Unit Name: Unit 2: Quadratic Functions and Modeling

Lesson Plan Number & Title: Lesson 5: I See Where You Are Coming From

Grade Level: High School Math II

Lesson Overview:
Given a linear, quadratic, or exponential context, students look at a series of steps to model the context and determine an explicit or recursive algebraic representations. Students will decide if a function is invertible, find the inverse if it exists and restrict the domain where necessary. They will combine standard function types using arithmetic of functions and relate these functions to the model.

Focus/Driving Question:
How can different representations of functions be useful in investigating patterns?

West Virginia College- and Career-Readiness Standards:
M.2HS.13
Write a function that describes a relationship between two quantities.

a. Determine an explicit expression, a recursive process or steps for calculation from a context.
b. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Instructional Note: Focus on situations that exhibit a quadratic or exponential relationship.

M.2HS.15
Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$. Instructional Note: Focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2$, $x > 0$.

Manage the Lesson:
Divide students into teams of 3 or 4 students.

Prepare a Resource/Learning Center for differentiating and tiering. Include the following possible tips or hints:
Explicit and Recursive Help
Inverse Functions Help

As a homework assignment at the end of each day, each student will use a word processor to keep a daily writing journal that includes accomplishments and a reflection of lessons learned. All entries will be in complete sentences.

Students will complete the activities in the Launch/Introduction and Investigate/Explore sections in 90 minute periods as follows:
Step 1 -- Launch/Introduction activity; Vocabulary and Concept Development Activity
Step 2 -- Explore Linear, Quadratic and Cubic Equations Activity
Step 3 -- Linear, Quadratic and Cubic Equations Assessment
Step 4 -- Investigating Quantity Relationships and Inverse Functions Activity
Step 5 -- Quantity Relationships and Inverse Functions Application Activity

After students have completed an activity, the teacher will lead a discussion either with individual groups or as whole class discourse.

Differentiation: Classroom format includes a mix of whole group, collaborative group, paired and individual activities. Functions are modeled in a wide variety of ways using physical and virtual manipulatives, graphing technology and Internet web sites. All explorations and discovery activities offer a variety of entry points. A Resource/Learning Center is provided that includes materials to meet the needs of all learners. Step-by-step instructions should be provided for the special needs student.

Academic Vocabulary Development:
common first, second, third, etc. differences
dependent variable
explicit function
independent variable
inverse function
invertible function
recursive function
Most of the vocabulary was developed and used extensively in the Math I course. The remaining terms will be addressed in the Lesson 5 - Vocabulary and Concept Development activity. The teacher will reinforce the vocabulary development from Math I and the activity by using the terms during the whole class discourse after each of the discovery and investigation activities. The students will demonstrate mastery of vocabulary by using each of the terms properly in their activities and daily journal entries.

Launch/Introduction:
The teacher will introduce the lesson with real-world applications. Follow tasks from Using Recursion to Explore Real-World Problems - http://www.dlt.ncssm.edu/stem/using-recursion-explore-real-world-problems. In their groups, students will discuss and individually complete *Using Recursion to Explore Real World Problems* handout. The teacher will project and discuss *Using Recursion to Explore Real World Problems presentation* to assist individual groups as needed.

Investigate/Explore:
Vocabulary and Concept Development Activity
Students will research the internet and complete the Lesson 5 - Vocabulary and Concept Development activity.
Suggested definitions and concepts can be found at Lesson 5 - Vocabulary and Concept Development Key.
Remind the students that they need to use each of the terms and concepts properly in their activities and daily journal entries.

Explore Linear, Quadratic and Cubic Equations
The activity helps students identify data sets as linear, quadratic and cubic by using first, second and third differences. The teacher will distribute Identifying Linear, Quadratic and Cubic Functions activity so that the students will be investigating six equations (2 linear, 2 quadratic and 2 cubic). The students will work in pairs and individually complete the activity sheet. Before given this activity, students will need to know how to enter an equation, graph an equation, use the list function, use regression, enter the regression equation and graph the regression equation on a graphing calculator. Special needs students may use Illuminations: Line of Best Fit - http://illuminations.nctm.org/ActivityDetail.aspx?ID=146.
Think-Pair-Share discussion questions after the activity should include the following:
What do common differences represent?
What would happen if you continue to find common differences after you have constant differences?
What are the regression equations for the following tables? Graph each regression.
What makes a set of data linear, quadratic, or cubic?

Linear, Quadratic and Cubic Equations Assessment
The teacher will use the template from Identifying Linear, Quadratic and Cubic Functions Assessment to prepare a test by selecting a variety of nine data sets from students' work and completing the f(x) column in each table (do not complete 1st, 2nd and 3rd differences columns - students will complete these columns). Use the following directions.
For each of the data sets:
State whether the data set is linear, quadratic or cubic.
Use the graphing calculator to find the regression equation.
Use the graphing calculator to graph the regression equation and copy the graph.

