Lesson Plan
Lesson 14: Using Trigonometric Ratios
Mathematics High School Math II

Unit Name: Unit 5: Similarity, Right Triangle Trigonometry, and Proof

Lesson Plan Number & Title: Lesson 14: Using Trigonometric Ratios

Grade Level: High School Math II

Lesson Overview: In a variety of problem situations, students develop a deeper understanding of the trigonometric functions and their applications. This lesson is designed for approximately 45 minutes.

Focus/Driving Question: How can the trigonometric ratios be applied in real world problem situations?

West Virginia College- and Career-Readiness Standards:

M.2HS.46 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures

M.2HS.48 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.

M.2HS.49 Explain and use the relationship between the sine and cosine of complementary angles.

M.2HS.50 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Manage the Lesson:

Students expand and apply their understanding of trigonometric ratios and the Pythagorean Theorem in problem solving situations.

Academic Vocabulary Development:

No new vocabulary is introduced. In a variety of investigations and explorations, students strengthen their understanding of the vocabulary related to trigonometric ratios. Students develop the ability to use appropriate vocabulary in problem solving situations. The related vocabulary includes sine, cosine, tangent, inverse functions, hypotenuse, opposite side, adjacent side, and complementary angles.

Launch/Introduction:

Introduce students to a problem situation *River Width* found at http://www.cpalms.org/Public/PreviewResourceAssessment/Preview/66153. In this problem, students are introduced to a situation similar to the exploration they will later undertake. Students are asked to determine the distance between two objects that are separated by a river.

Investigate/Explore:

To complete this activity, students will need to build a transit circle. Directions are provided in 14.1 Transit Circle**.** It would be advantageous for students to make and practice using their transit circles in advance of this investigation.

Distribute 14.2 Rapid Transit to student groups. In this problem, student groups determine the distance from a given point to an inaccessible point. Find a fairly level area, perhaps along the side of a road. Locate and mark a point, the "inaccessible point," at a distance across the road, perhaps with a wooden stake. Mark a second "accessible point" with a wooden stake. Ask students to determine the distance to the "inaccessible point." After students have completed this investigation, if it is possible, use a trundle wheel or tape measures to verify the distance students have calculated between the two points.

14.3 Trigonometric Ratios provides a skill set of problems and 14.4 Using Trigonometric Ratios provides a series of problem situations that involve the use of trigonometric ratios. In 14.5 Trigonometric Performance Task**,** students are challenged to make decisions about the construction of a new skateboard ramp after analyzing an original ramp and two options.

Summarize/Debrief:

14.6 Check Your Understanding presents four problem situations asking students to demonstrate their ability to apply trigonometric ratios.

Materials:

Transit Circles: construct each using a board (22 cm by 22 cm), a wood strip (15.5 cm), 4 finishing nails, and 1 washer
Trundle wheel or Tape measures
Wooden stakes
14.1 Transit Circle
14.2 Rapid Transit
14.3 Trigonometric Ratios
14.3 Trigonometric Ratios - Key
14.4 Using Trigonometric Ratios
14.4 Using Trigonometric Ratios - Key
14.5 Trigonometric Performance Task
14.6 Check Your Understanding
14.6 Check Your Understanding - Key

Cpalms – *River Width* : http://www.cpalms.org/Public/PreviewResourceAssessment/Preview/66153
Solving Triangles http://mathforum.org/mathimages/index.php/Solving_Triangles
Sine and Cosine Ratios:
http://images.google.com/imgres?q=trigonometric+ratios+%26+trees&start=215&num=10&hl=en&tbo=d&biw=1366&bih=667&tbm=isch&tbnid=mWbEyC6DfUAPbM:&imgrefurl=http://lwebs.ca/InterMath/Trigonometry/SineandCosineRatios/&docid=2vDTVLwakLkkBM&imgurl=http://lwebs.ca/InterMath/Trigonometry/SineandCosineRatios/paste_image123.png&w=614&h=305&ei=c54FUZGeM7CQ0QG8zYGgCw&zoom=1&iact=hc&vpx=712&vpy=327&dur=153&hovh=158&hovw=319&tx=123&ty=90&sig=111219021461538647280&page=7&tbnh=114&tbnw=230&ndsp=42&ved=1t:429,r:45,s:200,i:139
Trigonometry - http://lwebs.ca/InterMath/Module14/Trigonometry_print.html
Trigonometry for Solving Problems - This Illuminations lesson offers a pair of puzzles to enforce the skills of identifying equivalent trigonometric expression and problem situations involving the angle of elevation and angle of declination: http://illuminations.nctm.org/LessonDetail.aspx?id=L383

