Unit Name: Unit 1: Extending the Number System

Lesson Plan Number & Title: Lesson 8: Polynomials

Grade Level: High School Math II

Lesson Overview:
Students will be able to explain orally or in written format, the definition of a polynomial and apply the basic operations of addition, subtraction and multiplication. They should be able to present their solutions as simplified polynomials. Students' knowledge should include understanding that the solution sets are closed under the operations of addition, subtraction, and multiplication.

Focus/Driving Question:
If I start with a polynomial and combine it with another polynomial, do I always get a polynomial?

West Virginia College- and Career-Readiness Standards:
M.2HS.6
Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract and multiply polynomials.
Instructional Note: Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.

Manage the Lesson:
Step 1: Launch the lesson by sharing the following information with students: In manufacturing, package development is an important part of design. The surface area of a package provides additional space for advertising and product information. Draw a rectangular box on your paper and with a partner express the surface area in terms of height *h*, length *l*, and width *w*. You may want to label your picture for additional perspective. Calculate the total surface area. After providing students with ample think-time, initiate a discussion on their results. Use a drawing or rectangular box (such as a cereal box) to label and record a class answer for comparison to the partner activity. Discuss the importance of like terms to their solution.
Step 2: Begin the lesson with vocabulary by defining term, degree, polynomial, monomial, binomial, and trinomial. Discuss the requirements for a polynomial: that exponents on variables must be positive integers and coefficients must be real numbers. A review may be found at *Polynomials: Definitions/Evaluation* - http://www.purplemath.com/modules/polydefs.htm. A polynomial with only one term is called a monomial, with two terms is called a binomial, and three terms is referred to as a trinomial. In general, a term is joined by multiplication/division and separated by addition/subtraction. Be sure to remind the students that it is necessary for the terms to have the same degree to be combined and that when terms are multiplied the exponents on the variables are added.
Step 3: Build upon student knowledge by investigating with Polynomial Cards, which may be used in a variety of ways. Suggestions include: sorting activity – have the students sort the cards as to whether or not the term could be in a polynomial and the remaining cards could be sorted into piles of like terms; create your own monomials and then create binomials, trinomials, and/or polynomials and add/subtract/multiply them. Review combining like terms (to differentiate algebra tiles may be used). Remind students that terms whose variables have different exponents are not like terms and may not be combined.
Step 4: Students will demonstrate their knowledge through the incorporation of student practice utilizing a variety of materials. The combination of materials listed can be adapted to your students learning styles and abilities. For example, breaking the assignment into shorter tasks can guide your instruction and provide informal assessment on student mastery. All of the materials listed below may not be needed for

student mastery of the lesson objective. When planning lesson implementation, select the materials most appropriate for your student's needs.

Instructional Practice

Polynomial Cards**-** The cards can be adjusted to your lesson depending on the task needed. Students can sort, add, subtract, multiply or combine like terms.

Multiplying Polynomials **-** Instruct the 5 steps to multiplying polynomials using this organizer

Made4Math #5 Polynomial Station Activities - http://pamjwilson.wordpress.com/2012/07/30/made4math-5-polynomial-station-activities/ (ideas on creating activity stations for combining like terms, addition and subtraction of polynomials, multiplying a binomial and monomial, multiplying a binomial by a binomial)

Graphic Organizer-Adding, Subtracting and Multiplying Polynomials Excellent resource to review the steps for adding, subtracting and multiplying polynomials, follow the review with students writing their own summarizations of the process and providing examples

Bull's-eye Activity Using a bull's-eye to develop the concentric rings as students multiply, add or subtract the given polynomials

Equivalent Expressions- This activity will allow students to draw parallels between operations on algebraic expressions and operations on complex numbers.