Investigating Quantity Relationships and Inverse Functions
Students with the aid of a graphing calculator and/or CAS (Computer Algebra System) will investigate average rate of change of different types of functions. Color tiles may be useful in this activity. In their groups, students will discuss and individually complete Investigating Quantity Relationships and Inverse Functions activity. Suggested solutions can be found at Investigating Quantity Relationships and Inverse Functions Key.

Quantity Relationships and Inverse Functions Application
Students will apply quantity relationships and inverse functions to different types of functions in various forms. In pairs, students will discuss and individually complete Quantity Relationships and Inverse Functions Application activity. Suggested solutions can be found at Quantity Relationships and Inverse Functions Application Key.

Summarize/Debrief:

After students have completed each activity, the teacher will lead a discussion either with individual groups or as whole class discourse. The teacher will use students' responses from activities and their responses during discussion to determine concepts that need to be retaught and revisited.

Materials:

Graph paper
Color Tiles
Graphing calculator and/or CAS (Computer Algebra System)
Algebra Tiles and/or Algebra Tiles - http://nlvm.usu.edu/en/nav/frames_asid_189_g_1_t_2.html?open=activities
Using Recursion to Explore Real-World Problems - http://www.dlt.ncssm.edu/stem/using-recursion-explore-real-world-problems
Illuminations: Line of Best Fit - http://illuminations.nctm.org/ActivityDetail.aspx?ID=146
XP - Math Jobs - http://www.xpmath.com/careers/math_jobs.php
We Use Math - http://weusemath.org/
Explicit and Recursive Help
Inverse Functions Help
Lesson 5 - Vocabulary and Concept Development
Lesson 5 - Vocabulary and Concept Development Key
Identifying Linear Quadratic and Cubic Functions
Identifying Linear Quadratic and Cubic Functions Assessment
Investigating Quantity Relationships and Inverse Functions
Investigating Quantity Relationships and Inverse Functions Key
Quantity Relationships and Inverse Functions Application
Quantity Relationships and Inverse Functions Application Key

Career Connection:

Careers in the Science, Technology, Engineering and Mathematics cluster that use quantity relationships and inverse functions are actuaries, data analysts, mathematicians, computer programmers and statisticians along with several research careers in the Health Science cluster including biotechnology research and development. For more information on mathematics jobs and careers, see XP - Math Jobs - http://www.xpmath.com/careers/math_jobs.php and We Use Math - http://weusemath.org/.

Lesson Reflection:
The teacher will reflect on how the lesson went and determine the parts from the entire lesson that need to be revised or revisited.
As a final entry in their daily journal, students will respond to the following:
As you reflect on this lesson, in what ways were you most successful? In what areas do you still need improvement? Justify your responses.

Explicit and Recursive Help

Which Function Is It? -- http://moodle.tbaisd.org/mod/book/print.php?id=24175

Illuminations: Line of Best Fit -- http://illuminations.nctm.org/ActivityDetail.aspx?ID=146

Session 7, Part C: Figurate Numbers --
http://www.learner.org/courses/learningmath/algebra/session7/part_c/index.html

Inverse Functions Help

Introduction to Function Inverses|Functions Inverses|Khan Academy -- http://www.khanacademy.org/math/algebra/algebra-functions/function_inverses/v/introduction-to-function-inverses

Inverse Functions: Definition/Drawing Inverses -- http://www.purplemath.com/modules/invrsfcn.htm

Inverse Functions -- https://www.youtube.com/watch?v=5CPhScuRt_Y&index=34&list=PLBTUpQbZRE9Lzit1CVQW0ZQb8d1tMkOv3

Inverse Functions -- http://www.ehow.com/video_4756973_find-inverse-function.html

Inverse Functions -- http://dl.uncw.edu/digilib/mathematics/algebra/mat111hb/functions/inverse/inverse.html

Inverse Functions -- http://www.mathsisfun.com/sets/function-inverse.html

Vocabulary and Concept Development

Whether a function is linear, quadratic, cubic, etc. can be determined by the common first, second, third, etc. differences from its table of values. Explain how to determine the family of functions.

Define independent variable.

Define dependent variable.

Define explicit function.

Define recursive function.

Define inverse function.

Define invertible function.

Are all functions invertible? Explain.

Vocabulary and Concept Development Key

Whether a function is linear, quadratic, cubic, etc. can be determined by the common first, second, third, etc. differences from its table of values. Explain how to determine the family of functions.
If there is a common first difference of the y-values in its table of values for consecutive x values, the function is linear. If there is a common second difference of the y-values in its table of values for consecutive x values, the function is quadratic and so on. That is, if there is a common n^{th} difference of the y-values in its table of values for consecutive x values, the function belongs to x^n family of functions.

