Lesson Plan
Lesson 6: Deepening Understanding of Proof through Exploration of Special Quadrilaterals
Mathematics High School Math II

Unit Name: Unit 5: Similarity, Right Triangle Trigonometry, and Proof

Lesson Plan Number & Title: Lesson 6: Deepening Understanding of Proof through Exploration of Special Quadrilaterals

Grade Level: High School Math II

Lesson Overview: Students will continue to develop an understanding of proof while they investigate how congruent triangles can provide information about the sides, angles, and diagonals of special quadrilaterals. They will also consider relationships in isosceles triangles and perpendicular bisectors. They will be challenged to discover properties of quadrilaterals, isosceles triangles, and perpendicular bisectors and to justify their reasoning using a variety of methods of proof. This lesson is designed for approximately 60 to 90 minutes, but time may vary depending on the background of the students.

Focus/Driving Question: How can congruent triangles provide the means for discovering properties of special quadrilaterals, isosceles triangles and perpendicular bisectors while deepening an understanding of proof?

West Virginia College- and Career-Readiness Standards:

M.2HS.44 Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other and conversely, rectangles are parallelograms with congruent diagonals. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

Manage the Lesson:

Using congruent triangles, students will explore the relationships of the sides, angles, and diagonals of special quadrilaterals and also investigate isosceles triangles and perpendicular bisectors. They will develop conjectures and write convincing proofs in a variety of formats. The primary goal is to increase students' comfort level in multiple ways of writing proofs.

Academic Vocabulary Development:

Vocabulary addressed in this lesson will include:

rectangle

rhombus

square

isosceles triangle

base angles

perpendicular bisectors

Many of these terms are already familiar, but will be expanded to include the various characteristics of each figure. The special quadrilaterals will become part of the quadrilateral foldable developed in the lesson. A variety of opportunities will be provided in the lesson to increase the student's understanding of these terms.

Launch/Introduction:

Students will review properties of a parallelogram as they compile a foldable. Have students use one sheet of notebook paper and fold it lengthwise (hotdog fold), and then cut to create five tabs on each sheet. The directions are described in the attached 6.01 PowerPoint on Properties of a Parallelogram Foldable. The foldable will be expanded to include properties of special quadrilaterals later in the lesson.

Investigate/Explore:

Make and distribute to students a sheet with drawings of rectangles, rhombuses, squares, trapezoids and isosceles trapezoids. Instruct each student to carefully measure segments and angles and make conjectures based on these measures. Another way to explore these quadrilaterals is using the following website: http://teachers.henrico.k12.va.us/math/IGO/ Under Investigating Geometry Online, click on Quadrilaterals: Objectives. Particularly useful are Lessons 6.3, 6.4 and 6.5. The Hands-On Activities allow the student to discover the properties of a rectangle, a rhombus, and a trapezoid using GeoGebra. If time is short, another website that can be utilized allows the students to explore the properties quickly. http://mrskrummel.com/apps/Geometry/ch06_quadrilateral_properties.html

After students have made conjectures, compile a list on the chalkboard or chart paper of the properties that students have discovered. Theorems 12 & 13 from the **APPS MENU** address two of these properties. The other properties may be proven as a homework assignment. Students should be encouraged to utilize a variety of proof formats and to explore via GeoGebra ways to use transformations to verify the results as well. 6.02 Quadrilateral Proofs **(**6.03 Answers to Quadrilateral Proofs**)** incorporates some of the properties and will provide opportunities for deepening an understanding of proof as well as quadrilaterals. The fourth proof may prove challenging to students.

Using 6.04 Powerpoint on Special Quadrilaterals Foldable, students can continue their foldable recording the properties for a rectangle, rhombus, square, trapezoid and an isosceles trapezoid. If time is short, distribute 6.05 Properties of Quadrilaterals.

Distribute to students 6.06 Discoveries with GeoGebra and have students complete Investigation 1. Then prove the result (Perpendicular Bisector of a Segment) Theorem 14 using the APPS MENU for this theorem.

Continue with Investigation 2. Then prove the result (Isosceles Triangle Theorem) Theorem 15 using the **APPS MENU** for this theorem. Follow up with considering (Isosceles Triangles with Mutual Base) Theorem 16 also using the **APPS MENU** for this theorem.

