} \neq 1.4$

$\sqrt{2} \neq 1.41$

$\sqrt{2} \neq 1.414$

$\sqrt{2} \neq 1.414159$

$\sqrt{2} = 1.414159 \ldots$

Conclusion:

One can only use the = sign if two quantities are actually equal or equivalent. That is, if they live at the exact same spot on the number line.

So for example, while $\frac{1}{3}$ lives very close to 0.33333, it does not actually live at 0.33333.

Therefore, while $\frac{1}{3} \approx 0.33333$, $\frac{1}{3} \neq 0.33333$.

There are times where you need an exact answer, and times when you need an approximate the answer.

For example, $\sqrt{27}$ can only be written exactly as $\sqrt{27}$, but it can be approximated as 5, 5.2, 5.20, 5.196, etc., depending the accuracy or number of decimal places required.

Complete the following with a partner:
T or F

1. $\sqrt{4} = 2$ _____

2. $\sqrt{5} = 2.236067977$ _____

3. $\frac{1}{6} = .16$ _____

4. $\frac{1}{6} = 0.16$ _____

5. $\frac{1}{6} = 0.1666666$ _____

6. $\frac{1}{6} = 0.166...$ _____

7. $\frac{1}{6} = 0.16666...$ _____

8. $\frac{1}{6} = 0.16...$ _____

9. $\frac{9}{7} = 1.285714286$ _____

10. $\frac{9}{7} = 1 + \frac{2}{7}$ _____

Find and Fix – Square Roots

Name _____

Find and fix the errors in the following problems.

1. $\sqrt{30} = 15$

4. $6\sqrt{16} = 10$

2. $\sqrt{25} = \sqrt{5}$

5. $5\sqrt{45} = 5\sqrt{5 \cdot 3 \cdot 3} = 15$

3. $\dfrac{7\sqrt{35}}{5} = 7\sqrt{7}$

6. $\dfrac{4\sqrt{18}}{\sqrt{4}} = \sqrt{18} = 3\sqrt{2}$

My Quiz

Name _____

Name of student taking My Quiz _____

In this activity, on the first page create two types of problems; one radical problem and one exponential problem – leave the equivalent expression sections blank. On the second page of the activity, copy the radical problem and the exponent problem and provide 3 equivalent expressions for each of the problems. This will be your answer key. Keep your answer key hidden and hand your quiz problems on the first page to another student to solve. Use your answer key to grade their answers.

Radical Problem _____

Equivalent Expression

1. _____

2. _____

3. _____

Exponent Problem _____

Equivalent Expression

1. _____

2. _____

3. _____

Name _____

My Quiz Answer Key

Radical Problem _____

Equivalent Expression

    1. _____

    2. _____

    3. _____

Exponent Problem _____

Equivalent Expression

    1. _____

    2. _____

    3. _____