Lesson Plan
Lesson 6: Intro to Complex Numbers
Mathematics High School Math II

Unit Name: Unit 1: Extending the Number System

Lesson Plan Number & Title: Lesson 6: Intro to Complex Numbers

Grade Level: High School Math II

Lesson Overview: Students develop their understanding of the number system, building upon knowledge of rational and irrational numbers, to investigate complex numbers as a comparison to the real number system.

Focus/Driving Question: Why are complex numbers needed to supplement the real number system?

West Virginia College- and Career-Readiness Standards:

M.2HS.4 Know there is a complex number i such that $i^2 = -1$, and every complex number has the form $a + bi$ with a and b real.

Manage the Lesson:

Step 1: Establish student understanding by asking students if they can give an example of a complex number. What do they believe one to be? Why do we need complex numbers? Share the video to develop student perspective on the role of complex numbers in the number system. *Complex Numbers – Why We Need Them* - http://www.youtube.com/watch?feature=endscreen&NR=1&v=BDIv7r-X2kk and *Complex Numbers – Why We Need Them, Continued* - http://www.youtube.com/watch?v=rBOzwh5-iGc After watching the videos, ask students to develop their own examples of imaginary numbers.

Step 2: Develop a visualization of the vocabulary with the following video Introduction to i and Imaginary Numbers - http://www.youtube.com/watch?v=ysVcAYo7UPI, followed by students creating a Foldable for Complex Numbers. Once completed, have the students create class definitions and examples to add to the word wall.

Step 3: Build upon student knowledge through exploration using the Introduction to the Powers of i activity with printable and instructional guide on the properties of i to help your students develop the patterns used to calculate various exponential powers of i. After this is completed, share the Powers of i mini-poster with your students and post on the vocabulary wall.

Step 4: Students will demonstrate their knowledge through the incorporation of student practice utilizing a variety of materials. The combination of materials listed can be adapted to your students learning styles and abilities. For example, breaking the assignment into shorter tasks can guide your instruction and provide informal assessment on student mastery. When planning lesson implementation, select the materials most appropriate for your student’s needs.

Instructional Videos
Algebra I Help: Complex Numbers I - http://www.youtube.com/watch?v=XfMjlws58Do (how to simplify radicals with negative numbers and write in standard form. Stop video at 6:20 before addition and subtraction of complex numbers.)

Instructional Activities
Powers of i mini-poster detailing the powers of i
Real Life Context This document provides ideas on real-life applications for imaginary and complex numbers answering the question "Where will I ever use this?" for students
Find-n-Fix activity promotes student reasoning with error analysis
Complex Number Flowchart may be used for instruction, starter or short assessment
Imaginary Numbers Worksheet - http://www.mathworksheetsgo.com/sheets/algebra-2/complex-numbers/imaginary-numbers-worksheet.php (printable includes error analysis questions)

Complex Number Worksheet -
http://millermath.wikispaces.com/file/view/Complex+Numbers+Worksheet.pdf   (printable on simplifying imaginary numbers including those with variables)

Computer Practice
Imaginary Numbers - http://www.mathsisfun.com/numbers/imaginary-numbers.html (online lesson on imaginary numbers with questions at the bottom of the webpage)
Imaginary Unit and Standard Complex Form -
http://www.regentsprep.org/Regents/math/algtrig/ATO6/ImagineLes.htm (Instructional website and online practice)
Practice with Imaginary Unit and Standard Complex Form -
http://www.regentsprep.org/Regents/math/algtrig/ATO6/ImaginePrac.htm

Puzzles and Games
Complex Number Bingo - http://www.regentsprep.org/Regents/math/algtrig/ATO6/ComplexRes.htm
(Students play Bingo while practicing complex numbers from a printable)
"Roll out" Exponents of i - http://www.regentsprep.org/Regents/math/algtrig/ATO6/powerresouce.htm
(Using a bingo card, students roll and call out the simplified complex number)
Teacher Information
Question Corner – Complex Numbers in Real Life -
http://www.math.toronto.edu/mathnet/questionCorner/complexinlife.html (explanation of why and where used)
A Short History of Complex Numbers -
http://www.math.uri.edu/~merino/spring06/mth562/ShortHistoryComplexNumbers2006.pdf (may be too advanced for students, but provides background knowledge for the teacher)
Step 5: Summarize the lesson and assess student understanding with the Imaginary Numbers Exit Slip**.** Each student creates their own examples demonstrating individual learning. A short paragraph written by students permits opportunities to place the lesson context in their own words and provides instructors with information on student knowledge.
Step 6: Reflect with your students on the lesson in a Think-Pair-Share learning experience asking the question "Why are complex numbers needed to supplement the real number system?"