Define independent variable.
An independent variable in a functional relation is the variable that determines the value of the function.

Define dependent variable.
A dependent variable in a functional relation is the value that is determined by the variables of the function. It is the value of the function.

Define explicit function.
An explicit function is a function whose value may be computed from the independent variable.

Define recursive function.
A recursive function is a function that determines the next term of a sequence from one or more of the preceding terms. The base value of the function must be included.

Define inverse function.
The inverse function for a function f is a function g whose domain is the range of f and whose range is the domain of f. That is, $f\left(g\left(x\right)\right)=x \,\&\, g\left(f\left(x\right)\right)=x$.

Define invertible function.
An invertible function is a function that has an inverse function.

Are all functions invertible? Explain.
Not all functions are invertible. All functions have an inverse, but the inverse may not be a function. A function has an inverse function if and only if when its graph is reflected in $y=x$, the result is the graph of a function. That is, if the inverse of the function does not pass the vertical line test, the function is not invertible. In simpler terms, if the function does not pass the horizontal line test, it is not invertible.

Identifying Linear, Quadratic and Cubic Functions Activity Sheet

Name ______________________________

Directions: Write an equation of each type in the first column of the following table. Use a graphing calculator to graph each equation and copy the graph in the second column. Complete the table with the data set and common differences for each equation. After you have completed the above steps, enter your data set into a list in the graphing calculator and check your equation, data set and graph by using the proper regression.

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Identifying Linear, Quadratic and Cubic Functions Assessment

Name ______________________________

Directions: For each of the data sets:

1. State whether the data set is linear, quadratic or cubic in the first column.
2. Use the graphing calculator to find the regression equation.
3. Use the graphing calculator to graph the regression equation and copy the graph.

	Equation	Graph	Common Differences
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	Equation	Graph	Common Differences
			see table 1 below
			see table 2 below
			see table 3 below

x	f(x)	1^{st}	2^{nd}	3^{rd}
1		****	****	****
2			****	****
3				****
4				
5				
6				
7				

x	f(x)	1^{st}	2^{nd}	3^{rd}
1		****	****	****
2			****	****
3				****
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6				
7				

x	f(x)	1^{st}	2^{nd}	3^{rd}
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2			****	****
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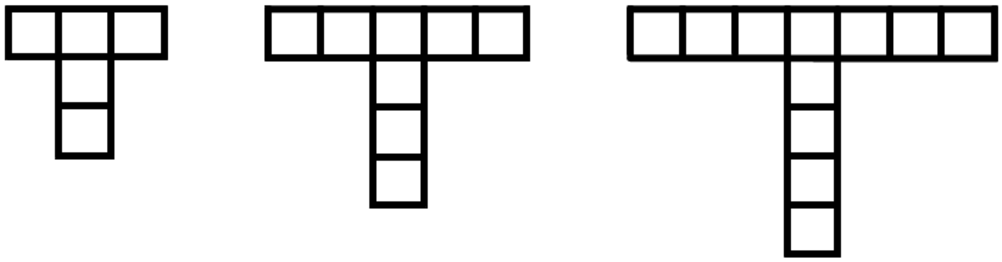
Investigating Quantity Relationships and Inverse Functions

Use the following figures that are constructed from square tiles to answer the following.



Size 1 Size 2 Size 3

1. Complete the following table of values where x is the size and $f(x)$ is the number of tiles for sizes 1, 2 and 3.

Table 1

x	$f(x)$	Common Differences		
		1st	2nd	3rd
0		****	****	****
1			****	****
2				****
3				
4				
5				

2. Extend the pattern by drawing Size 4 and Size 5 figures. Enter the values for $f(4)$ and $f(5)$ in Table 1.

3. It may be helpful to consider figure, Size 0. Size 0 is the figure that might come before Size 1. Draw Size 0 figure and enter the value for $f(0)$ in Table 1.

4. Look at the figures and Table 1 to complete the following.
   a. Write $f(5)$ as a function of $f(4)$.

   b. Write $f(4)$ as a function of $f(3)$.

   c. Write $f(3)$ as a function of $f(2)$.

   d. Write $f(2)$ as a function of $f(1)$.

   e. Write $f(1)$ as a function of $f(0)$.

   f. Find $f(0)$.

   h. Notice that you can work backwards using the equations (from parts a - f) to find the value for any $f(x)$ from $x=0$ to $x=5$. Observe the pattern in each equation and write f as a recursive function. Don't forget to include the base case.

