Lesson Plan
Lesson 10: Applications of Similarity
Mathematics High School Math II

Unit Name: Unit 5: Similarity, Right Triangle Trigonometry, and Proof

Lesson Plan Number & Title: Lesson 10: Applications of Similarity

Grade Level: High School Math II

Lesson Overview: Having established the conditions under which triangles are similar by formal proof, students will now apply these results in multiple contexts. This lesson is designed for approximately 90 minutes. Students will develop conjectures about the properties of similar triangles that will serve as the basis for the theorems developed throughout the unit. Throughout this unit, two geometric figures are similar if there exists a sequence of rigid transformations followed by a dilation that maps one figure onto the other.

Focus/Driving Question: How can the Similarity Theorems be utilized to determine inaccessible lengths and distances?

West Virginia College- and Career-Readiness Standards:

M.2HS.46 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Manage the Lesson:

Students have measured, described, and transformed triangles through a variety of investigations in previous lessons. In the last lesson they constructed logical and convincing arguments that supported the conjectures that they made about similarity. As students apply the Similarity Theorems to determine inaccessible lengths and distances, they will reinforce their understanding of similarity. It is important for students to realize that finally their knowledge of similarity will be helpful as they analyze and solve new applications. Students will be working in small groups in an outdoor setting to solve some of the application problems in this lesson.

Academic Vocabulary Development:

Students continue to strengthen their understanding of the vocabulary related to similar figures and develop the ability to use appropriate vocabulary in describing their understanding of similarity.

Launch/Introduction:

Create student groups and display 10.01 Similarity Challenge**.** In this PowerPoint, students are challenged to determine the length of the nose found on the Mount Rushmore sculpture of George Washington. Allow student groups a few minutes to discuss the challenge and decide what information might be necessary to determine the length of the nose of Washington on Mount Rushmore. Distribute one of the images of George Washington on Mount Rushmore that are found in 10.02 Similarity Challenge Pictures to each student group.

In a classroom discussion, encourage students to create a list of information that they might find valuable. After students have created a comprehensive list, provide them with two additional pieces of information: (1) the length of each of the faces found on Mount Rushmore is 60 feet and (2) a one-dollar bill.

As the students in each group collaborate, ask them to share any method they may have found to use the provided information to determine the length of Washington's nose on Mount Rushmore. It may be necessary to help them understand that they now have two images of George Washington and to prompt them to determine that it is now possible to create proportions relating the image of Washington found on the one-dollar bill with the image of Washington found on Mount Rushmore. As student groups share their strategies, remind them to clearly show all measurements and calculations so that anyone could understand and follow their thinking.

Students should be able to use the given information to determine that Washington's nose on the sculpture found on Mount Rushmore is approximately 21 feet in length.

Investigate/Explore:

As a prelude to the applications, it might prove helpful to show the video *Similarity* produced at Cal Tech as part of their Project Mathematics video series. (This DVD also includes *The Theorem of Pythagoras* and *The Story of Pi*.) A teacher resource booklet accompanies the video. This video extends the idea of similarity to solids which can provide some interesting discussion.

In 10.03 Thumbs Up**,** 10.04 Me and My Shadow and 10.05 Mirror, Mirror on the Ground, students to use their understanding of similar triangles to estimate heights and distances. In the first investigation, students estimate the distance to a given point by using their thumb. (It is important that students keep their thumb vertical.) This method is based upon the fact that for most people, the ratio between the length of their outstretched arm and the distance between their eyes is approximately 10:1. In the second investigation, students estimate the height of the school building. Students should determine that if the measurements are taken at the same time of day that similar triangles are created by the meter stick and its shadow and a building and its shadow. Therefore, the ratio comparing the height of the meter stick to the length of the meter stick's shadow is equivalent to the ratio comparing the height of the school building to the length of the school building shadow. Students should create and solve: the proportion height of meter stick/length of meter stick's shadow = height of school building/length of school building's shadow. In the third investigation, students apply similar reasoning as they use mirrors to create similar triangles to determine the height of a flag pole. (It is easier to use a large mirror in this third investigation.) Depending on your location, it may be advantageous to determine the height of other buildings, the football goal posts, trees, etc.