Career Connection:

As property is bought and sold, determining boundary points becomes essential. Surveyors use trigonometric relationships to accurately determine the location of points and the distance and angles between them. Road makers, bridge builders and those whose job it is to get buildings in the right place all use trigonometric functions in their daily work.

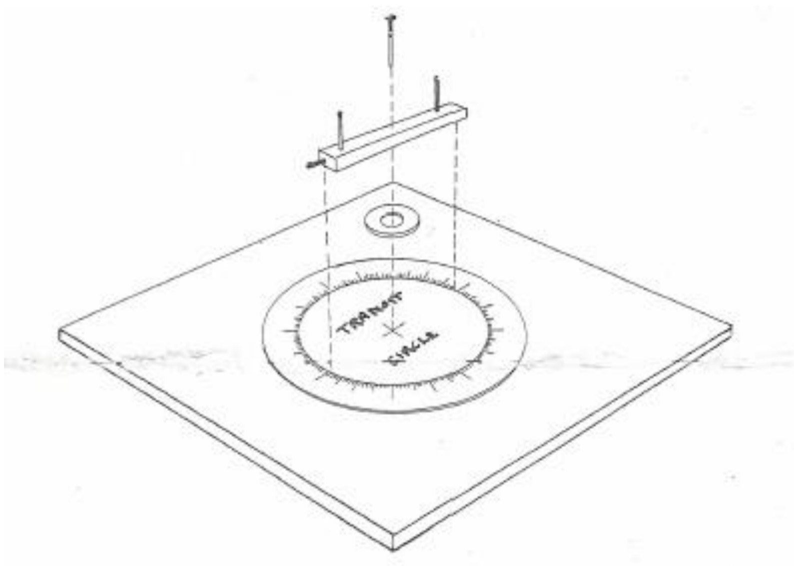
Lesson Reflection:

Students should understand how the trigonometric ratios are derived from the properties of similar triangles. Students should understand that to solve problems involving right triangles, to determine the measures of an angle or a side of a right triangle, it is necessary to be given only the measure of two sides or one side and an acute angle. Students should demonstrate skill in creating diagrams of problem situations, in identifying the appropriate trigonometric ratio to use, and in solving equations involving trigonometric functions.

In lesson 1, teachers were provided with a guide to aid them in reflecting upon the lesson as they seek to improve their practice. Chertainly, it may not be feasible to formally complete such a reflection after every lesson, but hopefully the questions can generate some ideas for contemplation.

Transit Circle

The transit circle is an instrument used for measuring horizontal angles. It is often called a transit, but technically a transit has the capability to measure both vertical and horizontal angles.



Materials:

Board (22 cm by 22 cm)

Wood strip (15.5 cm)