5. Find $f(7)$ recursively. Show all work.

6. The next set of questions refers to writing f as an explicit function, where $f(x)$ represents the number of tiles in each figure and x is the size.
   a. Use common 1^{st}, 2^{nd} and 3^{rd} differences from Table 1 to decide the function type.

   b. Write f as an explicit function by using information from Table 1, information from your work in Problem 5, type of function from Part a, and the relationship of of $f(x)$ and x. That is, think about how the number of tiles in each figure is related to the "size" of the figure. Don't forget to restrict the domain.

7. The next set of questions refers to the inverse of f.
   a. Use the definition of the inverse of f and complete the following table of values where x is the number of tiles and $f^{-1}(x)$ is the size.

Table 2

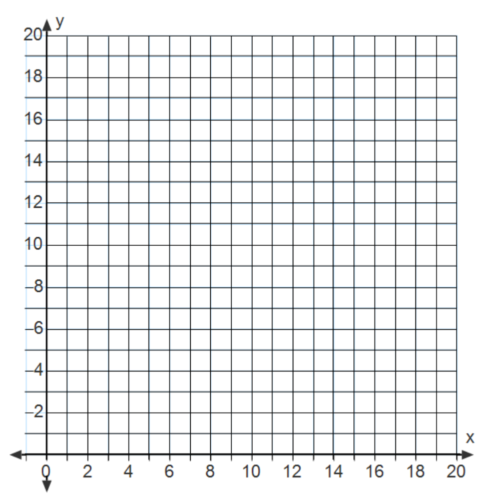
x	$f^{-1}(x)$
2	
5	
8	
11	
14	
17	

b. Is the inverse of f a function? Explain.

c. Compare Table 1 and Table 2. What do you notice about the domain and range?

d. Use the definition of an inverse function and the information gained from part c to find the equation for f^{-1}. Don't forget to restrict the domain.

e. Graph f , f^{-1} and $y = x$ on the following graph.



f. Describe the visual relationship of f and f^{-1} in the above graph. Do you believe the relationship will be true for all functions and their inverses?

8. Let $g(x)=2$. Find the values of each of the following. Describe each situation in the context of the problem.

a. $f(x)+g(x)=$ ________________

b. $f(x)-g(x)=$ ________________

c. $f(x)\cdot g(x)=$ ________________

d. For $x=0$ or positive even x, $\frac{f(x)}{g(x)}=$ ________________

e. $f^{-1}(x)+g(x)=$ ________________

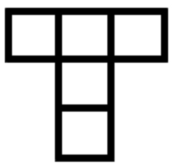
f. $f^{-1}(x)-g(x)=$ ________________

g. $f^{-1}(x)\cdot g(x)=$ ________________

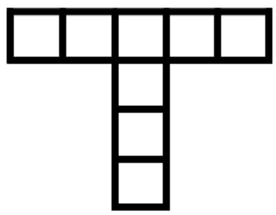
h. For $x\in\{n:n=6k+2 \text{ and } k \text{ is a whole number}\}$, $\frac{f^{-1}(x)}{g(x)}=$ ________________

Investigating Quantity Relationships and Inverse Functions Key

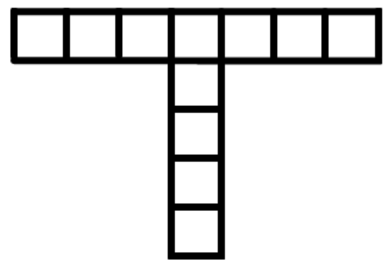
Use the following figures that are constructed from square tiles to answer the following.



Size 1



Size 2



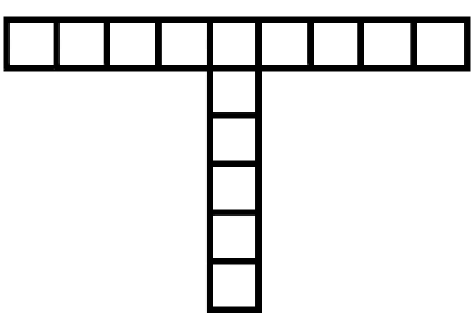
Size 3

1. Complete the following table of values where x is the size and $f(x)$ is the number of tiles for sizes 1, 2 and 3.

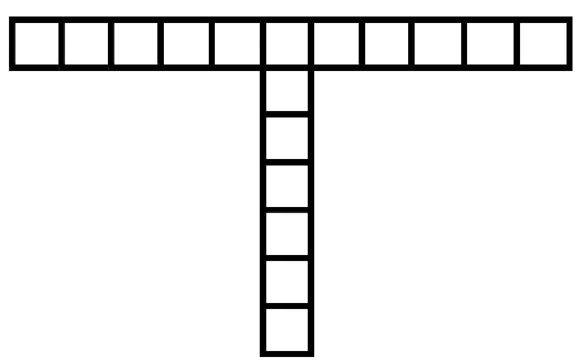
Table 1

x	$f(x)$	Common Differences		
		1st	2nd	3rd
0	2	****	****	****
1	5	3	****	****
2	8	3	0	****
3	11	3	0	0
4	14	3	0	0
5	17	3	0	0

2. Extend the pattern by drawing Size 4 and Size 5 figures. Enter the values for $f(4)$ and $f(5)$ in Table 1.

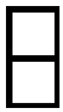


Size 4



Size 5

3. It may be helpful to consider figure, Size 0. Size 0 is the figure that might come before Size 1. Draw Size 0 figure and enter the value for $f(0)$ in Table 1.



