

**Unit Name:** Unit 1: Extending the Number System

**Lesson Plan Number & Title:** Lesson 5: Rational and Irrational Numbers

**Grade Level:** High School Math II

**Lesson Overview:**

Students should be able to explain, either orally or in written format, the relationship between sums and products between two rational numbers equals a rational solution, the sum of a rational and irrational will yield an irrational solution and a product of a nonzero rational number and irrational number produces an irrational solution.

**Focus/Driving Question:**

How are the solutions between sums/products of rational and irrational numbers different?

**West Virginia College- and Career-Readiness Standards:**

M.2HS.3

Explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational and that the product of a nonzero rational number and an irrational number is irrational. Instructional Note: Connect to physical situations, e.g., finding the perimeter of a square of area 2.

**Manage the Lesson:**

**Step 1:** Launch the lesson with [Real Number System Notes](#) (convert to a powerpoint). Use the Venn diagram to develop the vocabulary and assist students in the categorization of numerical examples.

**Step 2:** In order to create this foldable, each student will need 3 sheets of paper (different colors, if available). Offset the sheets of paper by approx.  $\frac{1}{2}$  inch. Fold the papers in half and staple at the top. You should now have a tablet with 6 pages. Use the [Irrational-Rational Number Foldable](#) from the <http://rvec.weldoncityschools.org> for what should be on each page. Connect the lesson vocabulary to the foldable.

**Step 3:** Build upon student knowledge by creating a [Wheel of Theodorus](#) (printable) to develop the definitions of rational vs. irrational numbers. Instructors may want to review the Pythagorean Theorem at the beginning of the activity. You will need 11 x 14 construction paper, scissors, colored pencils, and straight edge for each student.

**Step 4:** Students will demonstrate their knowledge through the incorporation of student practice utilizing a variety of materials. The combination of materials listed can be adapted to your students learning styles and abilities. For example, breaking the assignment into shorter tasks can guide your instruction and provide informal assessment on student mastery. Not all of these materials may be needed for student mastery of the objectives. All of the materials listed below may not be needed for student mastery of the lesson objective. When planning lesson implementation, select the materials most appropriate for your student's needs.

*Instructional Video*

The Closure Property - <http://www.youtube.com/watch?v=jRbRhplK2dE> (Explanation of the closure property)

Perimeter, Area, and Decimals – Can the perimeter of a rectangle be an irrational number? – [http://www.youtube.com/watch?v=gjwXiau\\_Q1c&safe=active](http://www.youtube.com/watch?v=gjwXiau_Q1c&safe=active)

### *Instructional Activities*

TI Classroom Activities: Number Sets –

<https://education.ti.com/en/us/activity/detail?id=BE58A201054949C5BDED03441E9CCD84> (Develop the relationships between number categories using a Venn Diagram and a graphing calculator, includes printables for student and teacher)

The Rational Number System WS –

[http://www.youtheducationsservices.ca/secure/subjects/number\\_concepts/pdfs/9\\_Rat\\_Numb\\_Wksht.pdf](http://www.youtheducationsservices.ca/secure/subjects/number_concepts/pdfs/9_Rat_Numb_Wksht.pdf)

(Develop the Number System using a Venn Diagram) - printable

Closure Property of Real Number Addition and Multiplication – <http://math.tutorcircle.com/number-sense/properties-of-irrational-numbers.html> (Teacher Resource for discussion on Closure regarding Rational and Irrational numbers. Adapt the instruction to include student provided numbers for demonstrating what is meant by the closure property.)

TI Classroom Activity – The Real Number System -

<https://education.ti.com/en/us/activity/detail?id=2780D45A78E24BADA22B8C161844909E>

(Students explore the real number system using graphing calculator technology followed by a quiz - Includes printables for student and teacher.)

Numbers Beyond Reason! – <http://www.beaconlearningcenter.com/Lessons/1026.htm> (Lesson plan and writing activity on comparing rational and irrational numbers using "Numbers in the Sand" from *Marvels of Math* by Kendall Haven, Teachers Idea Press, 1998.)

[Number System](#) Small Group Activity - Using manipulatives to explore the number system with integer chips, number line on board

word wall, and number line drawn on the floor, followed by a quiz on the activity

[Extending Irrational Numbers](#) (extension lesson based on Euclid and irrational numbers)

### *Games*

Number Set Tiles – [http://busynessgirl.com/files/games/number\\_set\\_tiles.pdf](http://busynessgirl.com/files/games/number_set_tiles.pdf) (Printable sheet for students to sort individually, in pairs or small groups. Suggestions are provided for the sorting activity.)

Rational and Irrational Numbers Game – <http://www.math-play.com/rational-and-irrational-numbers-game/rational-and-irrational-numbers-game.html> (Online games where students sort numbers into rational and irrational categories)

**Step 5:** Assess student mastery- Summarize/Debrief Use the [Rational and Irrational Graphic Organizer](#) to provide examples of each category. Ask the students if there any examples that they cannot think of a number for? Review the different types of numbers and the responses given by students. On the back of the printable, have the students explain why the sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational and that the product of a nonzero rational number and an irrational number is irrational using the numbers they provided for in the organizer.

**Step 6:** Read and reflect using [Rational and Irrational Reflection](#) (printable) Directions to the students- Write a paragraph in the space below using the two decimal forms of  $17/10$  and  $\pi$  remembering to justify your reasoning. Remind students to use appropriate vocabulary in their reflective writing. When everyone is complete, have students share their reflections with one another, either in partners, small groups or as whole group discussion.

### **Academic Vocabulary Development:**

Add to the word wall with your students and develop the following definitions-relate the vocabulary to their foldable on the number system. Ask them to create examples of the vocabulary on the word wall.