Summarize/Debrief:

Students will demonstrate their understanding with the following performance task. Distribute the 6.07 T-Shirt Investigation **(**6.08 Answers to the T-Shirt Investigation**)**. As much as possible, avoid leading

students to a particular tool or strategy as students will approach this task from a variety of perspectives. Students that are grappling with this task might be directed to consider patty paper, graph paper or GeoGebra to develop their conjectures. It is important that each student develop his/her own conjecture and then seek to prove its validity. The investigation should lead to students discovering that the quadrilateral created is a parallelogram and then proving this conjecture analytically. It would be particularly interesting to revisit this result after they have proven Corollary 32.1: The Mid-segment Theorem so they can explore an alternate proof.

If needed to reinforce the properties of special quadrilaterals, distribute 6.09 Geometrica Fights Back **(**6.10 Answers to Geometrica Fights Back**)** adapted from a lesson on the following website: http://alex.state.al.us/lesson_view.php?id=3009

The ALEX lesson suggests a CD with horror sounds could be played in the background as a student volunteer reads the story to the class. Students should use their foldable to organize their thoughts as they use the properties of quadrilaterals to solve the mystery.

Materials:
6.01 PowerPoint on Properties of a Parallelogram Foldable
6.02 Quadrilateral Proofs
6.03 Answers to Quadrilateral Proofs
6.04 Powerpoint on Special Quadrilaterals Foldable
6.05 Properties of Quadrilaterals
6.06 Discoveries with GeoGebra
6.07 T-Shirt Investigation
6.08 Answers to the T-Shirt Investigation
6.09 Geometrica Fights Back
6.10 Answers to Geometrica Fights Back

Career Connection:

Professions related to the following Career Listings: Architecture and Construction; Arts, A/V Technology and Communications; Manufacturing; Science, Technology, Engineering and Mathematics are particularly pertinent to the concepts learned in this lesson.

Lesson Reflection:

To assess student understanding utilize the website: http://teachers.henrico.k12.va.us/math/IGO/

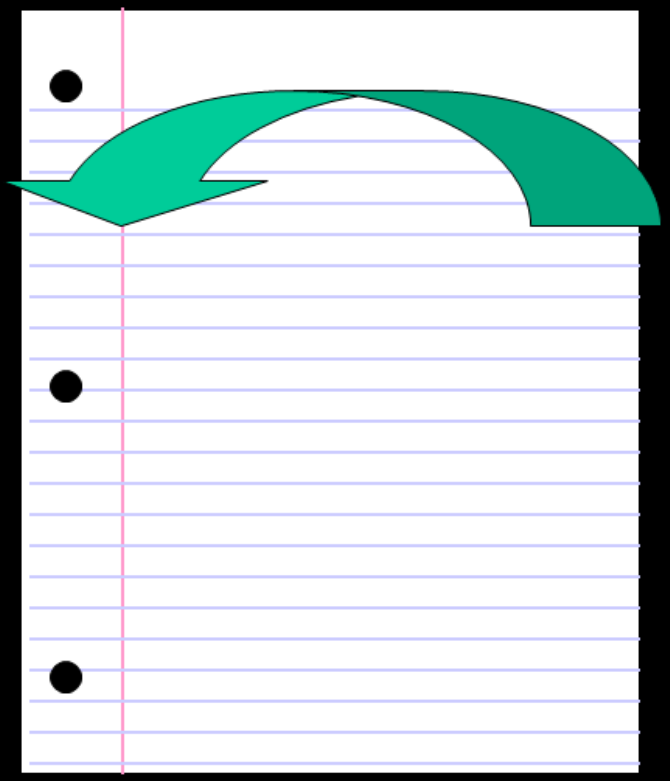
Under Investigating Geometry Online, click on Quadrilaterals: Objectives. The applets problem under Section 6.3 and the problem under Section 6.4 are particularly interesting for students to solve using dynamic construction.

In lesson 1, teachers were provided with a guide to aid them in reflecting upon the lesson as they seek to improve their practice. Certainly, it may not be feasible to formally complete such a reflection after every lesson, but hopefully the questions can generate some ideas for contemplation.