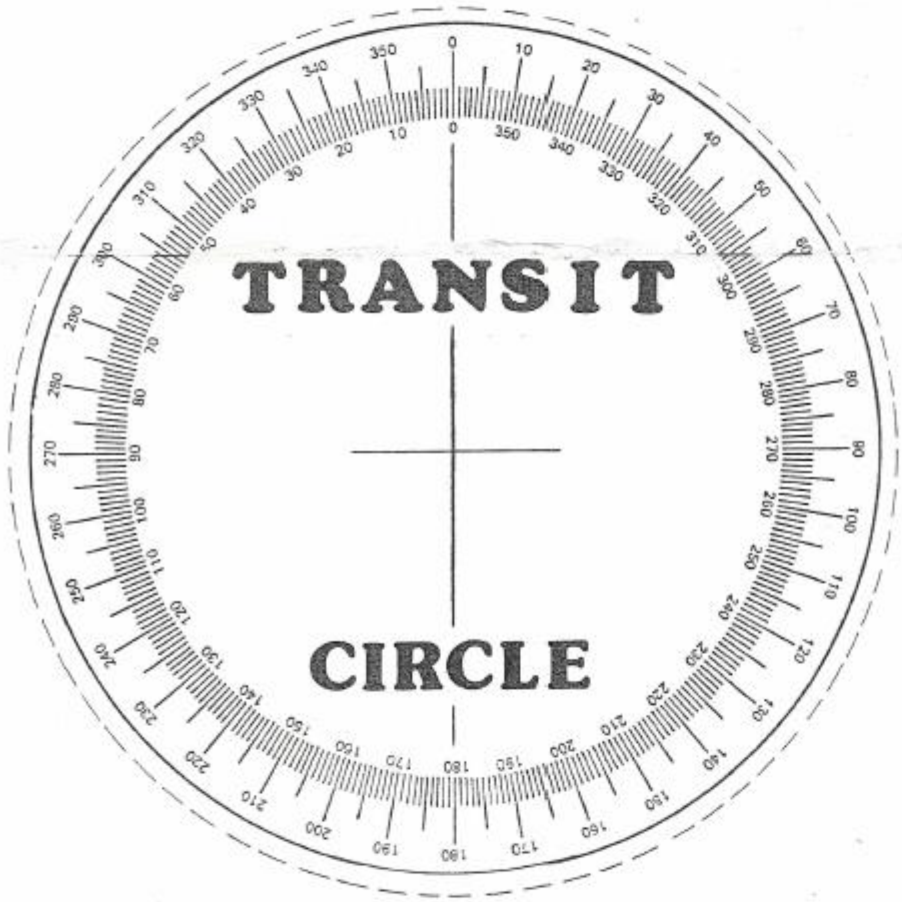
4 finishing nails

1 washer

Construction:

- Copy the figure below. Cut on the dashed lines and glue it to the board.
- Place two finishing nails 0.5 cm from the end of the wood strip, and 1 nail into one end of the strip.
- Nail the strip to the marked center of the figure with a washer between the strip and the board.

TRANSIT CIRCLE SCALE



Rapid Transit

Name: ____________________

- Set the transit circle to the zero mark.
- From the "accessible" wooden stake, sight across the transit circle to the "inaccessible" point.
- Turn the sighting strip at a right angle. On the line of sight, have a second member of the group place a stake at a distance of 10 meters.
- Position the transit at the new point and sight to the first stake. Measure the angle as you turn the sighting strip to the "inaccessible" point.
- Create a diagram of the problem situation and include all known measurements.
- Identify a trig ratio that can be used to find the distance between the original "accessible" wooden stake to the "inaccessible" point.
- Use the trig ratio to find the distance between the original "accessible" wooden stake to the "inaccessible" point. Show all of your work.

Trigonometric Ratios

Use your calculator and round your answers to the nearest tenth.

1. Find 6(cos 20°)

2. Find (cos 50°)

3. Find (sin 40°)

4. Find (cos 25°)(sin 35°)

5. Find $\frac{1}{2}$(sin 60°)

6. In △DEF, ∠E is a right angle. If sin D = 0.7, find the measure of ∠D to the nearest degree.

7. In △ABC, ∠B is a right angle. If *AB* = 7, *BC* = 24, and *AC* = 25, express sin *A* as a fraction.

8. In △MNP, ∠P is a right angle. If *MP* = 15, NP = 36, and *MN* = 39, express tan *M* as a fraction.

9. In △XYZ, ∠Z is a right angle. If *z* = 30 and ∠X = 25°, find *x* to the nearest integer.

10. In △GHK, ∠H is a right angle. If *GH* = 8, *GK* = 10, find HK.

11. △RST is a right triangle and tan R = 0.96. Find R to the nearest degree.