*irrational number*- A number that neither terminates nor repeats

*rational number*- A number that can be written as quotient of two integers

**Launch/Introduction:**

Launch the lesson with [Real Number System Notes](#). Use the Venn diagram to develop the vocabulary and assist students in the categorization of numerical examples.

Students will demonstrate their knowledge through the incorporation of student practice utilizing a variety of materials. Instructors combine materials listed by adapting to your students learning styles and abilities for investigation and practice of the objectives. For example, breaking the assignment into shorter tasks can guide your instruction and provide informal assessment on student mastery. Some materials may be used as online practice by the instructor for individualized remediation. Not all of these materials may be needed for student mastery of the objectives.

**Investigate/Explore:**

Build upon student knowledge- Investigate/Explore with an activity from TI Classroom Activities: Number Sets – <https://education.ti.com/en/us/activity/detail?id=BE58A201054949C5BDED03441E9CCD84>

Develop the relationships between number categories using a Venn Diagram and a graphing calculator with your students. If this is the first time to use one of the TI Activities, work through the activity using the teacher's guide to insure you understand the process. Try to anticipate what problems your students may have with the implementation of the activity in terms of calculator usage. You may need to download the activity apps to student calculators beforehand.

**Summarize/Debrief:**

Use the [Rational and Irrational Graphic Organizer](#) to provide examples of each category. Use the questions provided in Manage the Process for written student reflection and evaluation of their understanding. Share student responses in a whole group discussion.

**Materials:**

Integer Chips, Number Lines, Graphing Calculators, 11 x 14 construction paper, scissors, colored pencils, and straight edge, Word Wall Materials (construction paper, markers), foldable (white copy paper or notebook paper, markers or colored pencils, scissors), optional-computers

[Real Number System Notes](#) (use as a powerpoint)

[Rational and Irrational Graphic Organizer](#) (printable)

[Rational and Irrational Reflection](#) (printable)

[Number System Activity](#) (printable)

[Irrational-Rational Number Foldable](#) (printable)

[Wheel of Theodorus](#) (printable)

[Extending Irrational Numbers](#) (printable)

Websites:

<http://msdunphymath7.weebly.com/uploads/2/1/9/3/21931336/realnumbersystemnotes.pptx>

<https://education.ti.com/en/us/activity/detail?id=BE58A201054949C5BDED03441E9CCD84>

[http://www.youtheducationervices.ca/secure/subjects/number\\_concepts/pdfs/9\\_Rat\\_Numb\\_Wksht.pdf](http://www.youtheducationervices.ca/secure/subjects/number_concepts/pdfs/9_Rat_Numb_Wksht.pdf)

<http://math.tutorcircle.com/number-sense/properties-of-irrational-numbers.html>

<https://education.ti.com/en/us/activity/detail?id=2780D45A78E24BADA22B8C161844909E>

<http://www.beaconlearningcenter.com/Lessons/1026.htm>

[http://www.youtube.com/watch?v=gjwXiau\\_Q1c&safe=active](http://www.youtube.com/watch?v=gjwXiau_Q1c&safe=active)

**Career Connection:**

Knowledge of the Number System and Number Theory is necessary for converting units of measure, developing proportions, and geometry such as calculating perimeter, area and volume of solid. All of the following career clusters use the previously stated concepts: Arts and Humanities Cluster, Business and

Marketing Cluster, Engineering and Technical Cluster, Health Cluster, Human Services Cluster and Science and Natural Resources Cluster.

**Lesson Reflection:**

Read and reflect with your students using [Rational and Irrational Reflection](#) (printable). Remind students to use appropriate vocabulary in their reflective writing. When everyone is complete, have students share their reflections with one another, either in partners, small groups or as whole group discussion.

# The Real Number System

Adapted from:

<http://msdunphymath7.weebly.com/uploads/2/1/9/3/21931336/realnumbersystemnotes.pptx>

# Real Numbers

- Real numbers consist of all the rational and irrational numbers.
- The real number system has many subsets:
  - **Natural Numbers**
  - **Whole Numbers**
  - **Integers**

# Natural Numbers

***Natural numbers*** are the set of counting numbers.

$$\{1, 2, 3, \dots\}$$

# Whole Numbers

***Whole numbers*** are the set of numbers that include 0 plus the set of natural numbers.

$\{0, 1, 2, 3, 4, 5, \dots\}$

# Integers

***Integers*** are the set of whole numbers and their opposites.

$$\{\dots, -3, -2, -1, 0, 1, 2, 3, \dots\}$$

# Rational Numbers

- ***Rational numbers*** are any numbers that can be expressed in the form of  $\frac{a}{b}$ , where  $a$  and  $b$  are integers, and  $b \neq 0$ .
- They can always be expressed by using terminating decimals or repeating decimals.

# Terminating Decimals

- Terminating decimals are decimals that contain a finite number of digits.
- Examples:
  - 36.8
  - 0.125
  - 4.5

# Repeating Decimals

- Repeating decimals are decimals that contain a infinite number of digits.
- Examples:
  - $0.333\dots$
  - $1.\bar{9}$
  - $7.689689\dots$

FYI...The line above the decimals indicate that number repeats.