Properties of a Parallelogram Foldable

Foldable

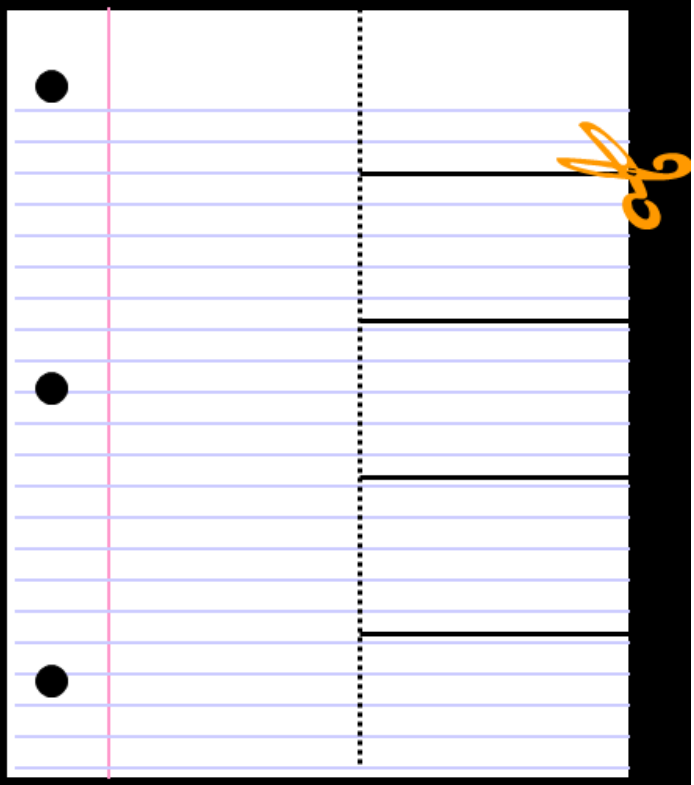
1. Take out a piece of notebook paper and make a hot dog fold over from the right side over to the pink line.



Foldable

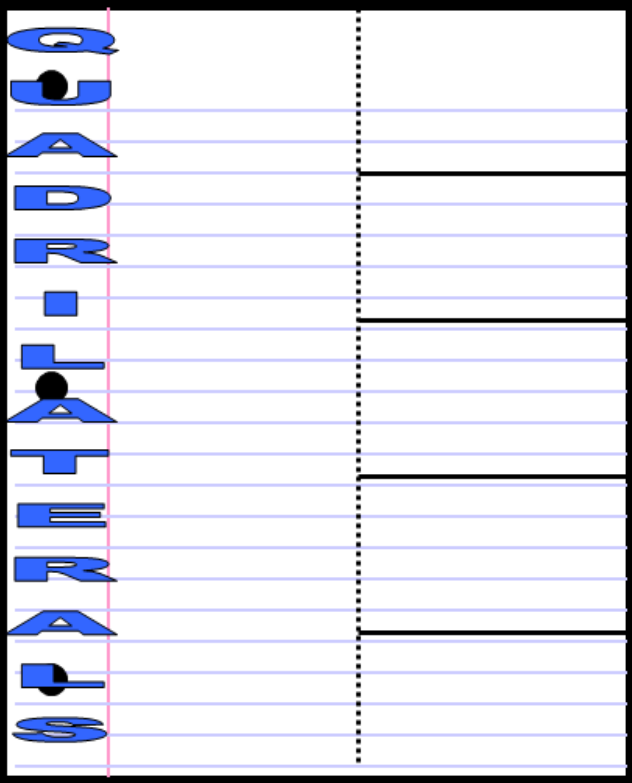
2. Now, divide the right hand section into 5 sections by drawing 4 evenly spaced lines.

3. Use scissors to cut along your drawn line, but ONLY to the crease!



Foldable

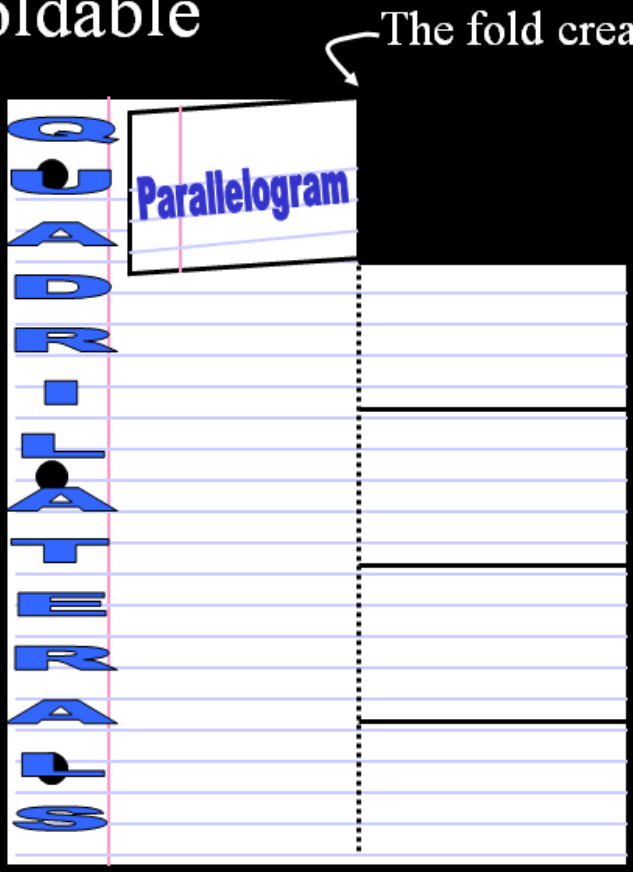
4. Write QUADRILATERALS down the left hand side.