12. △PQR is a right triangle and sin Q = 0.63. Find Q to the nearest degree.

Trigonometric Ratios - Key

Use your calculator and round your answers to the nearest tenth.

1. Find 6(cos 20°) 5.6

2. Find (cos 50°) 0.6

3. Find (sin 40°) 0.6

4. Find (cos 25°)(sin 35°) 0.5

5. Find $\frac{1}{2}$(sin 60°) 0.4

6. In △DEF, ∠E is a right angle. If sin D = 0.7, find the measure of ∠D to the nearest degree. 44°

7. In △ABC, ∠B is a right angle. If *AB* = 7, *BC* = 24, and *AC* = 25, express sin *A* as a fraction. $\frac{24}{25}$

8. In △MNP, ∠P is a right angle. If *MP* = 15, NP = 36, and *MN* = 39, express tan *M* as a fraction. $\frac{36}{15}$

9. In △XYZ, ∠Z is a right angle. If z = 30 and ∠X = 25°, find x to the nearest integer. 13

10. In △GHK, ∠H is a right angle. If *GH* = 8, *GK* = 10, find HK. HK = 6

11. △RST is a right triangle and tan R = 0.96. Find R to the nearest degree. 44°

12. △PQR is a right triangle and sin Q = 0.63. Find Q to the nearest degree. 39°

Using Trigonometric Ratios

Create a diagram. Identify the appropriate trigonometric ratio. Show all work.

1. The angle of elevation from a sailboat to the top of a 121 foot lighthouse on shore measure 16°. To the nearest foot, how far is the sailboat from shore?

2. According to Chinese legend, General Han Xin (Han dynasty 206 B.C - 220 A.D.) flew a kite over the palace of his enemy to determine the distance between his troops and the palace. If the general let out 1000 meters of string and the kite was flying at a 40° angle of elevation, how far away was the palace from General Han Xin's position?

3. A salvage ship's sonar locates wreckage at a 15° angle of depression. A diver is lowered 50 meters to the ocean floor. How far does the diver need to walk along the ocean floor to the wreckage?

4. A ship's officer sees a lighthouse at a 37° angle to the path of the ship. After the ship travels 2000 m, the lighthouse is at a 90° angle to the ship's path. What is the distance between the ship and the lighthouse at this second sighting?

5. An air traffic controller must calculate the angle of descent (the angle of depression) for an incoming jet. The jet's crew reports that their land distance is 53 km from the base of the control tower and that the plane is flying at an altitude of 6.2 km. Find the measure of the angle of descent.

6. A lighthouse is east of a sailboat. The sailboat's dock is 25 km north of the lighthouse. The ship's captain measure the angle between the sailboat and the dock and finds it to be 40°. How far is the sailboat from the dock?

7. A man on the ground observes the angle of elevation to a bird at the top of a telephone pole is 60°. If the man is standing 50 feet from the foot of the pole, how high is the telephone pole, to the nearest foot?

8. From the top of a 200-foot cliff, Sally spots the Loch Ness Monster at an angle of depression of 20°. To the nearest foot, how far is Sally from the Loch Ness Monster?

9. A zip-line is attached to a platform that is 80 feet above the ground. If the line has a 15° angle of elevation, to the nearest foot, how long is the zip-line?

Using Trigonometric Ratios - Key

Create a diagram. Identify the appropriate trigonometric ratio. Show all work.

1. The angle of elevation from a sailboat to the top of a 121 foot lighthouse on shore measure 16°. To the nearest foot, how far is the sailboat from shore?