Summarize/Debrief:

To review students' understanding and apply their learning related to similar triangles, conclude the lesson with the following problem. In Edward de Bono's book *Children Solve Problems*, children of various ages were asked to solve such problems as how to stop a cat and dog from fighting, how to weigh an elephant, and how to build a house quickly. One boy, when given the problem of how to build a house quickly, drew this diagram. (Show 10.06 Diagram of a House, top portion only) He figured that, since toy houses were sometimes made by cutting and folding cardboard, the same method might be applied to constructing a house from sheet metal. Ask students what the house would look like after it is "folded." (Show bottom portion of House Diagram.) The roof of the house slants down to one side so that someone walking in the front door would soon bump his head on the ceiling. Suppose that the right-hand wall of the house as shown here is 10 feet high and that the house is 30 feet wide. How far do you think the boy's dad, a 6-ft man, could walk into the house before his head would touch the ceiling? Encourage students to draw the design and use any previous learning to determine the point at which the man will bump his head. Have a student share their reasoning with the class. (The man could walk in 12 feet.) How did the students use similar triangles to arrive at their solution?

For homework, students could be assigned 10.07 Similarity Problems **(**10.08 Similarity Problems Key**)**, in which they are asked to use their understanding of similar triangles to design a plan that would allow NASA to determine the distance across a crater on Mars.

As an informal assessment of student understanding of similar figures and numeric problems, display the Regents Exam site multiple choice practice with similar figures and numeric problems http://www.regentsprep.org/Regents/math/geometry/GP11/PracSim.htm. Additional problems solving opportunities involving similar figures and proportions are provided in the Materials section of this template.

Similar Figures: Interactive Mathematics Teacher
http://www.mathsteacher.com.au/year10/ch06_geometry/05_similar/figures.htm

Worksheet on Similar Figures – Word Problems
http://www.kutasoftware.com/FreeWorksheets/PreAlgWorksheets/Similar%20Figure%20Word%20Problems.pdf

Similar Figures – Hotmath
http://hotmath.com/help/gt/genericprealg/section_9_8.html

Similar Figures - Glencoe
http://www.glencoe.com/sec/math/msmath/mac04/course2/add_lesson/similar_figures_mac2.pdf

Materials:
Meter sticks
Mirror (as large as available)
Patty paper
Trundle wheel or tape measures

10.01 Similarity Challenge
10.02 Similarity Challenge Pictures
10.03 Thumbs Up
10.04 Me and My Shadow
10.05 Mirror, Mirror on the Ground
10.06 Diagram of a House
10.07 Similarity Problems
10.08 Similarity Problems Key

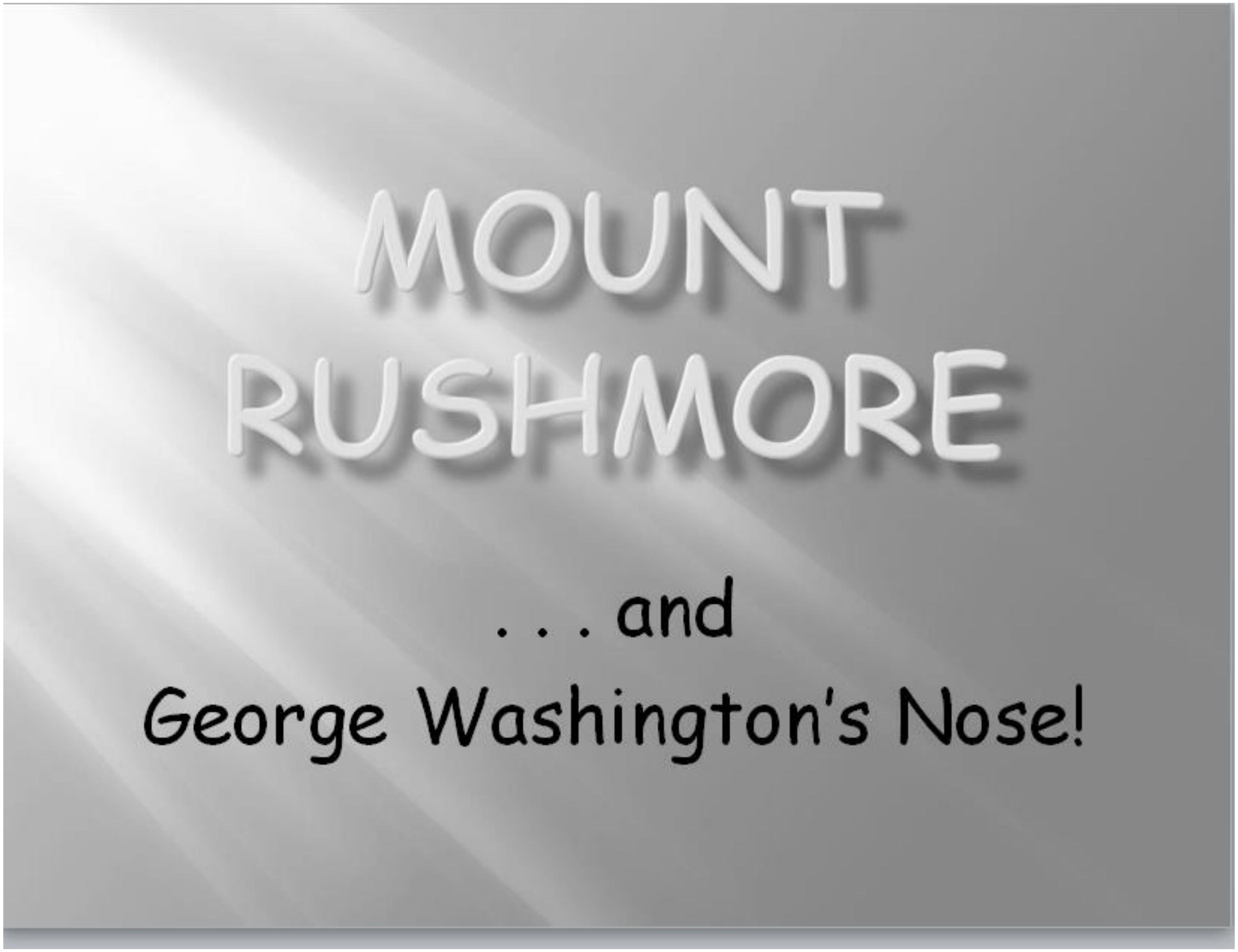
Career Connection:

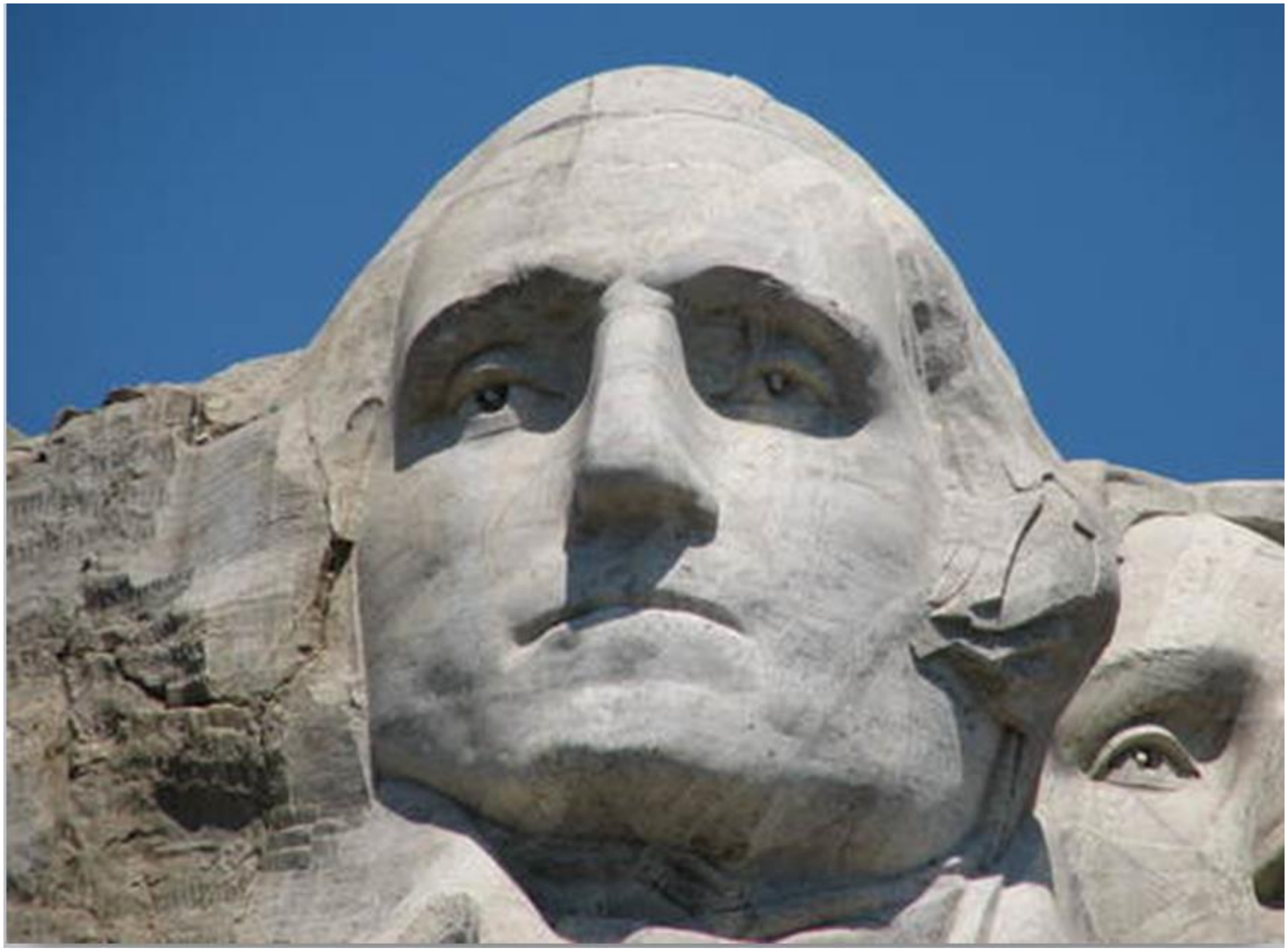
As noted in the previous lesson, an understanding of similarity will prove to be a valuable asset in many career pathways. All professions in which realistic models are an essential tool require an understanding of dilations. Most construction-based careers, such as architects, contractors, and electricians, require the ability to create or interpret and understand scale models. Advances in aviation and aerospace are based on research and testing that is generally carried out with scale models. These occupations are related to the following Career Listings: Architecture and Construction; Arts, A/V Technology and Communications; Manufacturing; Science, Technology, Engineering and Mathematics.