Foldable

5. Fold over the top cut section and write PARALLELOGRAM on the outside.

6. Reopen the fold.

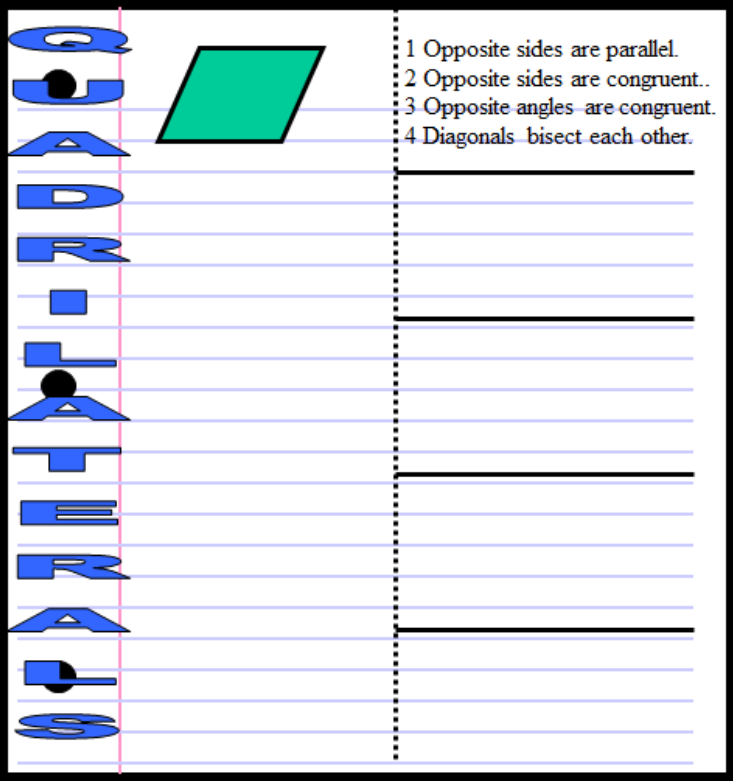


Foldable

7. On the left hand section, draw a parallelogram.

8. On the right hand side, list all of the properties of a parallelogram.

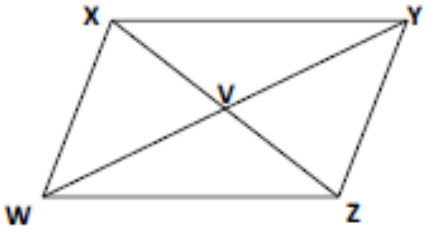
9. Place in your notebook and save for later in the lesson.



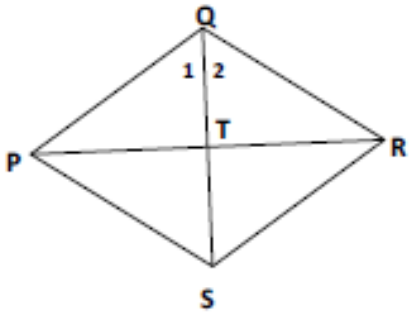
Quadrilateral Proofs

Write a proof for each of the following arguing from the given hypothesis to a given conclusion.

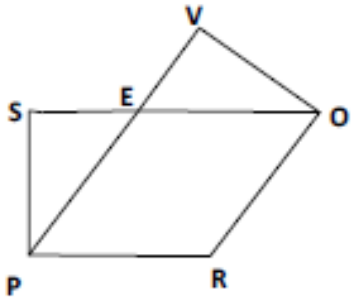
1. Given quadrilateral XYZW with diagonals $\overline{XZ}$ and $\overline{WY}$, which bisect each other at V, prove that $\triangle WXV \cong \triangle YZV$.