Size 0

4. Look at the figures and Table 1 to complete the following.

   a. Write $f(5)$ as a function of $f(4)$.

      $f(5) = f(4) + 3$

   b. Write $f(4)$ as a function of $f(3)$.

      $f(4) = f(3) + 3$

   c. Write $f(3)$ as a function of $f(2)$.

      $f(3) = f(2) + 3$

   d. Write $f(2)$ as a function of $f(1)$.

      $f(2) = f(1) + 3$

   e. Write $f(1)$ as a function of $f(0)$.

      $f(1) = f(0) + 3$

   f. Find $f(0)$.

      $f(0) = 2$

   i. Notice that you can work backwards using the equations (from parts a - f) to find the value for any $f(x)$ from $x = 0$ to $x = 5$. Observe the pattern in each equation and write f as a recursive function. Don't forget to include the base case.

      $f(x) = f(x-1) + 3$, $f(0) = 2$ and x is a positive integer

5. Find $f(7)$ recursively. Show all work.

$$\begin{aligned} f(7) &= f(6)+3 \\ &= (f(5)+3)+3 \\ &= ((f(4)+3)+3)+3 \\ &= (((f(3)+3)+3)+3)+3 \\ &= ((((f(2)+3)+3)+3)+3)+3 \\ &= (((((f(1)+3)+3)+3)+3)+3)+3 \\ &= ((((((f(0)+3)+3)+3)+3)+3)+3)+3 \end{aligned}$$

Using $f(x) = f(x-1)+3$ to work down to the base case.

$$= ((((((2+3)+3)+3)+3)+3)+3)+3$$

Substituting the base case, $f(0)=2$.

$$= 3(7)+2$$

Rearranging to help discover the explicit case.

$$= 23$$

6. The next set of questions refers to writing f as an explicit function, where $f(x)$ represents the number of tiles in each figure and x is the size.
   a. Use common 1st, 2nd and 3rd differences from Table 1 to decide the function type. Is f linear, quadratic or cubic? Explain.
      Since the common 1st differences is constant, f is linear.

   b. Write f as an explicit function by using information from Table 1, information from your work in Problem 5, type of function from Part a, and the relationship of $f(x)$ and x. That is, think about how the number of tiles in each figure is related to the "size" of the figure. Don't forget to restrict the domain.
      $f(x)=3x+2$, where x is a whole number

7. The next set of questions refers to the inverse of f.
   a. Use the definition of the inverse of f and complete the following table of values where x is the number of tiles and $f^{-1}(x)$ is the size.

Table 2

x	$f^{-1}(x)$
2	0
5	1
8	2
11	3
14	4
17	5

b. Is the inverse of f a function? Explain.

f^{-1} is a function, since each x maps to a unique $f^{-1}(x)$.

c. Compare Table 1 and Table 2. What do you notice about the domain and range? The domain and range values from Table 1 are swapped in Table 2.

d. Use the definition of an inverse function and the information gained from part c to find the equation for f^{-1}. Don't forget to restrict the domain.

$$f(x) = 3x + 2$$

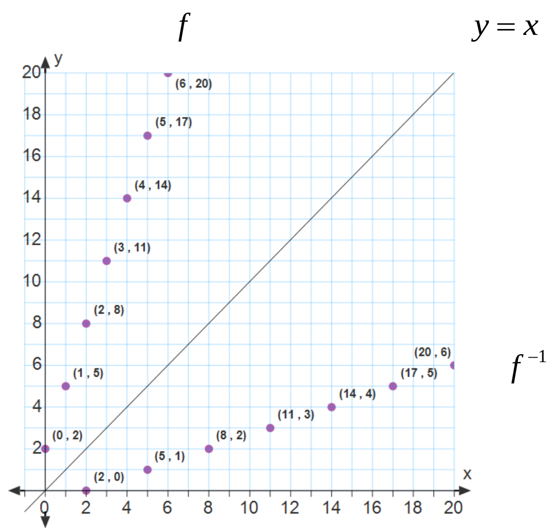
$$x = 3f^{-1}(x) + 2$$

$$x - 2 = 3f^{-1}(x)$$

$$\frac{x-2}{3} = f^{-1}(x)$$

$f^{-1}(x) = \dfrac{x-2}{3}$, where $x \in \{n : n = 3k + 2 \text{ and } k \text{ is a whole number}\}$

e. Graph f, f^{-1} and $y = x$ on the following graph.



f. Describe the visual relationship of f and f^{-1} in the above graph. Do you believe the relationship will be true for all functions and their inverses?

f and f^{-1} are symmetric about $y = x$.