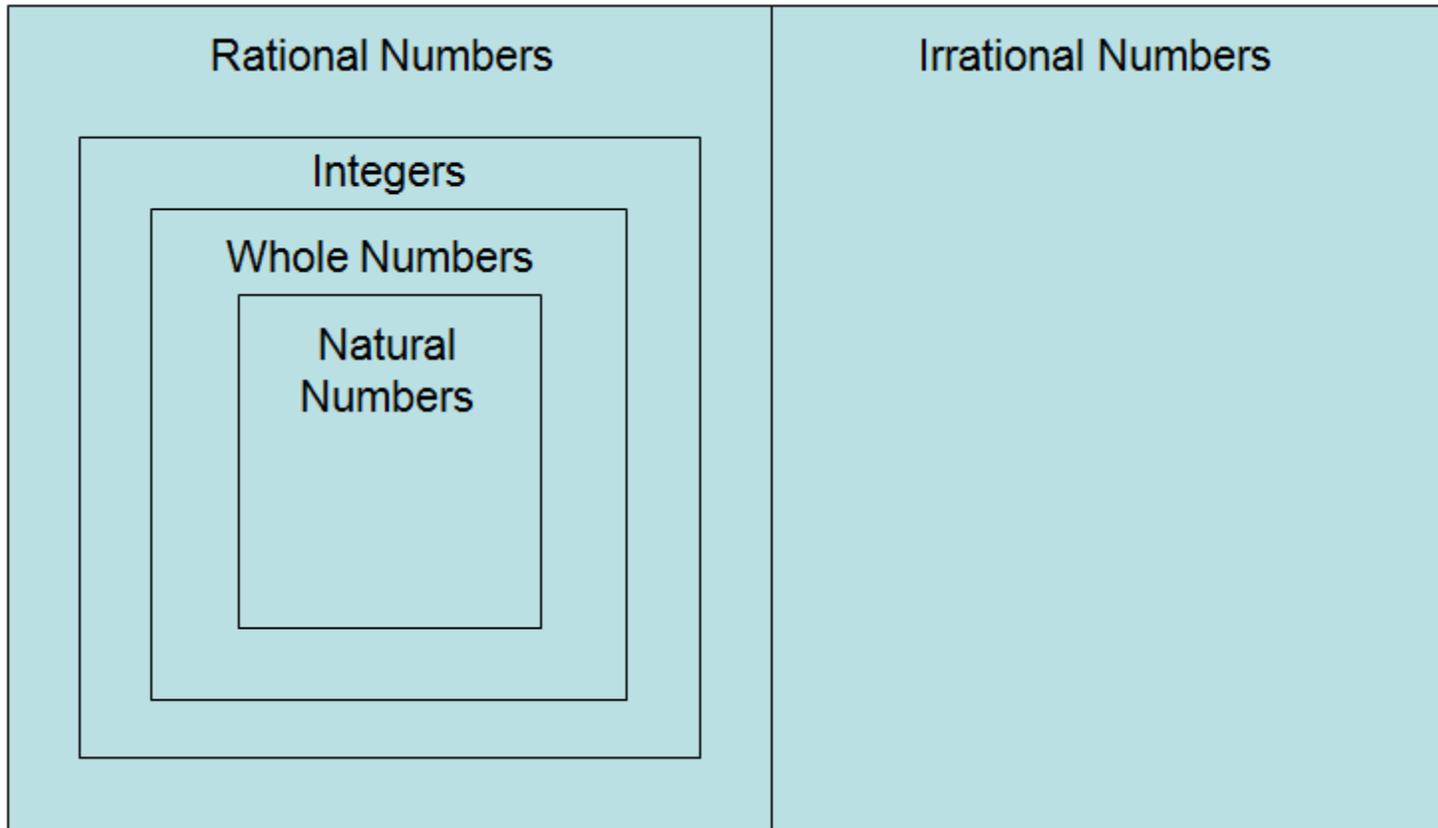
# Irrational Numbers

- ***Irrational numbers*** are any numbers that cannot be expressed as  $\frac{a}{b}$ .
- They are expressed as ***non-terminating, non-repeating decimals***; decimals that go on forever without repeating a pattern.
- Examples of irrational numbers:
  - 0.34334333433334...
  - 45.86745893...
  - $\pi$
  - $\sqrt{2}$

# Other Vocabulary Associated with the Real Number System

- ... (ellipsis)—continues without end
- { } (set)—a collection of objects or numbers. Sets are notated by using braces { }.
- Finite—having bounds; limited
- Infinite—having no boundaries or limits
- Venn diagram—a diagram consisting of circles or squares to show relationships of a set of data.

# Venn Diagram of the Real Number System



# Example

Classify all the following numbers as **natural**, **whole**, **integer**, **rational**, or **irrational**. List all that apply.

a. 117

b. 0

c. -12.64039...