Lesson Reflection:

As an exit slip, ask students to write a short paragraph related to their study of similarity. Any of the following questions could be included: What does it mean for two geometric figures to be similar? What are all the geometric features that must be considered in order to say that two figures are similar? If two figures are similar, what can you say about their perimeters or their areas? (Areas might have been discussed if you have shown the Project Mathematics Video.) Can you name a situation in which you might use similarity?

In lesson 1, teachers were provided with a guide to aid them in reflecting upon the lesson as they seek to improve their practice. Certainly, it may not be feasible to formally complete such a reflection after every lesson, but hopefully the questions can generate some ideas for contemplation.

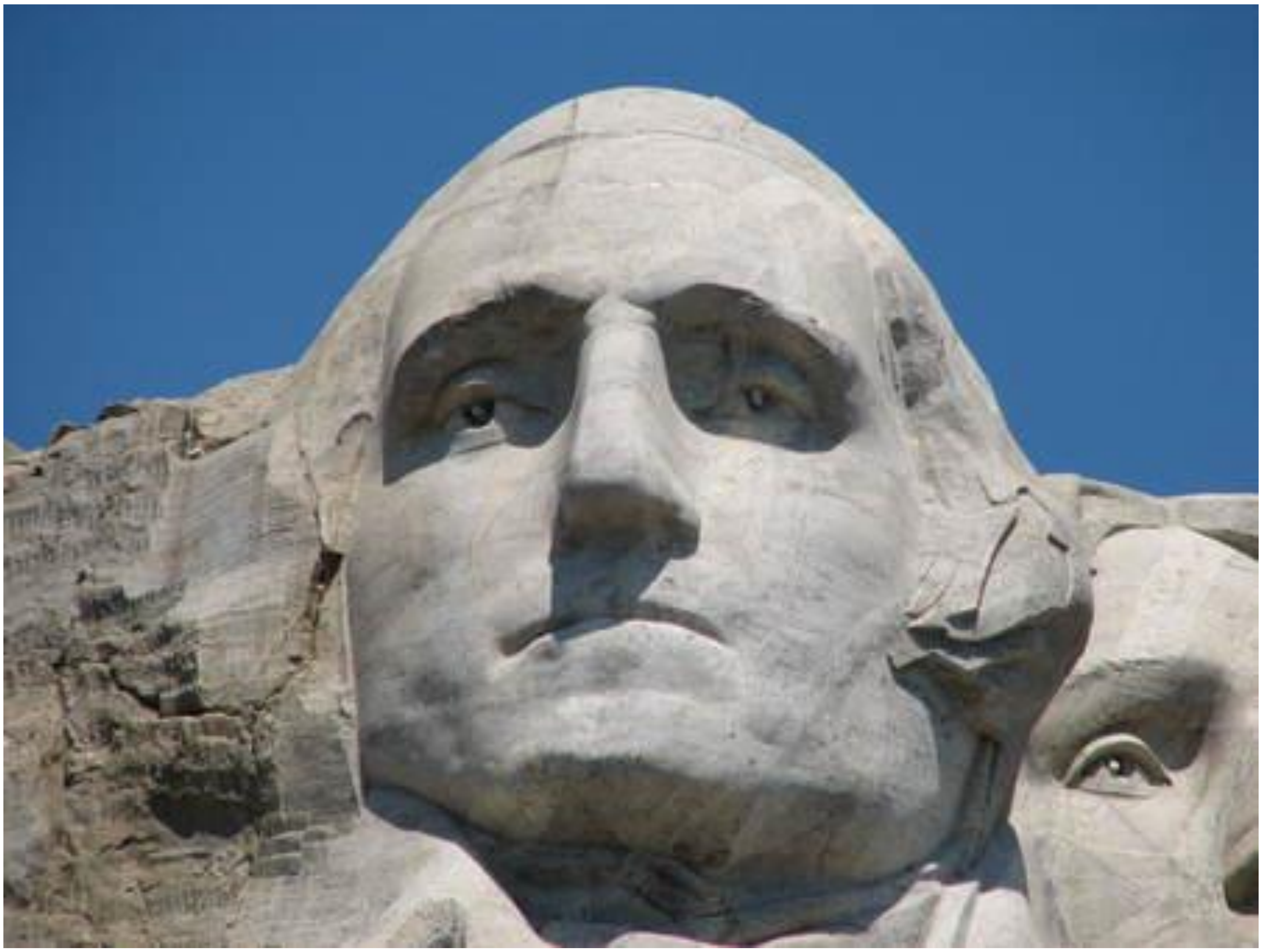

MOUNT RUSHMORE
. . . and
George Washington's Nose!



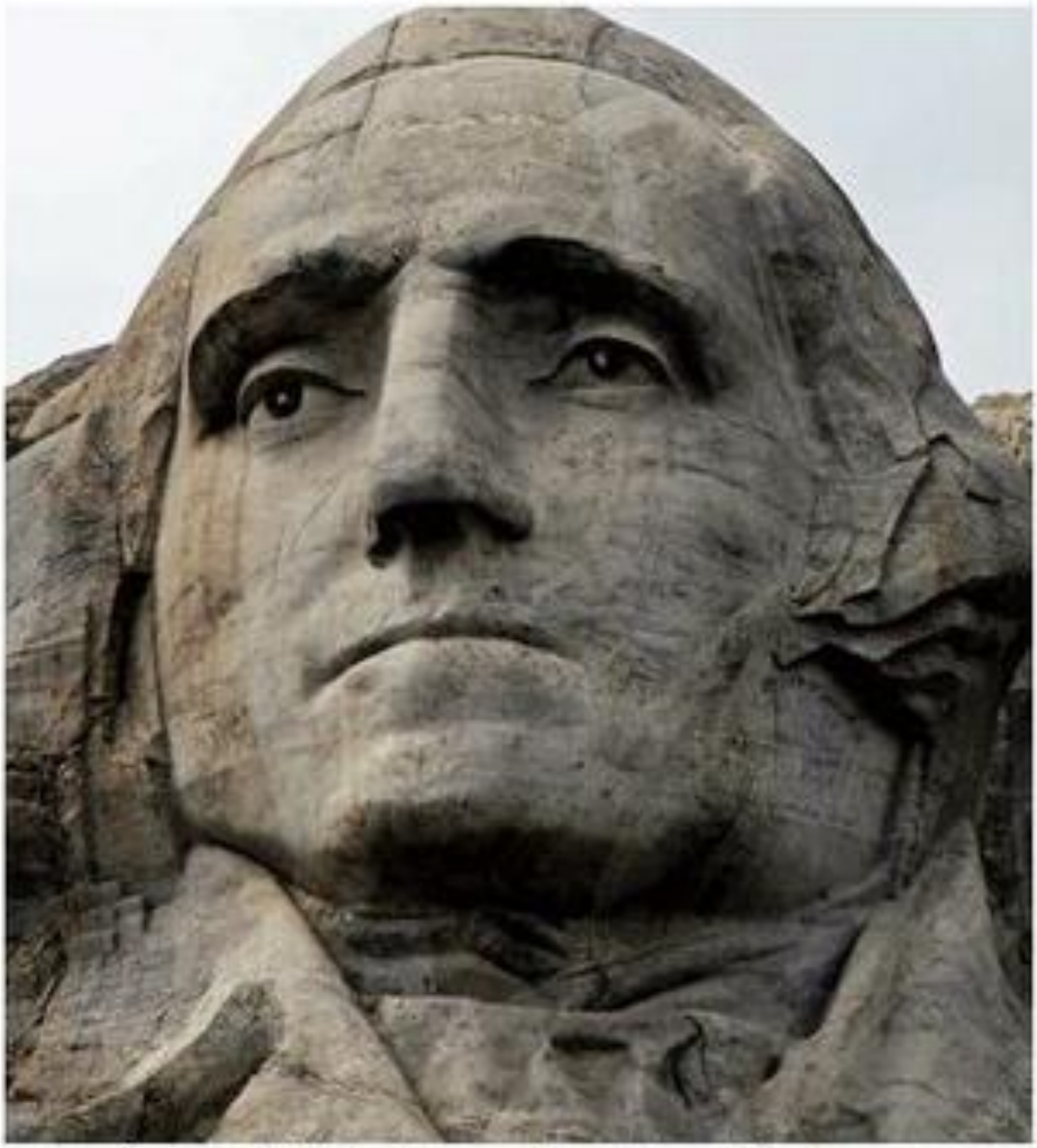
Your Challenge:

- Determine the length of George Washington's nose on the Mount Rushmore sculpture.

George Washington on Mount Rushmore



George Washington on Mount Rushmore



George Washington carving at Mount Rushmore

George Washington on Mount Rushmore

George Washington on Mount Rushmore

Thumbs Up

Student Name: ________________

To estimate the distance to a point, extend your right arm and close your left eye. Sight a point on the far wall over the center of your thumb.

Without moving your head or arm, close your right eye and again sight the object across the center of your thumb. Your thumb will appear to "jump" to the right.

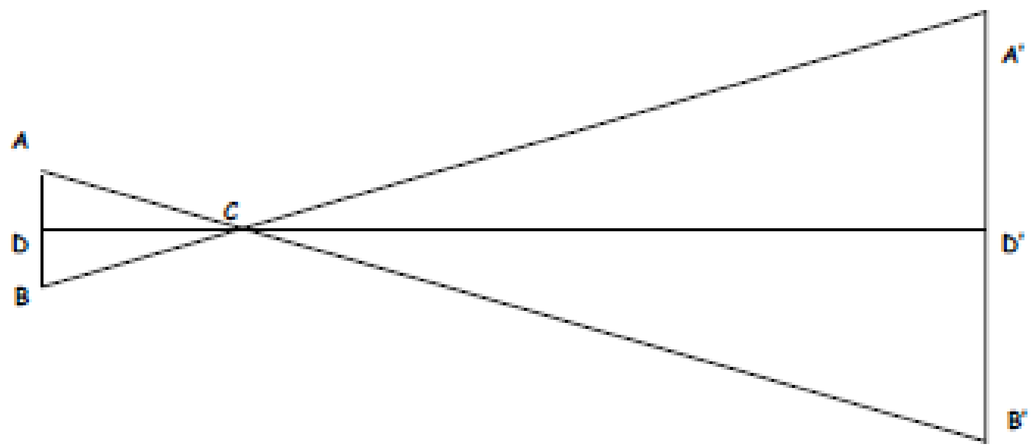
Estimate the distance of this "jump" by using the length of a brick or cinder block making up the far wall.

Estimated distance of "jump": ____________________

Multiply the estimated distance of this "jump" by 10. This will be the approximate distance to the point.

Approximate distance to point: ____________________

Mathematical Basis:



In the figure, A represents the student's left eye, B represents his/her right eye, and C represents his/her thumb.

It is assumed that the ratio of AB/DC is 1:10. (This is based on the fact that for most people, the ratio between the length of their outstretched arm and the distance between their eyes is approximately 10:1).