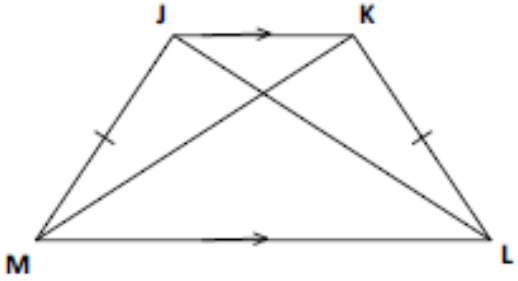
2. Given rhombus PQRS with diagonals $\overline{SQ}$ and $\overline{RP}$ which intersect at T, prove that $\angle 1 \cong \angle 2$.
Is this sufficient to verify that opposite angles of a rhombus are bisected?



3. Given that PROV is a rhombus and $\angle SPR \cong \angle VOR$, prove that $\overline{SE} \cong \overline{EV}$.



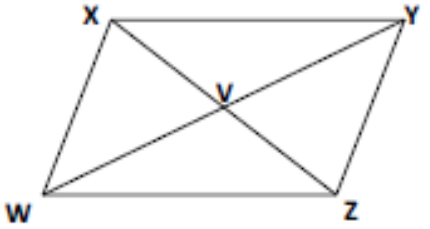
4. Given that JKLM is an isosceles trapezoid, prove that $\overline{JL} \cong \overline{KM}$.



Answers to Quadrilateral Proofs

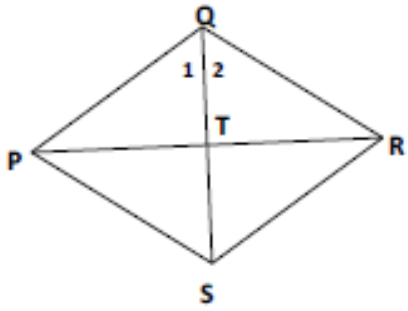
Write a proof for each of the following arguing from the given hypothesis to a given conclusion.

1. Given quadrilateral XYZW with diagonals $\overline{XZ}$ and $\overline{WY}$, which bisect each other at V, prove that $\triangle WXV \cong \triangle YZV$.



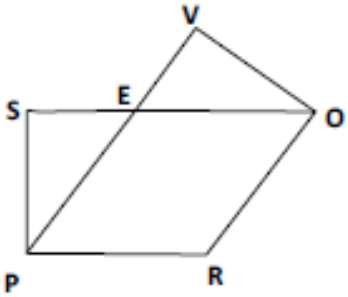
Pf: Since the diagonals bisect each other, $\overline{WV} \cong \overline{YV}$ and $\overline{XV} \cong \overline{ZV}$. Also $\angle XVW \cong \angle ZVY$ as they are vertical angles so $\triangle WXV \cong \triangle YZV$ by SAS.

2. Given rhombus PQRS with diagonals $\overline{SQ}$ and $\overline{RP}$ which intersect at T, prove that $\angle 1 \cong \angle 2$.
   Is this sufficient to verify that opposite angles of a rhombus are bisected?



Pf: Since PQRS is a rhombus, $\overline{PQ} \cong \overline{RQ}$. Since a rhombus is also a parallelogram, $\overline{PT} \cong \overline{RT}$. With $\overline{QT} \cong \overline{QT}$, $\triangle PQT \cong \triangle RQT$ by SSS. Then $\angle 1 \cong \angle 2$ as they are corresponding parts of the congruent triangles. Similarly, the other pairs of angles are bisected by the diagonals.