d.  $-\frac{1}{2}$

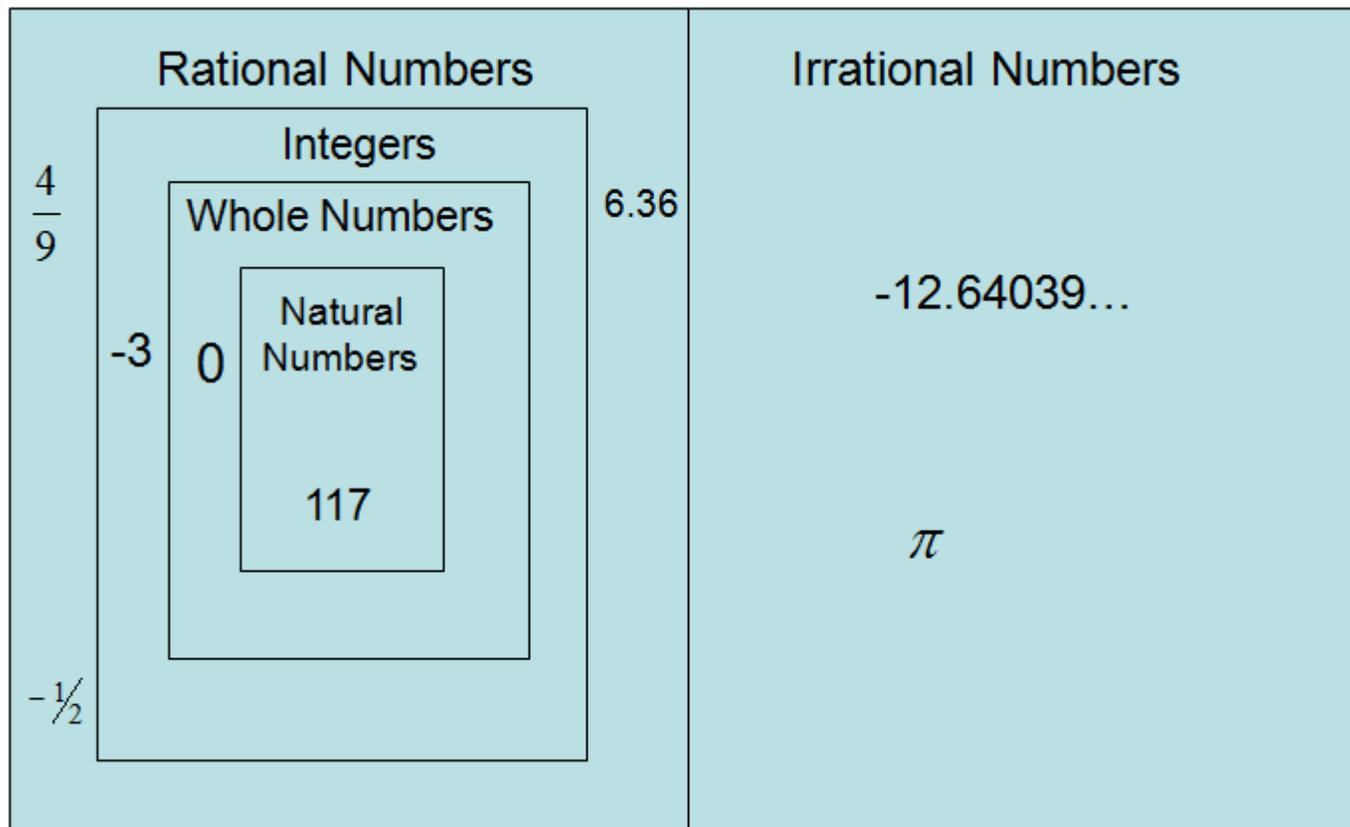
e. 6.36

f.  $\pi$

g. -3

h.  $\frac{4}{9}$

To show how these numbers are classified, use the Venn diagram. Place the number where it belongs on the Venn diagram.



# Solution

Now that all the numbers are placed where they belong in the Venn diagram, you can classify each number:

- 117 is a natural number, a whole number, an integer, and a rational number.
- $-\frac{1}{2}$  is a rational number.
- 0 is a whole number, an integer, and a rational number.
- -12.64039... is an irrational number.
- -3 is an integer and a rational number.
- 6.36 is a rational number.
- $\pi$  is an irrational number.
- $\frac{4}{9}$  is a rational number.

# FYI...For Your Information

When taking the square root of any number that is not a perfect square, the resulting decimal will be non-terminating and non-repeating. Therefore, those numbers are always irrational.

## Irrational – Rational Number Foldable

	Real Number System
1	Natural Numbers
2	Whole Numbers
3	Integers
4	Rational Numbers
5	Irrational

### Natural Numbers

Counting Numbers

1, 2, 3, 4, 5, ...

Example of where you have seen Natural Numbers used.

### Whole Numbers

0 + all the Natural Numbers

0, 1, 2, 3, 4, 5, ...

## Integers

Whole Numbers + all the opposites of the Natural Numbers

..., -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, ...

Example of where you have seen Integers used.

## Rationals

All Natural, Whole, and Integers

Any number you can write as a fraction  $\frac{a}{b}$  where  $a$  &  $b$  are integers with  $b \neq 0$

Any terminating decimal (0.5, 7.13, -6.876, -24.45)

Any repeating decimal (-10., 2., 3.6262..., -2.12333...)

Square roots of perfect square numbers (1, 4, 9, 16, 25, ...)

Example of where you have seen Rational Numbers used.