   $\tan 16 = \frac{121}{x}$ (where x = the distance from the sailboat to shore)

   $x \approx 422$ feet

2. According to Chinese legend, General Han Xin (Han dynasty 206 B.C - 220 A.D.) flew a kite over the palace of his enemy to determine the distance between his troops and the palace. If the general let out 1000 meters of string and the kite was flying at a 40° angle of elevation, how far away was the palace from General Han Xin's position?

   $\cos 40 = \frac{x}{1000}$ (where x = the distance between the troops and the palace)

   $x = 766$ meters

3. A salvage ship's sonar locates wreckage at a 15° angle of depression. A diver is lowered 50 meters to the ocean floor. How far does the diver need to walk along the ocean floor to the wreckage?

   $\tan 15 = \frac{50}{x}$ (where x = the distance the diver needs to walk to reach the wreckage)

   $x = 186$ meters

4. A ship's officer sees a lighthouse at a 37° angle to the path of the ship. After the ship travels 2000 m, the lighthouse is at a 90° angle to the ship's path. What is the distance between the ship and the lighthouse at this second sighting?

   $\tan 37 = \frac{x}{2000}$ (where x = distance between ship and lighthouse at the 2nd sighting)

   $x = 1507$ meters

5. An air traffic controller must calculate the angle of descent (the angle of depression) for an incoming jet. The jet's crew reports that their land distance is 53 km from the base of the control tower and that the plane is flying at an altitude of 6.2 km. Find the measure of the angle of descent.

   $\tan x = \frac{6.2}{53}$ (where x = angle of descent)

   $x = 6°$

6. A lighthouse is east of a sailboat. The sailboat's dock is 25 km north of the lighthouse. The ship's captain measure the angle between the sailboat and the dock and finds it to be 40°. How far is the sailboat from the dock?

$\sin 40 = \frac{25}{x}$ (where x is the distance from the sailboat to the dock)

x = 38.9 km

7. A man on the ground observes the angle of elevation to a bird at the top of a telephone pole is 60°. If the man is standing 50 feet from the foot of the pole, how high is the telephone pole, to the nearest foot?

$\tan 60 = \frac{x}{50}$ (where x = height of pole)

x = 87 feet

8. From the top of a 200-foot cliff, Sally spots the Loch Ness Monster at an angle of depression of 20°. To the nearest foot, how far is Sally from the Loch Ness Monster?

$\cos 20 = \frac{200}{x}$ (where x = distance from Sally to the Loch Ness Monster)

x = 213 feet

9. A zip-line is attached to a platform that is 80 feet above the ground. If the line has a 15° angle of elevation, to the nearest foot, how long is the zip-line?

$\sin 15 = \frac{80}{x}$ (where x = length of the zip-line)

x = 309 feet

Trigonometric Performance Task

Mark has constructed a skateboard ramp in his back yard. The triangular ramp is 8 feet in length (horizontal length) and reaches a height of 4 feet.

A. Create a diagram of the ramp and label the known dimensions. Determine the length of the skateboard ramp (diagonal length).

B. Determine the angle of elevation of the ramp, to the nearest degree.

C. Mark has decided to build a second ramp and increase the angle of elevation by 5°. This new ramp will also be 4 feet high. What is the length of this new skateboard ramp (diagonal length)? Is this second ramp longer or shorter than the original ramp?

D. Mark has decided to change his plans for the second ramp. He has decided to instead construct a ramp for his younger brother. He has decided to either decrease the height of the original ramp by one foot or increase the horizontal ramp by two feet, leaving all other measurements the same. Create a diagram of each of the two possible ramps.

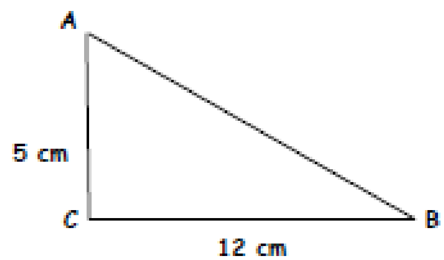
E. Find the angle of elevation for each of these two possible ramps to the nearest degree. Which ramp has the smallest angle of elevation?