3. Given that PROV is a rhombus and $\angle SPR \cong \angle VOR$, prove that $\overline{SE} \cong \overline{EV}$.



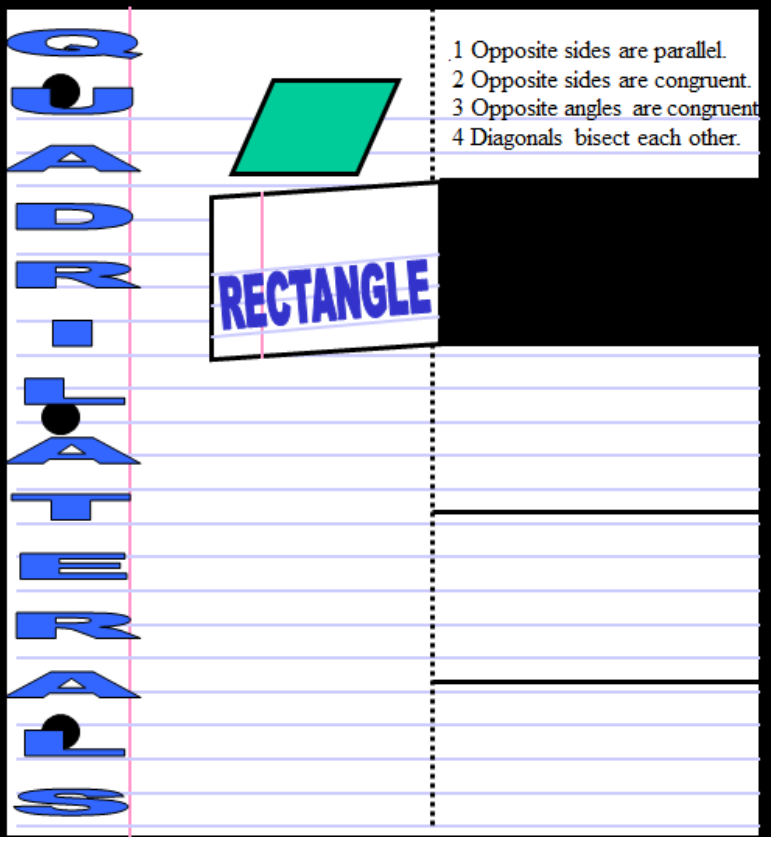
Pf: Since PROV is a rhombus $\overline{PE} \cong \overline{OE}$. Since a rhombus is also a parallelogram, $\angle EPR \cong \angle ROE$. Since $\angle SPR \cong \angle VOR$, by subtraction

Quadrilateral Foldable

Foldable

* Fold over the second cut section and write RECTANGLE on the outside.

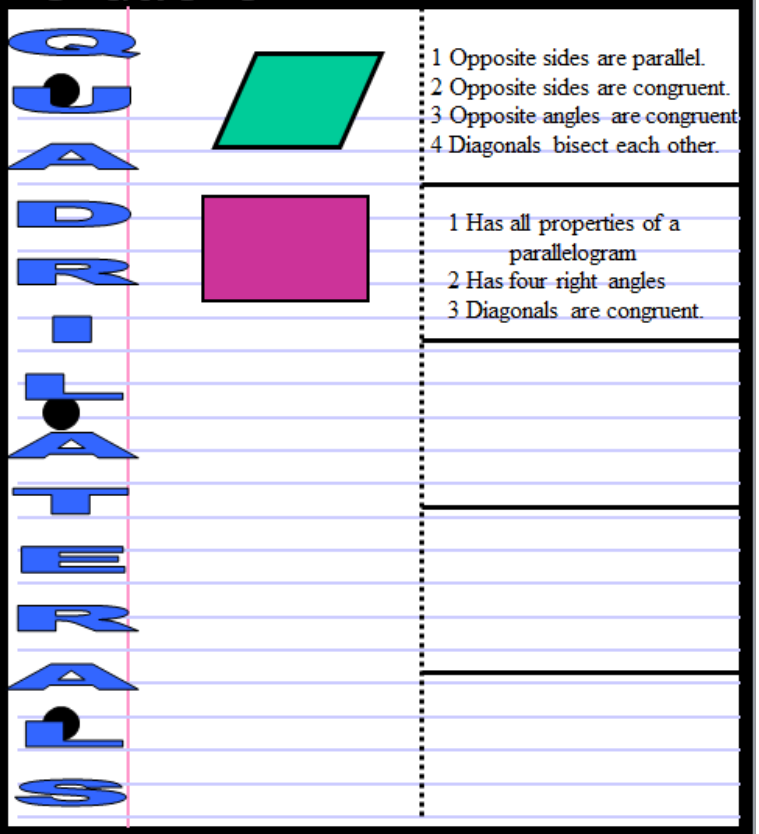
* Reopen the fold.