Computer Practice

Polynomials http://www.mathsisfun.com/algebra/polynomials.html (use to evaluate your students understanding of polynomials)

Adding Polynomials - http://www.purplemath.com/modules/polyadd.htm

Subtracting Polynomials - http://www.purplemath.com/modules/polyadd2.htm

Simple Polynomial Multiplication - http://www.purplemath.com/modules/polymult.htm

Polynomials - http://www.mathsisfun.com/algebra/polynomials.html (polynomial review activity)

Polynomial Operations - http://www.sascurriculumpathways.com/portal/ *Activity #87. Complete the accompanying handout and conduct online research to answer the question "How are polynomials added and subtracted?"*

Instructional Video

Polynomial Basics - http://www.khanacademy.org/math/algebra/polynomials/polynomial_basics/v/adding-polynomials (this link leads to a series of additional videos (found in the sidebar) on the topic of addition and subtraction of polynomials. Practice problems corresponding to the instruction can be accessed in the toolbar of website)

Multiplying Monomials, Binomials, and Polynomials in General - http://www.khanacademy.org/math/algebra/polynomials/multiplying_polynomials/v/multiplying-monomials (this link leads to a series of additional videos (found in the sidebar) on the topic of multiplication of polynomials. Practice problems corresponding to the instruction can be accessed in the toolbar of the website)

Games

Memory Multiplying Cards Game - Students take problem and answer pairs given on a sheet of paper, cut them apart, mix the cards up and turn them over to find the appropriate matches.

Trinomial Traverse - http://busynessgirl.com/files/games/trinomial_traverse.pdf **(**directions to making a board game on multiplying polynomials)

Step 5: After the students practice operations with polynomials; discuss whether or not polynomials are closed under addition, subtraction, and multiplication. End the class with the Find Someone Who Can Activity. The goal of the activity is for each student to sign off on each square to demonstrate knowledge. The twist is they may only do this for one person per square. They need to try to get their signature and solution in all of the different squares.

Step 6: Reflect with your students regarding whether or not all of their answers are polynomials? It is important for them to understand that polynomials are closed under addition, subtraction, and multiplication. Have your students write an exit slip (Polynomial Exit Slip) demonstrating their knowledge.

Academic Vocabulary Development:

Although there are many types of vocabulary strategies that may be used, a suggested method for vocabulary instruction involves the use of a modified Frayer model for Polynomial Vocabulary. The term could be put in the center and then examples, non-examples, and a written explanation could be used as section titles. The development of vocabulary can be guided by the instructor through the creation of a word wall or expanded throughout the unit's development.

closure – a set is defined as closed under an operation if you can combine any two things and the resulting answer is a member of the same set
coefficient - a constant that multiplies a variable
constant term - a value that does not change
degree - the number of times that a variable occurs as a factor in a monomial is the degree of the monomial. A nonzero constant does not have a degree. To find the degree of a polynomial calculate the greatest of all of the degrees of its terms after it has been simplified.
leading term - is the term with the highest degree in a polynomial placed in standard form
like terms - terms which have the same variable and exponents
monomial - an expression that is a numeral, a variable, or a product of a numeral and one or more variables
*binomia*l - a polynomial of two terms
polynomial - a sum of monomials
standard form - an arrangement of a simplified polynomial placed in descending powers of the variable
term - a mathematical expression using numerals or variables or a combination of both to indicate a product or a quotient
trinomial - a polynomial of three terms

Launch/Introduction:

Finding the surface area of a rectangular box can be a task used as an individual, partner, small group or whole group activity. Develop the concept of like terms from the definition of surface area.

Investigate/Explore:

Students can investigate using the Polynomial Cards. The cards can be adjusted to your lesson depending on the task needed. Students can sort, add, subtract, multiply or combine like terms.
The purpose of the lesson is the development of a polynomial and its basic operations of addition, subtraction, and multiplication. The launch activity encourages flexible instructional strategies for initiating the use of variables and combining like terms within a business application. The development of vocabulary can be guided by the instructor through the creation of a word wall. Students use the Frayer Model template in the lesson and develop their knowledge in the creation of corresponding examples/non-examples for each word. The instructor can choose to do this portion (create an example) as individual, small group or whole group instruction, depending on student abilities or necessary modifications. The process of instructional practice by students can be flexible instruction by the assignment of handouts, games, or online computer practice according to instructional resources and differentiation determined by the instructor. Assessment of the lesson is a continual part of the instructional process, whether informal or formal and should guide the instructor in the planning of instruction. A short assessment in step 5 of *Manage the Lesson* is available for students to use in the activity Find Someone Who Can**.** Both students and instructors are encouraged to reflect upon the lesson and knowledge gained with the Polynomial Exit Slip.

Summarize/Debrief:
Before ending the lesson, use the instructional technique called “Muddiest Point”. Hand each student a note card and ask them to write down their “muddiest point” or what they are unclear about regarding the lesson. Examples of problems not understood or questions are acceptable. Then place the note cards in a pile for review in whole group discussion or to be addressed the following day as a starter.