$(a,b) \in f \Leftrightarrow (b,a) \in f^{-1}$. A reflection of (a,b) in $y = x$ is (b,a) and vice versa. The line of reflection is also known as a line of symmetry. Therefore, the relationship is true for all functions and their inverse.

8. Let $g(x)=2$. Find the values of each of the following. Describe each situation in the context of the problem.

a. $f(x)+g(x)=3x+4$

Each figure has two more tiles.

b. $f(x)-g(x)=3x$

Each figure has two less tiles.

c. $f(x)\cdot g(x)=6x+4$

Each figure has twice as many tiles.

d. For $x=0$ or positive even x, $\frac{f(x)}{g(x)}=\frac{3x+2}{2}$

Each figure has half as many tiles.

e. $f^{-1}(x)+g(x)=\frac{x+4}{3}$

The size of the figure is increased by two.

f. $f^{-1}(x)-g(x)=\frac{x-8}{3}$

The size of the figure is decreased by two.

g. $f^{-1}(x)\cdot g(x)=\frac{2x-4}{3}$

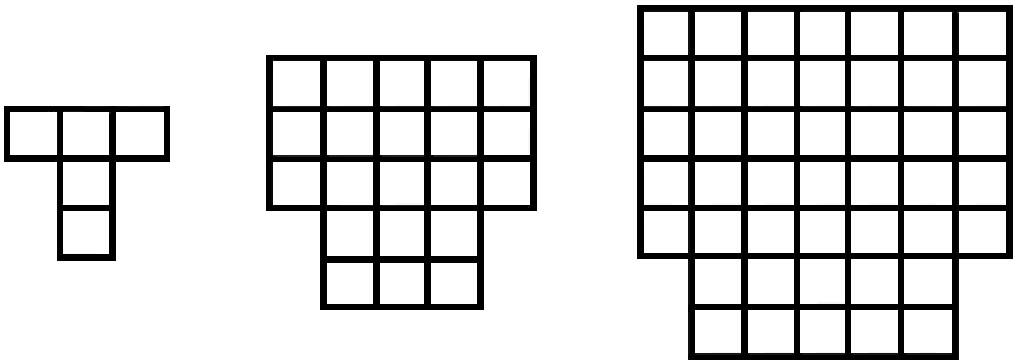
The size of the figure is doubled.

h. For $x\in\{n:n=6k+2 \text{ and } k \text{ is a whole number}\}$, $\frac{f^{-1}(x)}{g(x)}=\frac{x-2}{6}$

The size of the figure is half of $f^{-1}(x)$.

Quantity Relationships and Inverse Functions Application

Use the following figures that are constructed from square tiles to answer the following.



Size 1 Size 2 Size 3

1. Complete the following table of values where x is the size and $f(x)$ is the number of tiles for sizes 1 through 6.

Table 1

x	$f(x)$	Common Differences 1st	2nd	3rd
		****	****	****
			****	****
				****

4. Look at Table 1 to complete the following. [Hint: You may be helpful to express the constant term as a multiple of x.]
   a. Write $f(6)$ as a function of $f(5)$.

   b. Write $f(5)$ as a function of $f(4)$.

c. Write $f(4)$ as a function of $f(3)$.

d. Write $f(3)$ as a function of $f(2)$.

e. Write $f(2)$ as a function of $f(1)$.

f. Find $f(1)$.

j. Notice that you can work backwards using the equations (from parts a - f) to find the value for any $f(x)$ from $x=1$ to $x=6$. Observe the pattern in each equation and write f as a recursive function. Don't forget to include the base case.

5. Find $f(7)$ recursively. Show all work.

6. The next set of questions refers to writing f as an explicit function, where $f(x)$ represents the number of tiles in each figure and x is the size.
   a. Use common 1st, 2nd and 3rd differences from Table 1 to decide the function type. Is f linear, quadratic or cubic? Explain.

   b. Write f as an explicit function by using information from Table 1, information from your work in Problem 5, type of function from Part a, and the relationship of $f(x)$ and x. That is, think about how the number of tiles in each figure is related to the "size" of the figure. Don't forget to restrict the domain.

7. The next set of questions refers to the inverse of f.

a. Use the definition of the inverse of f and complete the following table of values where x is the number of tiles and $f^{-1}(x)$ is the size.