Foldable

*** On the left hand section, draw a rectangle.**

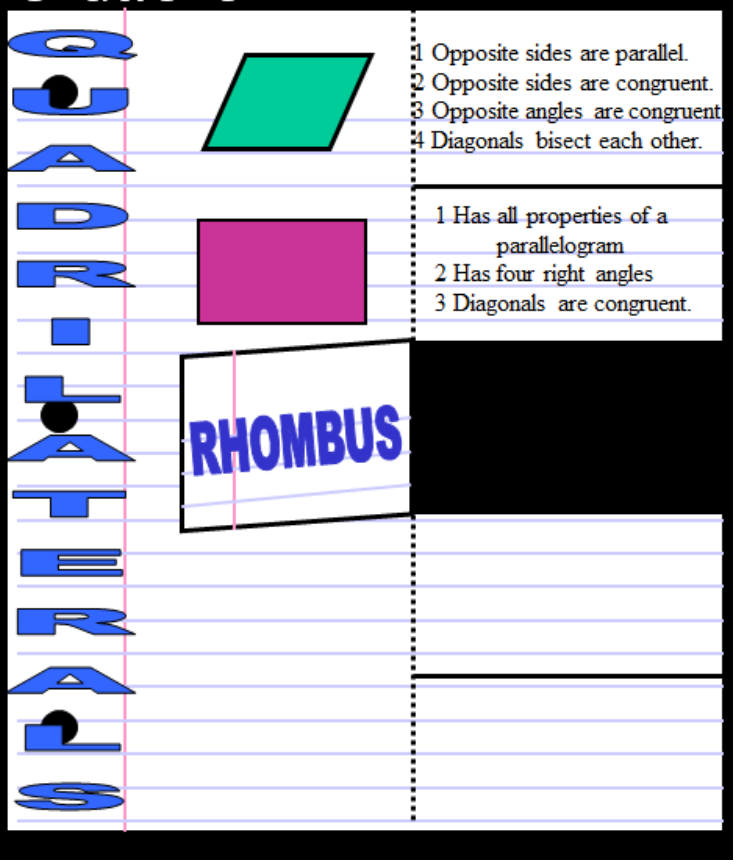
*** On the right hand side, list all of the properties of a rectangle.**



Foldable

* Fold over the third cut section and write RHOMBUS on the outside.

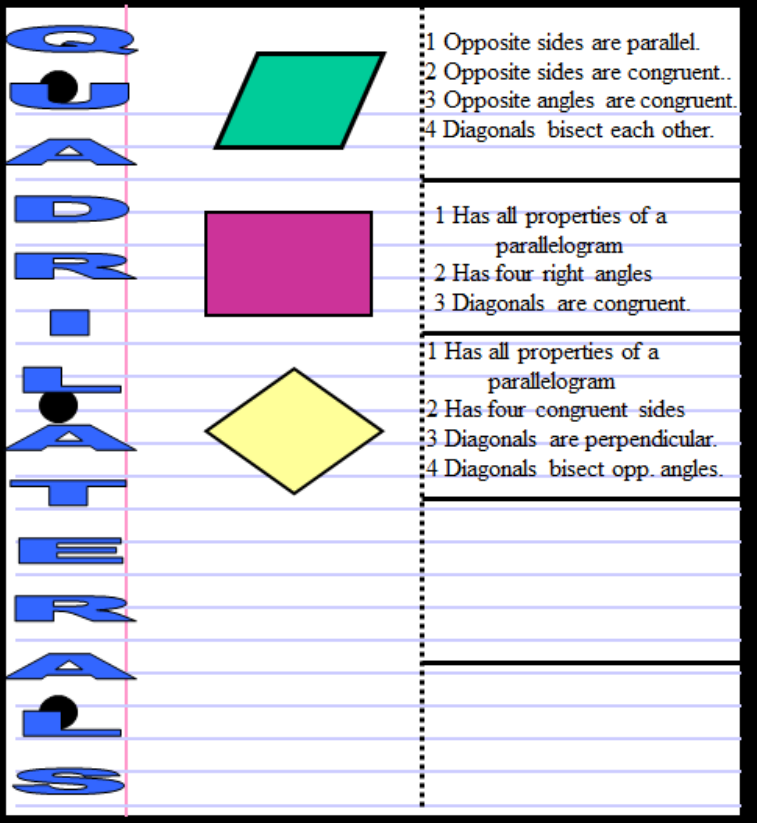
* Reopen the fold.



Foldable

*** On the left hand section, draw a rhombus.**

*** On the right hand side, list all of the properties of a rhombus.**



Foldable

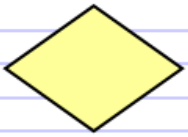
* On the left hand section, draw a square.

* On the right hand side, list all of the properties of a square.

QUADRILATERALS

1 Opposite sides are parallel.
2 Opposite sides are congruent..
3 Opposite angles are congruent.
4 Diagonals bisect each other.

1 Has all properties of a Parallelogram
2 Has four right angles
3 Diagonals are congruent.



1 Has all properties of a Parallelogram
2 Has 4 Congruent sides
3 Diagonals are perpendicular.
4 Diagonals bisect opp. angles.