It is also assumed that DCD' is a straight line. (It is the line from the student's nose to the object.) Then vertical angles DCB and D'CA' are congruent. Angles CBD and CD'A' are right angles, and therefore congruent. Therefore, triangle CDB and triangle CD'A' are similar.

Likewise, triangle CDA and triangle CD'B' are similar. It follows that triangle ABC and triangle B'A'C are similar.

Therefore, $\frac{AB}{DC} = \frac{A'B'}{D'C}$

$$\frac{1}{10} = \frac{A'B'}{D'C}$$

and $D'C = 10\ A'B'$

Me and My Shadow Name: ______________________

- Hold the meter stick perpendicular to the ground and measure its shadow.
- Measure the shadow of the school building.
- Create diagrams to model the situation (one diagram of the meter stick and its shadow; a second diagram of the building and its shadow). Include all known measurements in the diagrams.
- Create a proportion relating the measurements in the diagrams.
- Calculate the height of the school building. Show all work.

Mirror, Mirror on the Ground

Name: ____________________

- Lay the mirror flat on the ground in front of the flag pole whose height is to be determined.
- Slowly back away from the mirror until you can see the top of the flag pole in the mirror. Mark this point.
- Measure the distances from the marked point to the mirror and from the mirror to the base of the flag pole.

  Distance from marked point to mirror: _____________

  Distance from mirror to base of flag pole: _____________
- Create a diagram to model the situation. Include all known measurements in the diagram.
- Create a proportion relating the measurements in the diagram.
- Calculate the height of the school building. Show all work.

Diagram of the house designed by a six-year-old:

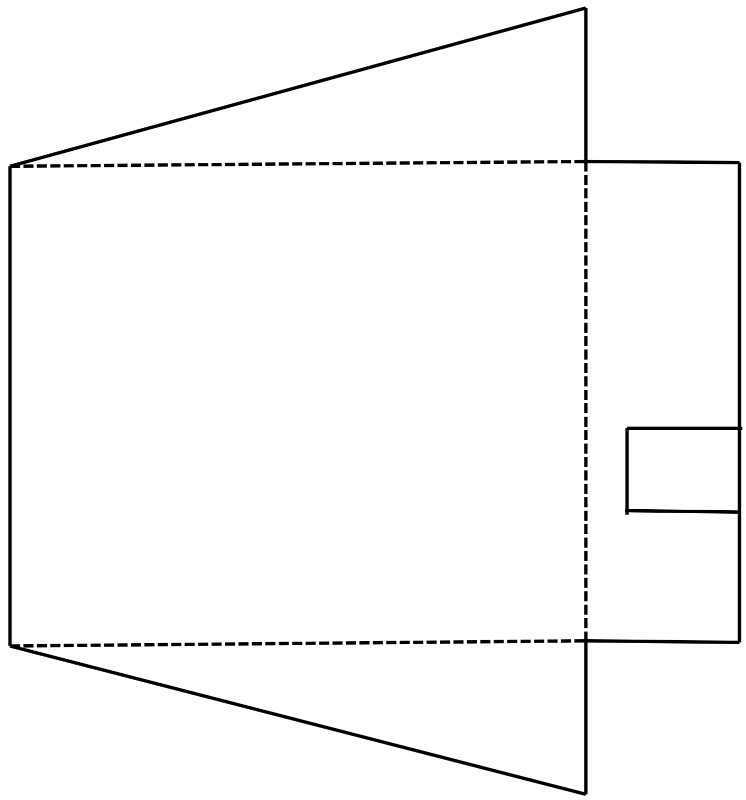
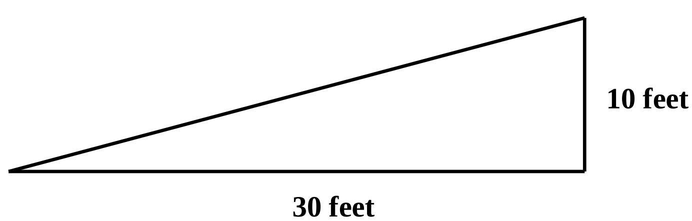