Table 2

x	$f^{-1}(x)$

b. Is the inverse of f a function? Explain.

c. Compare Table 1 and Table 2. What do you notice about the domain and range?

d. Use the definition of an inverse function and the information gained from part c to find the inverse function, f^{-1}. Don't forget to restrict the domain. [Hint: Use Algebra Tiles to Complete the Square in order to find $f^{-1}(x)$. Also, you may have to limit the values of f^{-1} so that it is a function and the values of $f^{-1}(x)$ are meaningful in the context of the problem.]

8. Let $g(x)=4$. Find the values of each of the following. Describe each situation in the context of the problem.
   a. $f(x)+g(x)=$ ____________________

   b. $f(x)-g(x)=$ ____________________

   c. $f(x)\cdot g(x)=$ ____________________

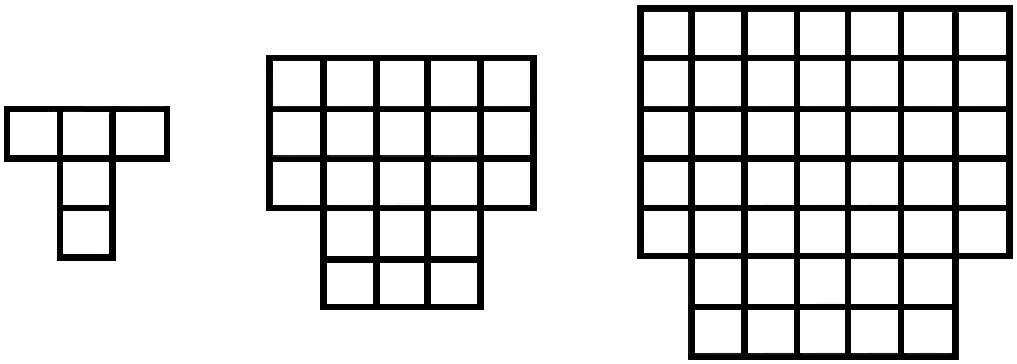
   d. $f^{-1}(x)+g(x)=$ ____________________

e. $f^{-1}(x)-g(x)=$ ______________________

f. $f^{-1}(x)\cdot g(x)=$ ______________________

Quantity Relationships and Inverse Functions Application Key

Use the following figures that are constructed from square tiles to answer the following.



Size 1 Size 2 Size 3

1. Complete the following table of values where x is the size and $f(x)$ is the number of tiles for sizes 1 through 6.

Table 1

x	$f(x)$	Common Differences 1st	2nd	3rd
1	5	****	****	****
2	21	16	****	****
3	45	24	8	****
4	77	32	8	0
5	117	40	8	0
6	165	48	8	0

4. Look at Table 1 to complete the following. [Hint: You may be helpful to express the constant term as a multiple of x.]
   a. Write $f(6)$ as a function of $f(5)$.

   $f(6) = f(5) + 48$

   $f(6) = f(5) + 8x$

   b. Write $f(5)$ as a function of $f(4)$.

   $f(5) = f(4) + 40$

   $f(5) = f(4) + 8x$

c. Write $f(4)$ as a function of $f(3)$.

$f(4) = f(3) + 32$

$f(4) = f(3) + 8x$

d. Write $f(3)$ as a function of $f(2)$.

$f(3) = f(2) + 24$

$f(3) = f(2) + 8x$

e. Write $f(2)$ as a function of $f(1)$.

$f(2) = f(1) + 16$

$f(2) = f(1) + 8x$

f. Find $f(1)$.

$f(1) = 5$

k. Notice that you can work backwards using the equations (from parts a - f) to find the value for any $f(x)$ from $x = 1$ to $x = 6$. Observe the pattern in each equation and write f as a recursive function. Don't forget to include the base case.

$f(x) = f(x-1) + 8x$, $f(1) = 5$ and x is a positive integer

5. Find $f(7)$ recursively. Show all work.

$$
\begin{aligned}
f(7) &= f(6)+8\cdot 7\\
&= \big(f(5)+8\cdot 6\big)+8\cdot 7\\
&= \Big(\big(f(4)+8\cdot 5\big)+8\cdot 6\Big)+8\cdot 7\\
&= \bigg(\Big(\big(f(3)+8\cdot 4\big)+8\cdot 5\Big)+8\cdot 6\bigg)+8\cdot 7\\
&= \Bigg(\bigg(\Big(\big(f(2)+8\cdot 3\big)+8\cdot 4\Big)+8\cdot 5\bigg)+8\cdot 6\Bigg)+8\cdot 7\\
&= \Bigg(\Bigg(\bigg(\Big(\big(f(1)+8\cdot 2\big)+8\cdot 3\Big)+8\cdot 4\bigg)+8\cdot 5\Bigg)+8\cdot 6\Bigg)+8\cdot 7
\end{aligned}
$$