Academic Vocabulary Development:

Imaginary number-numbers involving the imaginary unit "*i*" which is defined to be the square root of -1
Real numbers-any number that is a positive number, a negative number or zero
Standard Form of a Complex Number- a complex number a + b*i* is imaginary provided b is not equal to 0

Launch/Introduction:

Establish student understanding by asking students if they can give an example of a complex number. What do they believe one to be? Why do we need complex numbers? Share the video listed in Step One of Manage the Process to develop student perspective on the role of complex numbers in the number system.

Investigate/Explore:

Use the Introduction to the Powers of i activity with printable and instructional guide on the properties of "*i*" to help your students develop the patterns used to calculate various powers of "*i*".
Students will demonstrate their knowledge through the incorporation of student practice utilizing a variety of materials. Instructors combine materials listed by adapting to your students learning styles and abilities for investigation and practice of the objectives. For example, breaking the assignment into shorter tasks can guide your instruction and provide informal assessment of student mastery. Some materials may be used as online practice by the instructor for individualized remediation. Not all of these materials may be needed for student mastery of the objectives.

Summarize/Debrief:

Students use the Imaginary Numbers Exit Slip to place the lesson context in their own words and provide instructors with information on student knowledge.

Materials:
Graphing Calculator, Word Wall Materials (construction paper, markers), foldable (white copy paper or notebook paper, markers or colored pencils, scissors), optional-computers, handouts, websites

Career Connection:

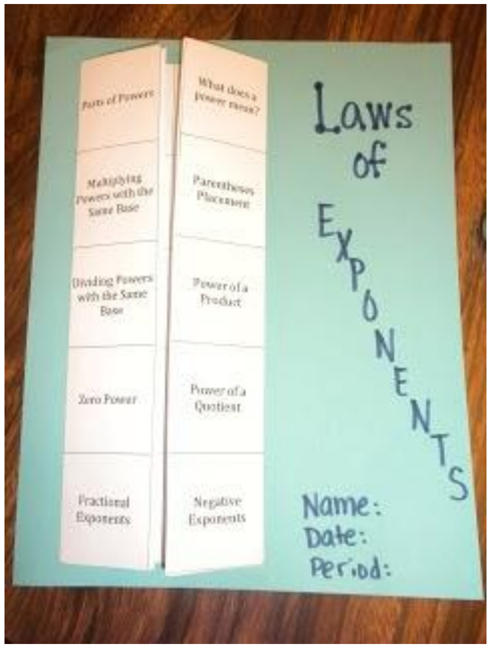
Engineering, Science and Natural Resources use complex and imaginary numbers when calculating electrical impedance.

Lesson Reflection:
Reflect with your students on the lesson in a Think-Pair-Share learning experience asking the question "Why are complex numbers needed to supplement the real number system?".