## Irrationals

Any number that is NOT Rational

Decimals that do not terminate AND do not repeat

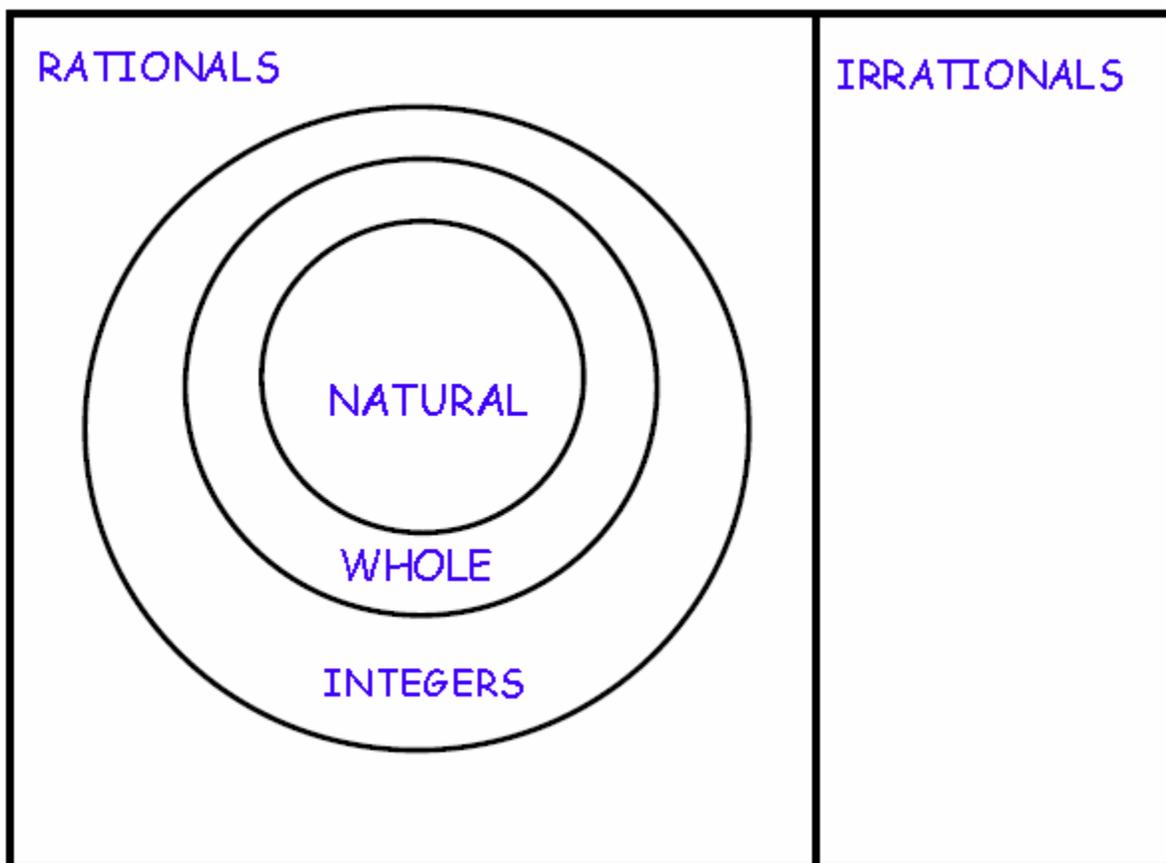
“CRAZY NUMBERS”

Square roots of non-perfect square numbers ( $\sqrt{2}$ ,  $\sqrt{23}$ ,  $\sqrt{30}$ ,  $\sqrt{55}$ )

Can be positive or negative

Example of where you have seen Irrational Numbers used.

## Real Number System



### A1 Real Number System Practice

Classify the following values.

N = Natural; W = Whole; I = Integer; R = Rational  
Irr = Irrational

1.) -5

2.) 8

3.)  $\frac{2}{3}$

4.)  $-5 \frac{1}{8}$

5.)  $\sqrt{9}$

6.) 6.888...

7.) -14.12

## Wheel of Theodorus Art

### Irrational Numbers

Adapted from <http://www.montanamath.org/?p=lessons/msplans>

Submitted by Melissa Romano

Directions for the teacher

**The mathematics:** This lesson allows students to investigate irrational numbers and how they are different from rational numbers. Students create a product that displays irrational numbers.

**Before the task:** Students should be familiar with what a rational number is. Briefly discuss with students the definition of an irrational number. Numbers are irrational if its digits do not terminate or contain repeating patterns, and it cannot be represented as a fraction of integers.

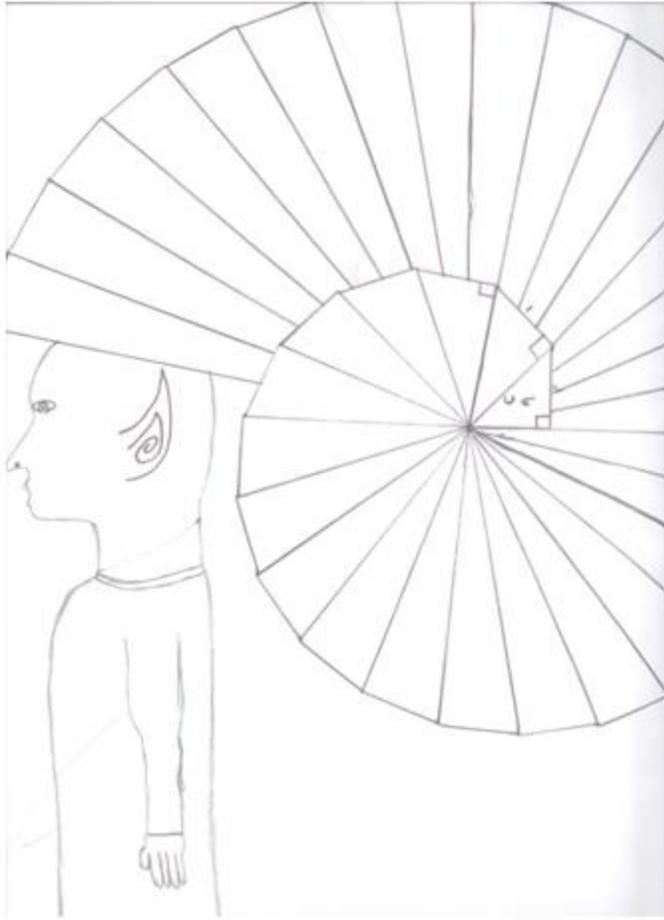
**The Task:** Students will create an image known as the wheel of Theodorus by carefully constructing a series of right triangles with one leg remaining 1 unit long and the other leg being the previous hypotenuse. Using the Pythagorean Theorem (at which point, you may need to review the Pythagorean Theorem and discuss how to calculate the value of the hypotenuse), students will then calculate the length of the hypotenuse of an isosceles right triangle with the length of each leg is 1 inch. Student should draw this triangle in the middle of their paper. Students can see the visual representation of  $\sqrt{2}$  units and compare it with a length of 1 unit in the drawing. Students continue to add a triangle with one leg being 1 unit long. Encourage students to calculate and reason that this new hypotenuse must be  $\sqrt{3}$  units long. Students will continue to add triangles, making a spiral image known as the Wheel of Theodorus. They are free to create their images with as much or as little detail or artistry as they choose. (See examples) Students can continue to label calculated side lengths. Once students have at least ten isosceles triangles drawn, inquire if anyone can observe a pattern regarding the radicals generated in each triangle. Look at the solutions with the students and develop a connection that some square roots are rational ( $\sqrt{4}=2$ ) and some are not.