Using $f(x)=f(x-1)+8x$ to work down to the base case.

$$= \Bigg(\Bigg(\bigg(\Big(\big(5+8\cdot 2\big)+8\cdot 3\Big)+8\cdot 4\bigg)+8\cdot 5\Bigg)+8\cdot 6\Bigg)+8\cdot 7$$

Substituting base case, $f(1)=5$

$$
\begin{aligned}
&= 5+8(2+3+4+5+6+7)\\
&= 5+8\big[(2+7)+(3+6)+(4+5)\big]\\
&= 8(3\cdot 9)+5
\end{aligned}
$$

Rearranging to help discover the explicit case.

$$= 221$$

6. The next set of questions refers to writing f as an explicit function, where $f(x)$ represents the number of tiles in each figure and x is the size.
   a. Use common 1st, 2nd and 3rd differences from Table 1 to decide the function type. Is f linear, quadratic or cubic? Explain.
      Since the common 2nd differences is constant, f is quadratic.

   b. Write f as an explicit function by using information from Table 1, information from your work in Problem 5, type of function from Part a, and the relationship of $f(x)$ and x. That is, think about how the number of tiles in each figure is related to the "size" of the figure. Don't forget to restrict the domain.
      $f(x)=8\left(\dfrac{x-1}{2}\right)(x+2)+5$
      $f(x)=4(x-1)(x+2)+5$
      $f(x)=4x^2+4x-3$, where x is a positive integer

7. The next set of questions refers to the inverse of f.

a. Use the definition of the inverse of f and complete the following table of values where x is the number of tiles and $f^{-1}(x)$ is the size.

Table 2

x	$f^{-1}(x)$
5	1
21	2
45	3
77	4
117	5
165	6

b. Is the inverse of f a function? Explain.

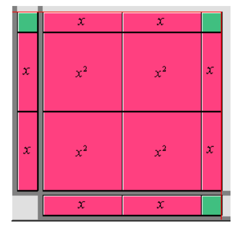
f^{-1} is a function, since each x maps to a unique $f^{-1}(x)$.

c. Compare Table 1 and Table 2. What do you notice about the domain and range?
The domain and range values from Table 1 are swapped in Table 2.

d. Use the definition of an inverse function and the information gained from part c to find the inverse function, f^{-1}. Don't forget to restrict the domain. [Hint: Use Algebra Tiles to Complete the Square in order to find $f^{-1}(x)$. Also, you may have to limit the values of f^{-1} so that it is a function and the values of $f^{-1}(x)$ are meaningful in the context of the problem.]

$$f(x) = 4x^2 + 4x - 3$$
$$x = 4\left[f^{-1}(x)\right]^2 + 4f^{-1}(x) - 3$$
$$x + 3 = 4\left[f^{-1}(x)\right]^2 + 4f^{-1}(x)$$
$$x + 4 = 4\left[f^{-1}(x)\right]^2 + 4f^{-1}(x) + 1$$
$$x + 4 = \left[2\left[f^{-1}(x)\right] + 1\right]^2$$

} Completing the Square with Algebra Tiles



$$\left[2\left[f^{-1}(x)\right] + 1\right]^2 = x + 4$$
$$2\left[f^{-1}(x)\right] + 1 = \pm\sqrt{x+4}$$
$$2\left[f^{-1}(x)\right] = -1 \pm \sqrt{x+4}$$
$$f^{-1}(x) = \frac{-1 \pm \sqrt{x+4}}{2}$$

$f^{-1}(x) = \dfrac{-1 + \sqrt{x+4}}{2}$, where $x \in \{n : n = 4k^2 + 4k - 3 \text{ and } k \text{ is a positive integer}\}$

8. Let $g(x) = 4$. Find the values of each of the following. Describe each situation in the context of the problem.
   a. $f(x) + g(x) = 4x^2 + 4x + 1$
      Each figure has four more tiles. That is, each figure is a perfect square with sides of length $2x + 1$.

   b. $f(x) - g(x) = 4x^2 + 4x - 7$
      Each figure has four less tiles.

   c. $f(x) \cdot g(x) = 16x^2 + 16x - 12$
      Each figure has four times as many tiles.

   d. $f^{-1}(x) + g(x) = \dfrac{7 + \sqrt{x+4}}{2}$
      The size of the figure is increased by four.

e. $f^{-1}(x) - g(x) = \dfrac{-9 + \sqrt{x+4}}{2}$

The size of the figure is decreased by four.

f. $f^{-1}(x) \cdot g(x) = -2 + 2\sqrt{x+4}$

The size of the new figure is four times the size of f^{-1}.