Materials:
Graphing calculator, Word Wall materials (construction paper, markers), foldable (white copy paper or notebook paper, markers or colored pencils, scissors), optional-computers
Polynomial Cards – will need to be copied and cut out for each student/group.
Multiplying Polynomials – will need to be copied and cut out for each student/group.
Polynomial Exit Slip – will need to be copied and cut out for each student/group.
Graphic Organizer-Adding, Subtracting and Multiplying Polynomials – will need to be copied and cut out for each student/group.
Bull’s-eye Activity – will need to be copied and cut out for each student/group.
Memory Multiplying Cards Game - teachers may want to enlarge the “board” for better game play
Polynomial Vocabulary - a modified Frayer Model for the lesson vocabulary
Find Someone Who Can - will need to be copied and cut out for each student/group.
Equivalent Expressions - printable
Websites:
http://www.purplemath.com/modules/polydefs.htm
http://www.purplemath.com/modules/polyadd.htm
http://www.purplemath.com/modules/polyadd2.htm
http://www.purplemath.com/modules/polymult.htm
http://www.mathsisfun.com/algebra/polynomials.html
http://www.khanacademy.org/math/algebra/polynomials/polynomial_basics/v/adding-polynomials
http://www.khanacademy.org/math/algebra/polynomials/multiplying_polynomials/v/multiplying-monomials
http://busynessgirl.com/files/games/trinomial_traverse.pdf
http://www.sascurriculumpathways.com/portal/

Career Connection:
Business and Marketing Cluster, Engineering and Technical Cluster, Health Cluster- All have careers where multiple unknowns need representation which can be accomplished by using variable expressions in the form of polynomials.

Lesson Reflection:
Reflect with your students regarding polynomials. Are there any concepts that they are unclear on? Consider your teaching practices and how the lesson organization worked with your students. As a "pair and share" activity, have each student write a sentence or two reacting to the following the driving question individually and then turn to a partner to share their responses. Complete the activity by inviting everyone to share to the group their thoughts regarding polynomials.

Polynomial Cards

Directions: Sort these by using like terms, ask students to add, subtract or multiply with the cards

$\frac{1}{5}x^2$	$\frac{2}{3}x$	$\frac{5}{x}$
$x^{1/2}$	$-x$	$3x^2$
$-4x^2$	x^2	$13x$
$8xy$	$-3x^{-2}$	$\sqrt{x}$
8	-13	7

-1	$-7x$	$4x^3$

Multiplying Polynomials

Ex: $(x + 1)(x + 2)$

1st term 2nd term 1st term 2nd term

1st – create a grid

Identify number of terms in 1ST polynomial to determine # rows you need
Identify number of terms in 2ND polynomial to determine # columns you need

	Column 1	Column 2	
Row 1			
Row 2			

2nd – separate your terms and place in rows and columns

$$(x + 1)(x + 2)$$

Goes on left side Goes on top

	X	2
X		
1		

3rd – multiply to fill in boxes

	X	2
X	X^2	2X
1	1X	2

4th– combine like terms

	X	2
X	x^2	2X
1	1X	2

5th – write in standard form

$$x^2 + 3x + 2$$

(highest degree) (next highest) (constant at end)

Graphic Organizer – Adding, Subtracting and Multiplying

Adapted from an activity by Emily and Katie via http://www.ilovemath.org

Guidelines for Adding, Subtracting, and Multiplying Complex Numbers

Adding

Parentheses do not change the problem. Combine like terms (real with real and imaginary with imaginary) – by, combining coefficients.

- Examples -

(7 + 6*i*) + (8 - 3*i*) =

(-5 - 2*i*) + (-8*i*) =

Subtracting

All signs for each term must be switched in the set of parentheses that follow the subtraction sign. Then follow the rules for adding complex numbers.