Steps:

1. Create a template for a particular unit length and a right angle, forming an isosceles right triangle.
2. Using your template again, add another unit length and right angle to the hypotenuse of your original right triangle.
3. Make a right triangle out of the new unit lengths and the previous hypotenuse.
4. Keep adding a new unit length to the previous hypotenuse at right angles to build new right triangles.
5. When you get to the stage where your right triangles will overlap previous right triangles, draw your hypotenuse toward the center of the spiral but do not mark over the previous drawings.

Materials: 11x14 paper, pencil, colored pencils, markers, or other art materials students choose





Name \_\_\_\_\_

### Student Directions

#### Steps:

1. Create a template for a particular unit length and a right angle, forming an isosceles right triangle. Students will then calculate the length of the hypotenuse of an isosceles right triangle with the length of each leg is 1 inch.
2. Draw this triangle in the middle of your paper.
3. Using your template again, add another unit length and right angle to the hypotenuse of your original right triangle.
4. Make a right triangle out of the new unit lengths and the previous hypotenuse. Continue to add triangles, making a spiral image known as the Wheel of Theodorus.
5. Keep adding a new unit length to the previous hypotenuse at right angles to build new right triangles.
6. When you get to the stage where your right triangles will overlap previous right triangles, draw your hypotenuse toward the center of the spiral but do not mark over the previous drawings.

### Student Checklist

- |   |              |
|---|--------------|
| I have drawn a template of an isosceles right triangle composed of legs measuring 1 unit.     | _____ 4pts.  |
| I have calculated the value of the hypotenuse for this triangle and labeled it in my drawing. | _____ 4pts.  |
| I have calculated the value of 19 additional hypotenuses, following each triangle pattern.    | _____ 19pts. |
| I have created a minimum of 19 additional isosceles triangles and labeled them in my drawing. | _____ 19pts. |
| I have created a wheel of Theodorus by carefully constructing a series of right triangles.    | _____ 4pts.  |
| Total   | _____ 50pts. |

Wheel of Theodorus

Name \_\_\_\_\_

Student Calculation Chart

	Leg	Leg	Pythagorean Theorem	Hypotenuse	Rational vs. Irrational
1					
2					
3					
4					
5					
6					
7					
8					
9					
10					
11					
12					
13					
14					
15					
16					
17					
18					
19					
20					

## Number System Activity

### Students' directions:

- Be aware that ovals, circles, and squares or rectangles can be used to represent the universe, or largest, set and its subsets. However, the size and shape of a Venn diagram is unimportant.
- Each felt circular piece represents a different component, or subset, of the real number system.
- The class will be divided into groups of four.
- Manipulative:
  - integer chips (Group 1)
  - a number line drawn on the board (Group 2)
  - a number line made on the floor (Group 3)
- Each group will rotate around the classroom so that all of them will get a chance to use each manipulative.
- Rule for Number line on the floor: the student must begin at 0 when given new instructions and face the positive direction.
- Group 3 will initially begin the activity; Group 1 and Group 2 will represent the movement made by Group 3 by using integer chips and the number line.
- Group 1 will complete the problems below using the integer chips. The green integer chips are negative and the red integer chips are positive chips representing one unit each.
- All other groups will complete the problems below using a number line.
  - +5 =
  - 3 =
  - +5 + (-3) =
  - +6 - (-6) =
  - 1/5 =
  - 2/7 =
  - 10/6 =

Name \_\_\_\_\_

Name the set or sets of numbers to which each real number belongs. Use **N** for natural numbers, **W** for whole numbers, **Z** for integers, **Q** for rational numbers, and **I** for irrational numbers.

1.  $-\sqrt{49}$  \_\_\_\_\_

2. 0.3333333 \_\_\_\_\_

3. 0.6666666 \_\_\_\_\_

4. 3.14 \_\_\_\_\_

5. 0 \_\_\_\_\_

6.  $-1/2$  \_\_\_\_\_

7.  $10/5$  \_\_\_\_\_

8.  $3/5$  \_\_\_\_\_

9. 0.4583 \_\_\_\_\_

10.  $\sqrt{49}$  \_\_\_\_\_

## Teacher Directions

### Activity purpose:

The purpose of these Venn Diagrams is to **assist** students in moving away from a limited focus on isolated segments to include a greater emphasis on integrating concepts. These diagrams promote valid inferences about mathematics learning. They also inform students what relationships they are expected to learn.

### Objectives:

- Students will be able to classify numbers using Venn Diagrams for reference.
- Students will be able to compare and order rational numbers.
- Students will be able to explore problem situations.
- Students will be able to state the coordinate of a point on a number line.
- Students will be able to graph integers on a number line.
- Students will be able to add and subtract integers by using a number line.
- Students' comprehension and performance will exceed 80 % as measured by various activities and assessment tools.