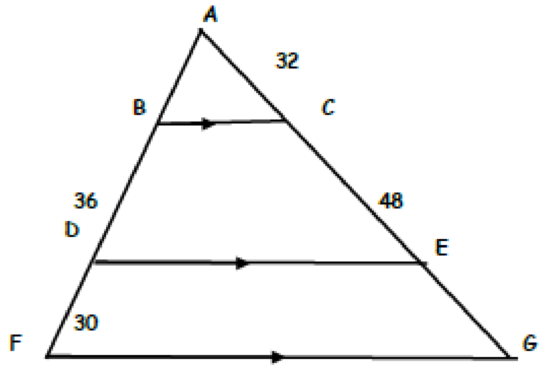
Diagram of a side view of the house after it is folded:



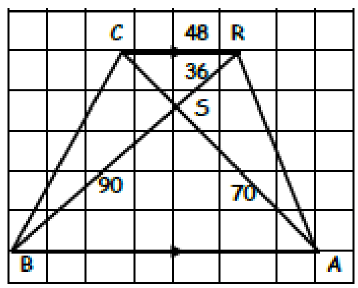
Similarity Problems

1. Segment BC, segment DE, and segment FG are parallel. Find the lengths of the missing segments.

   Length of AB = ______ Length of EG = ______

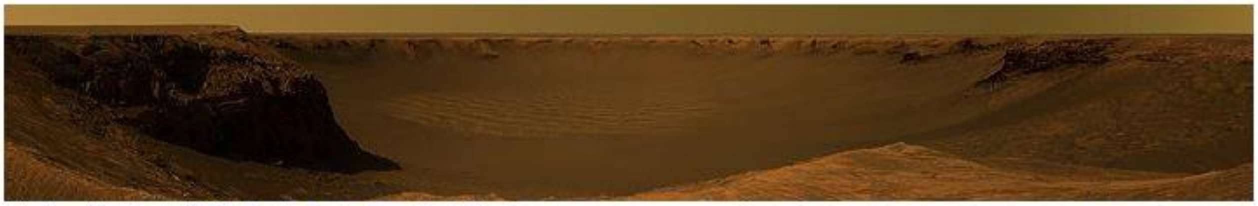


2. CRAB is a trapezoid, with segment CR parallel to segment BA.
   Is angle BRC congruent to angle ABR? Why or why not?
   Is angle RCA congruent to angle BAC? Why or why not?
   Why is △RCS ~ △BAS?
   Find the length of segment CS __________; find the length of segment BA __________.



NASA scientists wish to use the Mars Rover to determine the distance across a deep crater. Use your understanding of similar triangles to design a plan that would allow NASA to determine the distance across the crater.

Two views of the same Martian crater are provided:



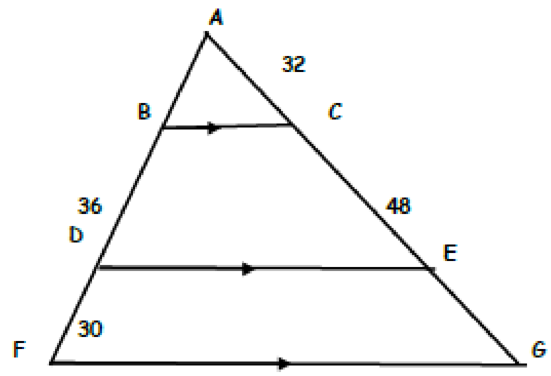
http://en.wikipedia.org/wiki/File:Victoria_Crater,_Cape_Verde-Mars.jpg

crater_mars_victorial.jpg

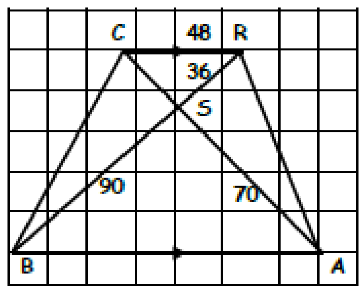
Similarity Problems Key

1. Segment BC, segment DE, and segment FG are parallel. Find the lengths of the missing segments.

   Length of AB = 24 Length of EG = 40



2. CRAB is a trapezoid, with segment CR parallel to segment BA.
   Is angle BRC congruent to angle ABR? Why or why not?
   Is angle RCA congruent to angle BAC? Why or why not?
   Why is $\triangle RCS \sim \triangle BAS$?
   Find the length of segment CS = 28 ; find the length of segment BA = 120.



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http://en.wikipedia.org/wiki/File:Victoria_Crater,_Cape_Verde-Mars.jpg

crater_mars_victoria1.jpg

Possible Plan:

Have the Mars Rover travel the paths of the similar triangles. Create a proportion relating the missing length and a side of the triangle that contains it with the corresponding sides of the second triangle.