- Examples -

(12 + *i*) – (-4 + 5*i*) =

(2) – (8 – 7*i*) =

Multiplying

When multiplying two complex numbers, you are just using the distributive property multiple times. Simplify the resulting expression. Remember $i^2 = -1$

- Examples -

(3 + 5*i*) (6 – 9*i*) =

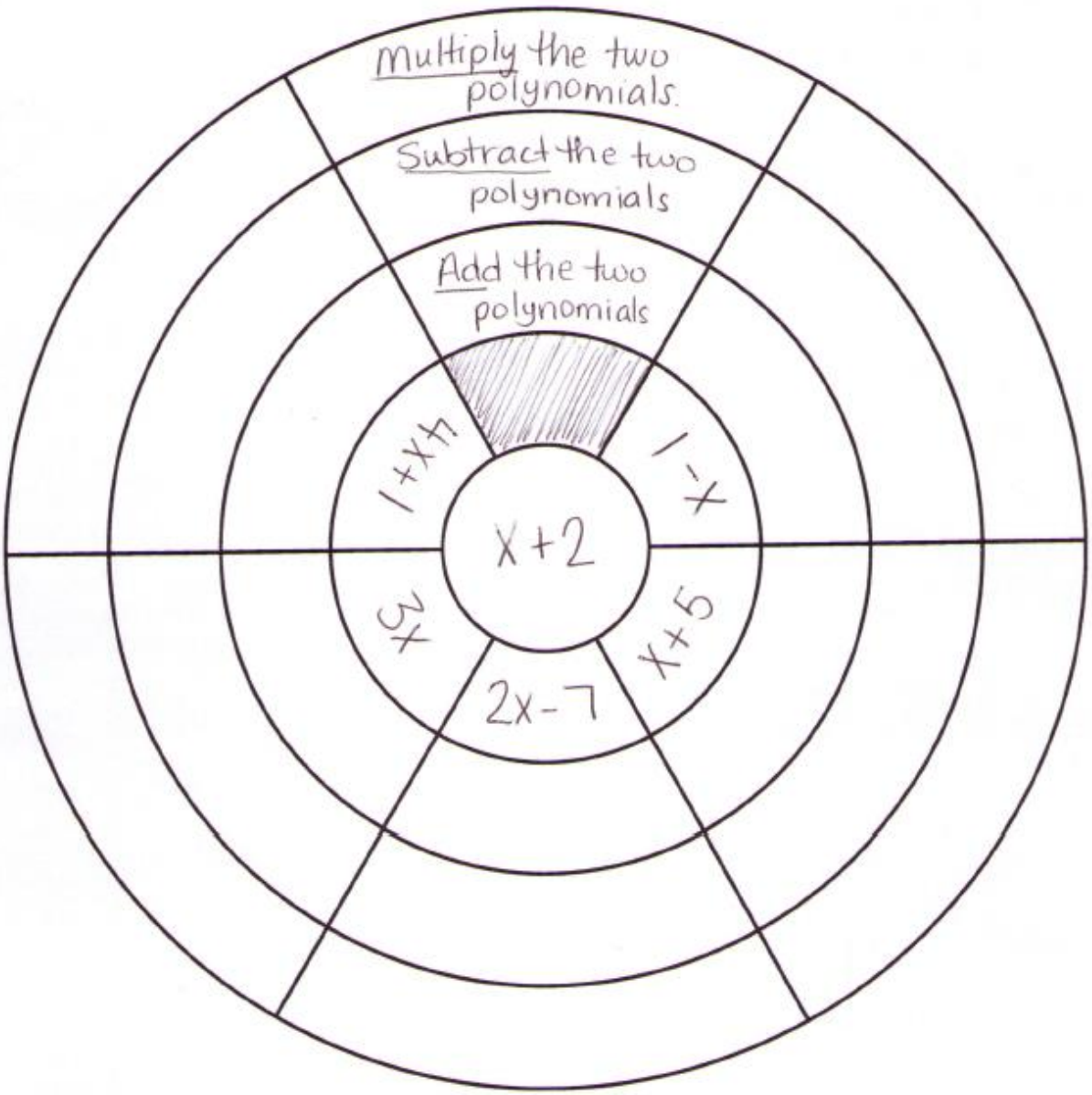
(6 – 3*i*) (6 + 3*i*) =

On a sheet of notebook paper, summarize in your own words the process for adding, subtracting, and multiplying polynomials. Create an example for each type and find each answer.

Bull's Eye
Working with Polynomials

Name _____
Date _____ Hr_____

In the center of the Bull's Eye, there is a polynomial. In the first ring, there is another polynomial. In the following rings, add the two polynomials. In the next ring, subtract the two polynomials, and in the last ring, multiply the two polynomials.