### Procedures for Developing the Activity:

#### Teacher's directions:

- The two main purposes of this activity are to teach students about integers (addition, subtraction,  $<$ ,  $>$ ,  $=$ ) from the previous grade and to get students cognitively focused on the topic.
- A model that's actually a Venn diagram will be constructed of felt circular pieces. They will be prominently labeled and will be used to explain the real number system and pertinent terminology.
- A Venn diagram is a figure often used to represent sets of numbers. They are diagrams that use circles or ovals inside a rectangle to show relationships of sets.
- It will be noted that ovals, circles, and squares or rectangles can be used to represent the universal, or largest, set and its subsets. However, the size and shape of a Venn diagram is unimportant.
- Each felt circular piece will represent a different component, or subset, of the real number system. Each piece will be explained in detail.
  - Real number - The set of rational numbers and the set of irrational numbers together form the set
  - Natural numbers - The set of numbers which is a subset of whole numbers which are greater than zero
  - Whole numbers - The set of whole numbers is represented by  $\{0, 1, 2, 3, \dots\}$ .
  - Integers - The set of numbers used on the number line that compose the set that can be written  $\{\dots, -6, -5, -4, -3, -2, -1, 0, 1, 2, 3, 4, 5, 6, \dots\}$ .
  - Rational numbers - A rational number is a number that can be expressed in the form  $a/b$ , where "a" and "b" are integers and "b" is not equal to 0.
  - Irrational numbers - An irrational number is a number that cannot be expressed in the form of  $a/b$ , where "a" and "b" are integers and  $b \neq 0$ .

- Examples and mathematical problems will be completed to illustrate all components, or subsets, of the real number system.
- The class will be divided into 3 groups (or 6 small groups). This activity will motivate students to think about integers.
  - Manipulative: - integer chips (Group 1)
  - Number line drawn on the board (Group 2)
  - Number line made on the floor (Group 3)
- Each group will rotate around the classroom so that all of them will get a chance to use each manipulative.
- Rule for Number line on the floor: the student must begin at 0 when given new instructions and face the positive direction.
- Group 3 will initially begin the activity; Group 1 and Group 2 will represent the movement made by Group 3 by using integer chips and the number line.
- Group 1 will complete the problems below using the integer chips. The green integer chips are negative and the red integer chips are positive chips representing units of one each.
- Once students have completed their problems, answers will be compared as a class to determine that all methods are valid ways of determining answers for problems involving integers.
- Within each of the three groups, students will be given their own replica of the Venn diagram depicting the real number system made of colorful construction paper.
- Students will be given various questions and must decide within their group which subset, rational numbers, integers, whole numbers, natural numbers, or irrationals, best represent the answer of each problem. After one minute, each group should raise the subset(s) that they have selected.

**Teacher's Key (Various Problems):**

1.  $0.80000 + 0.03333 = 0.83333$ , rational number
2.  $-\sqrt{16} = -4$ , integer and a rational number
3.  $\frac{14}{2} = 7$ , integer, a natural number, a whole number, and a rational number
4.  $\sqrt{120} \doteq 10.95445115$ , irrational
5.  $-\sqrt{81} = -9$ , integer and rational number

**Teacher Instructions**

- Once students have completed their problems, answers will be compared as a class to determine that all methods are valid ways of determining answers for problems involving integers. Within each of the three groups, students will be given their own replica of the Venn diagram depicting the real number system made of colorful construction paper.
- Students will be given various questions on the chalkboard and must decide within their group which subset, rational numbers, integers, whole numbers, natural numbers, or irrationals, best represent the answer of each problem. After one minute, each group should raise the subset(s) that they have selected.
- Students will complete the post quiz consisting of ten questions about the real number system.

**Assessment Tool:**

Students are to answer the ten questions below using the tools and knowledge presented today. Name the set or sets of numbers to which each real number belongs. Use **N** for natural numbers, **W** for whole numbers, **Z** for integers, **Q** for rational numbers, and **I** for irrational numbers.

1.  $-\sqrt{49}$  \_\_\_\_\_
2. 0.3333333 \_\_\_\_\_
3. 0.6666666 \_\_\_\_\_
4. 3.14 \_\_\_\_\_
5. 0 \_\_\_\_\_
6.  $-1/2$  \_\_\_\_\_
7.  $10/5$  \_\_\_\_\_
8.  $3/5$  \_\_\_\_\_
9. 0.4583 \_\_\_\_\_
10.  $\sqrt{49}$  \_\_\_\_\_

**(Teacher's Key) Assessment Tool:**

Students are to answer the ten questions below using the tools and knowledge presented today. Name the set or sets of numbers to which each real number belongs. Use **N** for natural numbers, **W** for whole numbers, **Z** for integers, **Q** for rational numbers, and **I** for irrational numbers. An overall class average of 80 % has been selected as the score to signify sufficient comprehension.

1.  $-\sqrt{49}$  **Z, Q**
2. 0.3333333 **Q**
3. 0.6666666 **Q**
4. 3.14 **Q**
5. 0 **W, Z, Q**
6.  $-1/2$  **Q**
7.  $10/5$  **N, W, Z, Q**
8.  $3/5$  **Q**
9. 0.4583 **Q**
10.  $\sqrt{49}$  **N, W, Z, Q**

## Extending Irrational Numbers

Euclid proved in the tenth book of his Elements that  $\sqrt{2}$  is an irrational number. He used an indirect proof. That is, he assumed that  $\sqrt{2}$  was rational. Then he showed that this assumption led to a contradiction.

Assume that  $\sqrt{2}$  is a rational number. Then  $\sqrt{2} = \frac{a}{b}$ , where  $a$  and  $b$  are integers,  $b \neq 0$ . Also, assume that  $a$  and  $b$  are relatively prime. That is, they have no common integral factor other than 1.

$$\begin{array}{ll} \text{If} & \sqrt{2} = \frac{a}{b} \\ \text{then} & 2 = \frac{a^2}{b^2} \quad (\text{Square both sides.}) \\ \text{and} & 2b^2 = a^2 \end{array}$$

Since  $2b^2$  is an even number,  $a^2$  is even. Since  $a^2$  is even,  $a$  is even. Therefore, for some integer  $c$

$$\begin{array}{ll} a = 2c & \\ a^2 = 4c^2 & (\text{Square both sides.}) \\ 2b^2 = 4c^2 & (\text{Recall that } a^2 = 2b^2.) \\ b^2 = 2c^2 & \end{array}$$

Since  $2c^2$  is an even number,  $b^2$  is even. Since  $b^2$  is even,  $b$  is even. However, two even numbers cannot be relatively prime, so  $a$  and  $b$  are not relatively prime. This contradicts the original assumption, so it is not true that  $\sqrt{2}$  is rational. Thus,  $\sqrt{2}$  is irrational.