F. Determine the length of each of these possible skateboard ramps (diagonal length). Which ramp is closest in length to Mark's original ramp?

G. Help Mark decide which ramp to build for his brother. Explain your reasons.

Check Your Understanding

- Given the triangle below, find sin *A*, sin B, cos *A*, cos B, tan *A*, tan B, the measure of angle *A* and the measure of angle B.



sin *A* =

sin B =

cos *A* =

cos B =

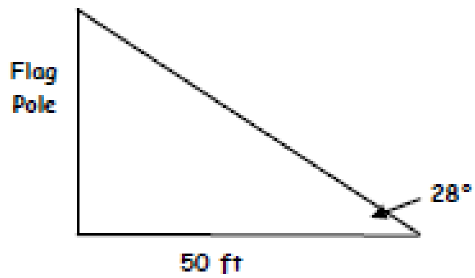
tan *A* =

tan B =

measure of angle *A* =

measure of angle B =

- Find the height of a flag pole to the nearest tenth if the angle of elevation of the sun is 28° and the length of the shadow of the flag pole is 50 feet.

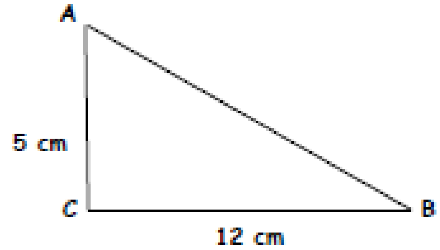


- A teenager whose eyes are five feet above ground level is looking into a mirror on the ground and can see the top of the building that is 30 feet away from the teenager. The angle of elevation from the center of the mirror to the top of the building is 70°. How tall is the building? How far away from the teenager's feet is the mirror?

- While traveling across flat land, you see a mountain directly in front of you. The angle of elevation to the peek is 3.5°. After driving 14 miles closer to the mountain, the angle of elevation is 9.5°. Explain how you would set up the problem, and find the approximate height of the mountain.

Check Your Understanding - Key

- Given the triangle below, find sin *A*, sin B, cos *A*, cos B, tan *A*, tan B, the measure of angle *A* and the measure of angle B.



sin *A* = $\frac{12}{13}$

sin B = $\frac{5}{13}$

cos *A* = $\frac{5}{13}$

cos B = $\frac{12}{13}$

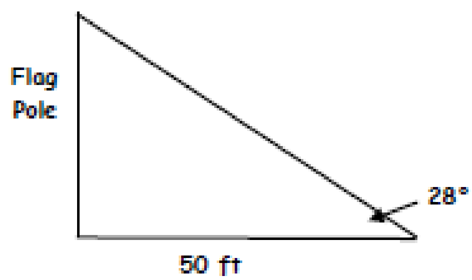
tan *A* = $\frac{12}{5}$

tan B = $\frac{5}{12}$

measure of angle *A* = 67°

measure of angle B = 23°

- Find the height of a flag pole to the nearest tenth if the angle of elevation of the sun is 28° and the length of the shadow of the flag pole is 50 feet. 26.6°



- A teenager whose eyes are five feet above ground level is looking into a mirror on the ground and can see the top of the building that is 30 feet away from the teenager. The angle of elevation from the center of the mirror to the top of the building is 70°. How tall is the building? How far away from the teenager's feet is the mirror? 77.5 feet; 1.8 feet

- While traveling across flat land, you see a mountain directly in front of you. The angle of elevation to the peek is 3.5°. After driving 14 miles closer to the mountain, the angle of elevation is 9.5°. Explain how you would set up the problem, and find the approximate height of the mountain. Create a system of equations in two variables: tan 3.5 = c/x and tan 9.5 = c/x-14 where c is height of mountain and x is the distance to mountain. (x = 22 miles) Height of mountain = 1.3 miles.