Equivalent Expressions

Name_____

Solve the problems in each "Simplify" column. Once completed, examine them horizontally to make comparisons between each problem. How are they the same? How are they different? Write a sentence explaining your observations.

Simplify	Simplify	How are they the same?	How are they different?
$(2+x)(3-2x)$	$(2+i)(3-2i)$		
$(2+x)+(3-2x)$	$(2+i)+(3-2i)$		
$(2+x)-(3-2x)$	$(2+i)-(3-2i)$		

Memory Multiplying Cards Game

$-6x^5 + 16x^3 - 11x^2$	$(x - 10)(x + 1)$	$x^2 - 9x - 10$	$(x + 4)(x^2 - 2x + 3)$
$x^3 + 2x^2 - 5x + 12$	$(x + 7)(x + 7)$	$x^2 + 14x + 49$	$(x - 2)(x^2 + 6x - 7)$
$x^3 + 4x^2 - 19x + 14$	$(x + 11)(x - 3)$	$x^2 + 8x - 33$	$(x^2 + 2x - 9)(x - 4)$
$x^3 - 2x^2 - 17x + 36$	$(x - 15)(x - 4)$	$x^2 - 19x + 60$	$(4x^2 - 3x - 2)(x + 12)$
$4x^3 + 45x^2 - 38x - 24$	$(x + 2)(x + 7)$	$x^2 + 9x + 14$	$(-2x)(4x + 7)$
$-8x^2 - 14x$	$(x + 1)(x + 1)$	$x^2 + 2x + 1$	$2x(x^2 + x - 5)$
$2x^3 + 2x^2 - 10x$	$(x + 6)(x - 10)$	$x^2 - 4x - 60$	$-4x^2(3x^2 + 2x - 6)$
$-12x^4 - 8x^3 + 24x^2$	$(x - 15)(x + 4)$	$x^2 - 11x - 60$	$(2x - 5)(-4x)$
$-8x^2 + 20x$	$(x + 2)(x^2 + 3x + 5)$	$x^3 + 5x^2 + 11x + 10$	$3x^2(7x - x^3 - 3)$
$-3x^5 + 21x^3 - 9x^2$	$(x - 5)(x^2 - 2x - 6)$	$x^3 - 7x^2 + 4x + 30$	$(-x)(6x^2 + 5x)$
$-6x^3 - 5x^2$	$(x - 3)(x^2 - 4x - 6)$	$x^3 - 7x^2 + 6x + 18$	$4x^2(3x^3 - 2x^2 - x)$
$12x^5 - 8x^4 - 4x^3$	$(2x + 3)(3x^2 - 4x + 2)$	$6x^3 + x^2 - 8x + 6$	$-x^2(6x^3 - 16x + 11)$

Directions

1. Cut out each of the rectangles above and separate them into two different categories: answers and problems.

2. Turn them face down leaving them in two separate categories.

3. One person will turn over 1 problem, but both partners will multiply it out.

4. After he/she has finished, turn over 1 rectangle from the answer side.

5. If it's a match, keep the pair and choose again.

6. If it's not a match, turn them both back over and now the other person chooses.

7. The winner is the person with most matches!!!

Find Someone Who Can – Students use sheet similar to the one provided to find someone who can answer each question dealing with polynomials and the laws of exponents. Students answering questions must write the correct answer in the square and sign/initial the square. Good movement, bell-ringer, or exit activity.

FIND SOMEONE WHO CAN...

Your mission is to find a different person among your classmates to simplify each expression in the table below. Assume that no denominator is equal to zero. Your teammates must write the correct answer in the square and then sign the square. You may not have a person sign/answer more than one square.

$4x^4 \cdot x^6$	$(6n^2p^4)(-3n^3p^3)$	$(r^6u^3)^4$
$\frac{r^5v^3}{r^3v}$	$\frac{32e^2f^{-6}}{4e^{-2}f^2}$	$(4u^2 - u - 3) + (3u^2 - u + 4)$
$(-x^2 + 3x) - (5x + 2x^2)$	$2m^2(2m^2 + 3m - 5)$	$(4c + 1)(2c + 1)$