Your Turn:

Use an indirect proof to show that  $\sqrt{3}$  is irrational.

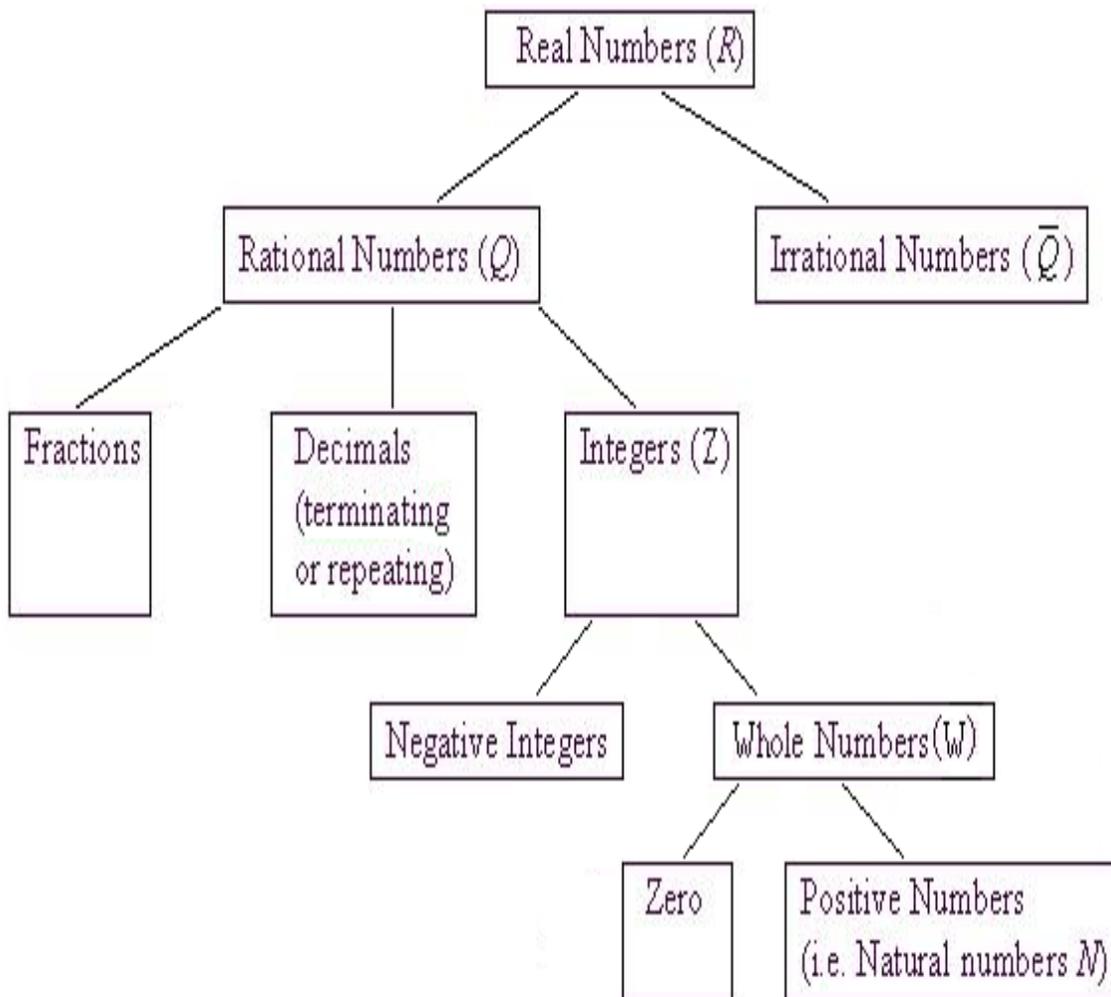
# Rational and Irrational Number Relationships

# Graphic Organizer

Name \_\_\_\_\_

**Real Numbers**  $R = \{N, W, Z, Q, \bar{Q}\}$

Provide a numerical example of each type of number next to each box and circle your response.



Adapted from: <http://msskehill.weebly.com/unit-3-exponents--radical.html>

## Rational and Irrational Reflection

Name \_\_\_\_\_

~ HANDLES ~

A 'handle' in mathematics is a way of thinking about a mathematical topic that makes sense to you. A handle is a way of explaining a math topic. You should try to get a handles on math topics that don't make sense to you.

~ A HANDLE FOR IRRATIONAL & RATIONAL NUMBERS ~

**RATIO**nal numbers are numbers that can be written as the **RATIO** of two integers.

$\frac{2}{3}$ ,  $\frac{7}{5}$ ,  $\frac{13}{1}$ , 13, &  $\sqrt{9}$  are examples of rational numbers. Irrational numbers cannot be written as a

RATIO of two integers.  $\pi$  &  $\sqrt{10}$  are examples of irrational numbers.

Here is a handle for the difference between rational and irrational numbers: When written in equivalent decimal form, rational numbers either repeat or terminate, whereas irrational numbers neither repeat nor terminate.

Here is another handle for the difference between rational and irrational numbers:

You can put your finger at the exact spot on a number line where a rational number lives, whereas it is somewhere between difficult and impossible to put your finger on the exact spot where an irrational number lives!!

Reflection: Write a paragraph in the space below using the two decimal forms of  $\frac{17}{10}$  and  $\pi$  remembering to justify your reasoning.