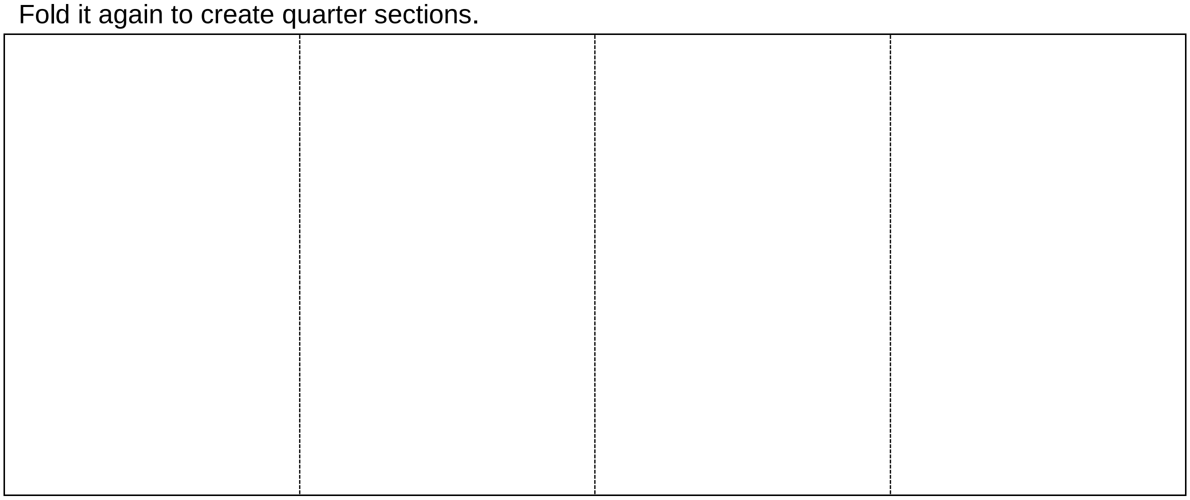
The teacher should reflect on how the lesson went, what parts went well or what parts need to be revised.

Foldable for Complex Numbers

Although this is not the content of the foldable the students are creating, here is an illustration of the style of the foldable. It is not necessary to glue the foldable to a separate sheet.



- Have each students take a sheet of paper (consider using colorful paper so that the foldable is more easily identified by the student) and fold it half “hamburger” style.
- Fold it again to create quarter sections.



- Then have students fold it again lengthwise to create quarter sections horizontally. Each student should now

- Each student should now have a total of 16 sections. The section on the far left and right should be folded in. These sections will be the titles on the "outside", which will hide the middle sections until the students lifts the tab.

Have the student cut as indicated.

(cut here – only to vertical fold)			(cut here – only to vertical fold)
(cut here – only to vertical fold)			(cut here – only to vertical fold)
(cut here – only to vertical fold)			(cut here – only to vertical fold)

➢ Complete the foldable as shown. Remember the 1st and 4th columns are written on the "outside" of the tabs.

Complex Numbers		$i = \sqrt{-1}$ $i^2 = -1$ $i^3 = -i$ $i = 1$	Powers of *i*
Have students put their name here.	Written by and for:	Any number that can be made by dividing one integer by another. The word comes from "ratio". Examples: • 1/2 is a rational number (1 divided by 2, or the ratio of 1 to 2) • 0.75 is a rational number (3/4) • 1 is a rational number (1/1)	Rational Number
Standard Form	$a + bi$	A real number that cannot be written as a simple fraction - the decimal goes on forever without repeating. Example: π is an irrational number.	Irrational Number
Definition	A combination of a real and an imaginary number in the form $a + bi$, where a and b are real, and *i* is imaginary.	$5 + 6i$ $\sqrt{3} - 7i$ $8i \rightarrow (0 + 8i)$ $7 \rightarrow (7 + 0i)$ $\sqrt{4} + i\sqrt{5}$	Examples of Complex Numbers

Introduction to the powers of *i*

Name ___________________________________

Complete the following table using the powers of *i*.

Powers of i	i	-i	1	-1
i				
-i				
1				
-1				

Based upon your findings, predict the following values for *i*.

i^7 __________ i^{12} __________ i^3 __________ i^2 __________ i^{10} __________

Explain the "rule" that you applied to develop your predictions.

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Now investigate infinite set {*i*, 2*i*, 3*i*, 4*i*…} with the operation of addition. Ask student pairs to investigate for closure, identity and inverses.

Give an example of an identity for the above set. ____

Give an example of an inverse for the above set. ____

Explain whether or not the set is considered closed. ______________________________________

Modified from http://dnet01.ode.state.oh.us/ims.itemdetails/lessondetail.aspx?id=0907f84c8053191c

Instructor Notes

Instructional Tip:

Discuss with students that doing many examples is only considered a proof if all possible examples can be considered. In the case of an infinite set, such as the natural numbers, a more formal proof would be needed to prove closure and the existence of an identity.