Created by Debbie Bunch and Hope Hoggard

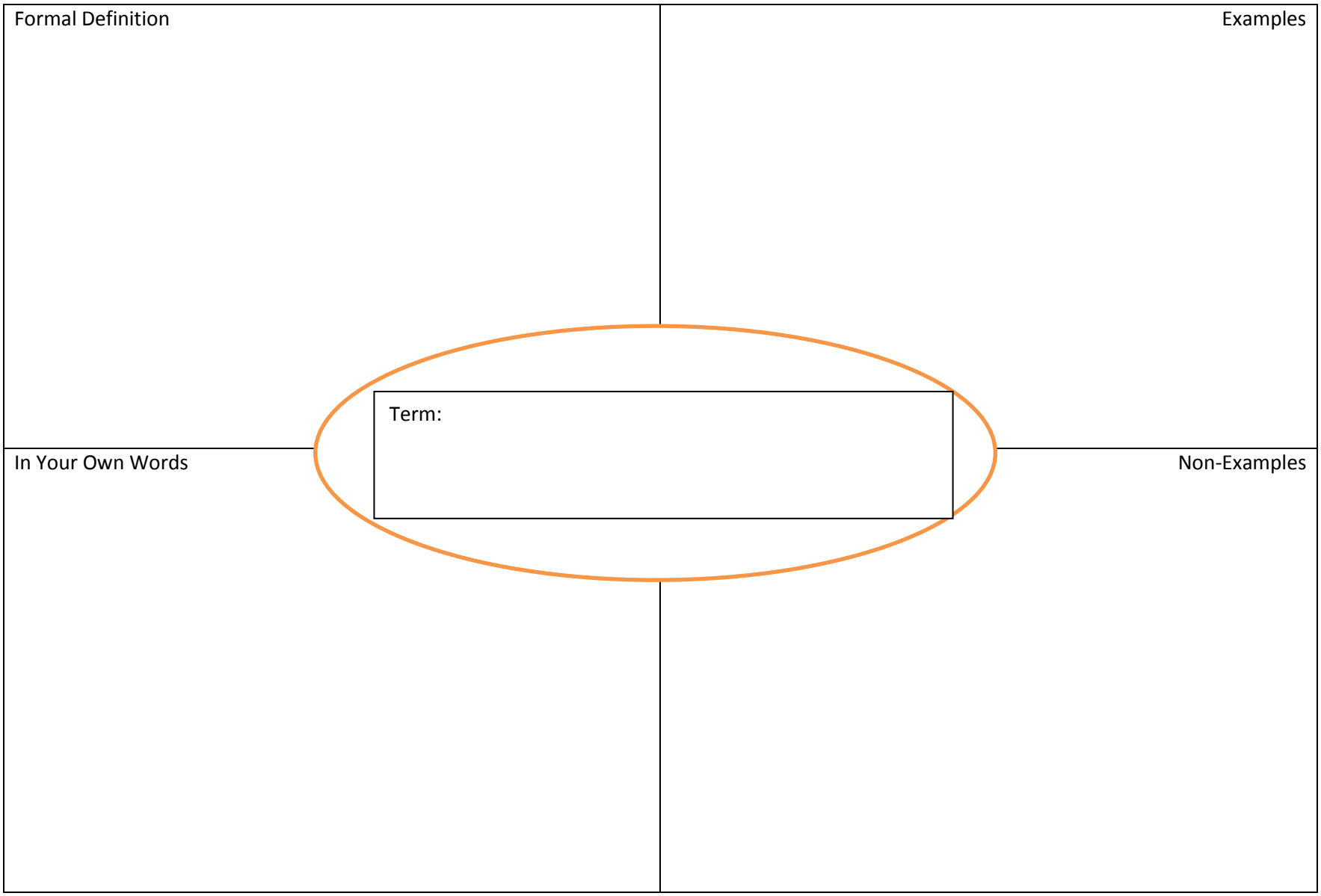
Adapted from *Tidewater Team for Math Education* at William & Mary's School of Education

Polynomial Exit Slip

Create an example demonstrating the set of polynomials is closed with respect to addition:

Create an example demonstrating the set of polynomials is closed with respect to subtraction:

Create an example demonstrating the set of polynomials is closed with respect to multiplication:


Formal Definition
Examples
Term:
In Your Own Words
Non-Examples

Term: Coefficient

Formal Definition:

a coefficient is a constant by which a variable is multiplied

Examples

The coefficient of 5x is 5.

The coefficient of a is 1.

In Your Own Words

The number in front of the variable

Non-Examples

The coefficient of x^2 is not 2.

The coefficient of $\frac{m}{3}$ is not 3.