Explain that students are going to investigate the infinite set {*i*, 2*i*, 3*i*, 4*i*…} with the operation of addition. Ask student pairs to investigate for closure, identity and inverses.

a. Ask students to explain whether or not the set has an identity element.

b. Ask students to use the definition of inverses and to determine if the set contains inverses. (In this case, inverses are two elements of the set that add together to make the identity. Since the identity is not in the set, identifying inverses becomes irrelevant.) Ask students to explain why not having an identity automatically means there cannot be inverses.

c. Ask students to explain whether or not the set is closed. Remind students that the operation is addition. (The set is closed. Adding any two imaginary numbers together yields another imaginary number.) Ask student if the set would be closed over multiplication. (The set is not closed under multiplication. One easy example is $i^2 = i * i = -1$. Negative one is not a member of the set.

Extension: Include zero, or 0*i*, in the set. Ask students what would have to be added to include inverses in the set. (Add the negative of each term.)

Discuss how mathematicians would prove commutativity.

a. As a class, discuss identity, closure, inverses, the commutative property and the associate property of real numbers. (All of these hold for the real numbers.)

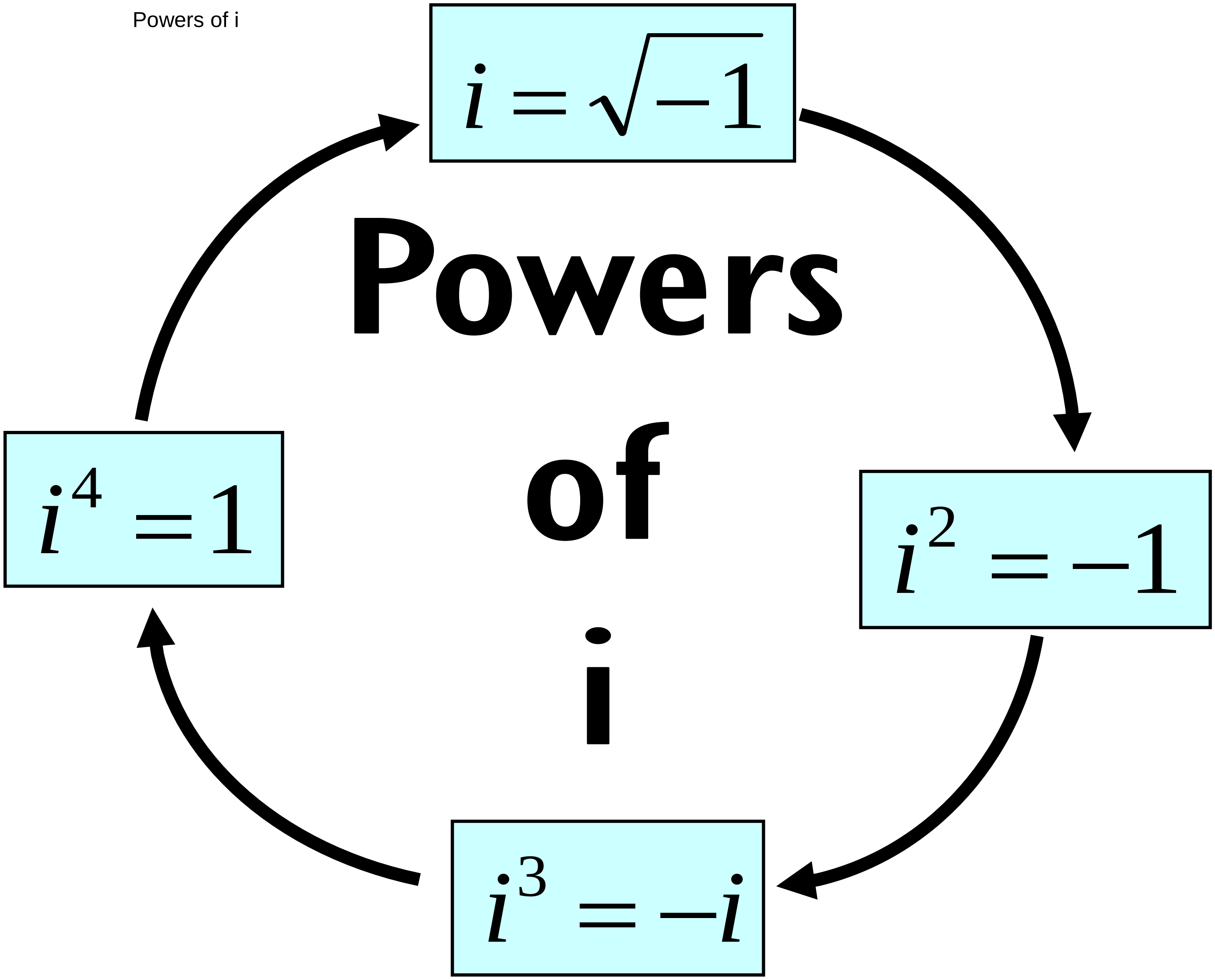
b. Ask the student pairs to look at *ai* and *bi* where *a* and *b* are any real numbers. Write *ai* + *bi* on the board or overhead. Simplify *ai* + *bi* as *ai* + *bi* = (*a* + *b*)*i* .

c. Point out that as a and b are both real numbers, the real number properties apply, so *a* + *b* must be a real number because of closure.

d. Next look at *bi* + *ai* = (*b* + *a*)*i*. Since a and b are real, and the commutative property is true for real numbers, then *a* + *b* = *b* + *a*; hence, *ai* + *bi* = (*a* + *b*)*i* = (*b* + *a*)*i* = *bi* + *ai*.

e. Explain that because this sample is not a specific example using numbers, but rather a general case, that this has proved that addition of pure imaginary numbers is commutative.

Modified from http://dnet01.ode.state.oh.us/ims.itemdetails/lessondetail.aspx?id=0907f84c8053191c



Always divide the exponent by 4.

If it divides evenly, then the answer is 1.

If you get a remainder of 1, then the answer is *i*.

If you get a remainder of 2, then the answer is –1.

If you get a remainder of 3, then the answer is –*i*.

Real Life Context

Complex Numbers are useful in representing a phenomenon that has two parts varying at the same time, for example an alternating current. Also, radio waves, sound waves and microwaves have to travel through different media to get to their final destination. There are many instances where, for example, engineers, doctors, scientists, vehicle designers and others who use electromagnetic signals need to know how strong a signal is when it reaches its destination. The two parts in this context are: the rotation of the signal and its strength. The following are examples of this phenomenon:

- A microphone signal passing through an amplifier
- A mobile phone signal travelling from the mast to a phone a couple of miles away
- A sound wave passing through the bones in the ear
- An ultrasound signal reflected from an unborn child in the womb
- The song of a whale passing through miles of ocean water

Complex Numbers are also used in:

- The prediction of eclipses
- Computer game design
- Computer generated images in the film industry
- The resonance of structures (bridges, etc.)
- Analyzing the flow of air around the wings of a plane in aircraft design

Adapted from Projectmaths

Find- n- Fix

Review Game

Problem: Students ***hate*** showing work. They think they can look at equations and guess the solution. They don't understand the importance of following steps in solving problems.

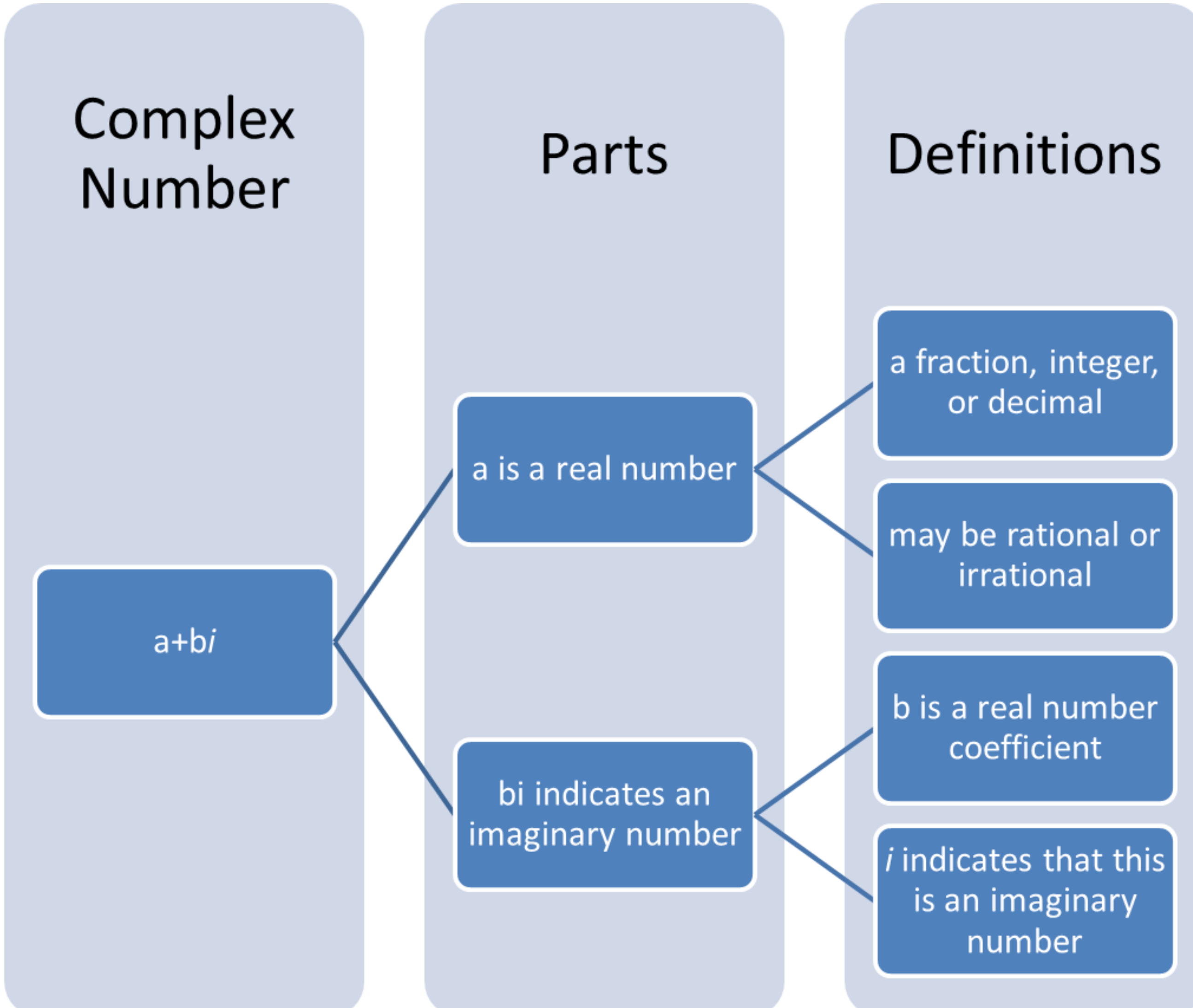
Purpose: Find-n-Fix requires students to understand the process of and demonstrate their ability to solve equations and other mathematic problems; step by step.

How it works: This can be an individual opportunity for students to earn extra points on an upcoming quiz or test; or some other positive reinforcement (candy, small snack, etc.). However, I've found it to be more effective when the students are placed in to small groups of 3 or 4 (chosen randomly). This way it's easier to pinpoint the students that aren't contributing. Because there is a much desired end result, all students tend to get involved in the activity.

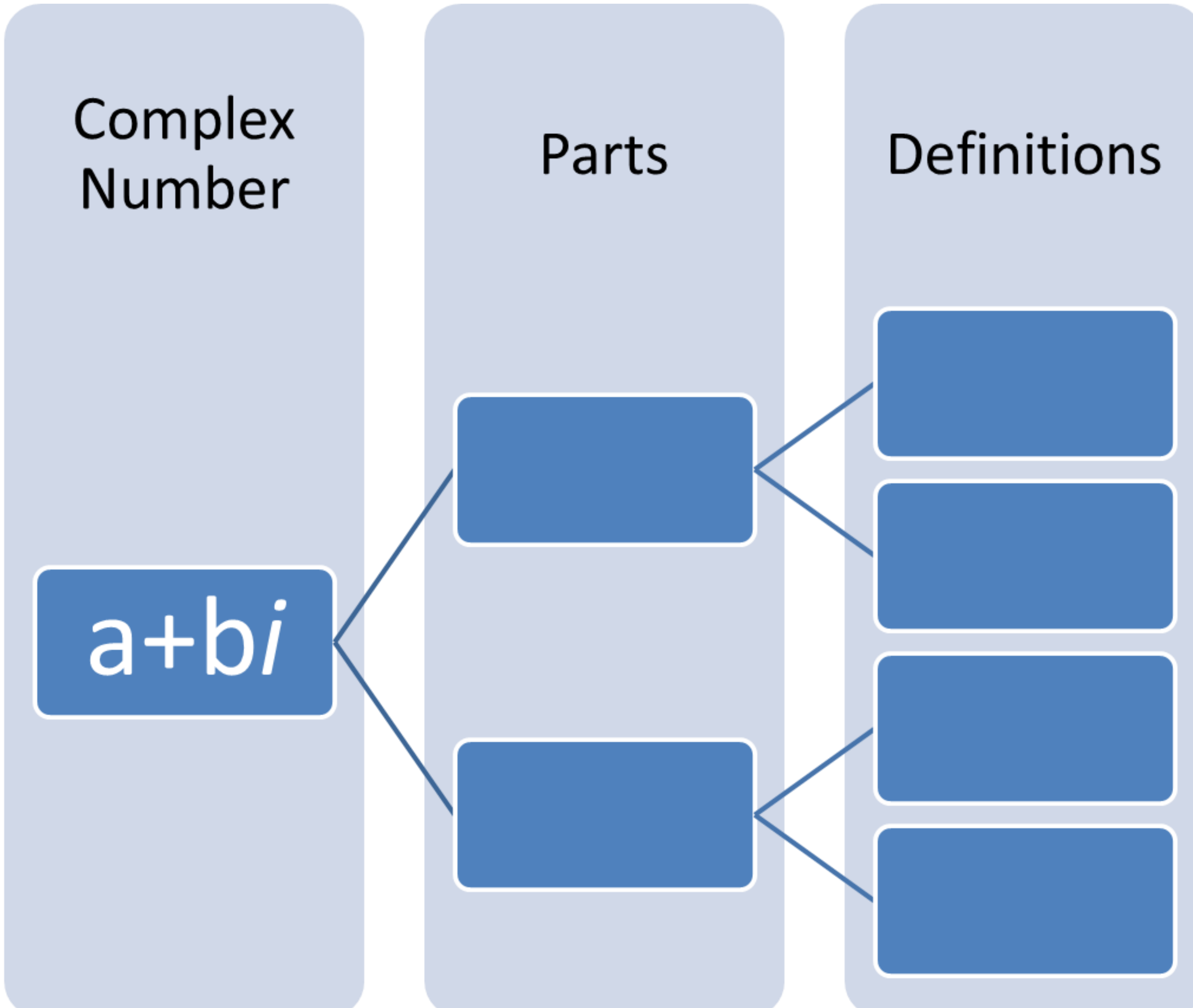
Procedure: The teacher begins the activity by telling the students that the following problems contain mistakes (there is no limit to how many mistakes there can be). It is up to the students to find and correct the mistakes in order to receive the points. The groups will go in their numbered order (delegated at the start of the game). If the group responsible for answering is incorrect or does not have a response, a group will be chosen at random (from a hat???) to respond. This selection process is repeated until the correct answer is given.

The teacher prepares several slides of incorrect problems for viewing. One by one the teacher places the problem on the overhead and allows each group to work for 2-3 minutes on the problem; time depending on level of difficulty. When time is up (determined by the buzzing of an egg timer), the group whose turn it is, must reply. Procedures will follow as aforementioned.

Complex Numbers



Complex Numbers



Imaginary Numbers Exit Slip

Create an example demonstrating the set of imaginary numbers is closed with respect to addition:

Create an example demonstrating the set of imaginary numbers is closed with respect to subtraction:

Create an example demonstrating the set of imaginary numbers is closed with respect to multiplication:

Think-Pair-Share My Partner's Thoughts

Like the Think-Pair-Share strategy, the Think-Pair-Share My Partner's Thoughts strategy is used to encourage reflection during a classroom activity. It is a great way to check for understanding.

During the "think" stage, the teacher tells students to ponder a question or problem. This allows for wait time and helps students control the urge to impulsively shout out the first answer that comes to mind. Next, individuals are paired up and discuss their answer or solution to the problem. Students know they will have to share their partner's thoughts, rather than their own, so they will listen more intently during the discussion.

The teacher can use this activity as a formative assessment by listening to the conversations of each pair of students.