1 Has all the properties of a parallelogram, a rectangle, and a rhombus
2 Four congruent sides and four right angles

Foldable

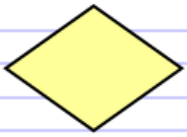
*** On the left hand section, draw a square.**

*** On the right hand side, list all of the properties of a square.**

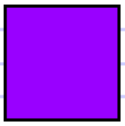
QUADRILATERALS

1 Opposite sides are parallel.
2 Opposite sides are congruent..
3 Opposite angles are congruent.
4 Diagonals bisect each other.

1 Has all properties of a Parallelogram
2 Has four right angles
3 Diagonals are congruent.



1 Has all properties of a Parallelogram
2 Has 4 Congruent sides
3 Diagonals are perpendicular.
4 Diagonals bisect opp. angles.



1 Has all the properties of a parallelogram, a rectangle, and a rhombus
2 Four congruent sides and four right angles

Foldable

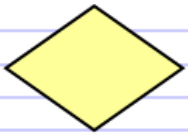
*** Fold over the cut section and write TRAPEZOID on the outside.**

*** Reopen the fold.**

QUADRILATERALS

1 Opposite sides are parallel.
2 Opposite sides are congruent..
3 Opposite angles are congruent.
4 Diagonals bisect each other.

1 Has all properties of a Parallelogram
2 Has four right angles
3 Diagonals are congruent.



1 Has all properties of a Parallelogram
2 Has 4 Congruent sides
3 Diagonals are perpendicular.
4 Diagonals bisect opp. angles.

1 Has all the properties of a parallelogram, a rectangle, and a rhombus
2 Four congruent sides and four right angles

TRAPEZOID

Foldable

* On the left hand section, draw a trapezoid.

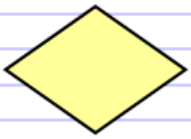
* On the right hand side, list all of the properties of a trapezoid.

* Place in your notebook and save for future reference.

QUADRILATERALS

1 Opposite sides are parallel.
2 Opposite sides are congruent..
3 Opposite angles are congruent.
4 Diagonals bisect each other.

1 Has all properties of a Parallelogram
2 Has four right angles
3 Diagonals are congruent.

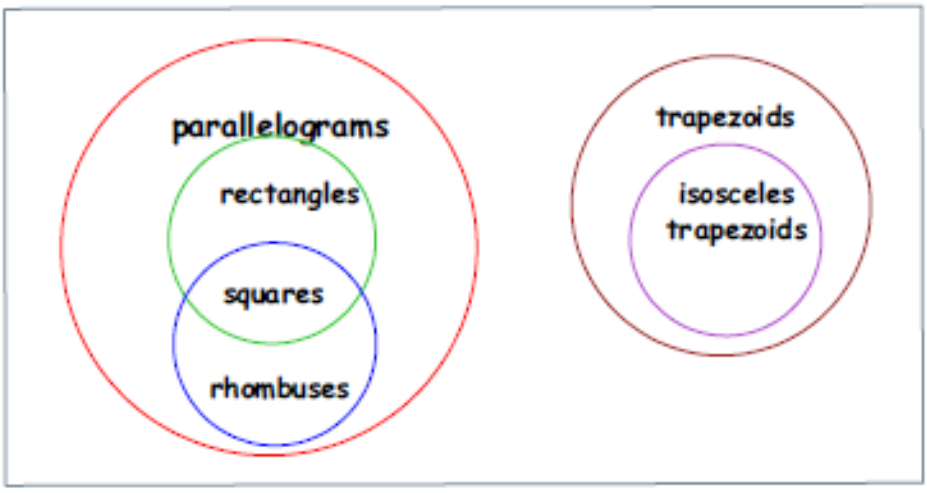


1 Has all properties of a Parallelogram
2 Has 4 Congruent sides
3 Diagonals are perpendicular.
4 Diagonals bisect opp. angles.

1 Has all the properties of a parallelogram, a rectangle, and a rhombus
2 Four congruent sides and four right angles

Exactly two sides parallel and median ½ sum of bases.
Isosceles trapezoids have
1. Congruent legs
2. Congruent diagonals

Properties of Quadrilaterals



Parallelogram
1—Opposite sides are ∥.
2—Opposite sides are ≅.
3—Opposite ∠s are ≅.
4—Diagonals bisect each other.

Trapezoid
1—Exactly one pair of ∥ sides.
2—Median is ½ the sum of bases.

Isosceles Trapezoid
1—Legs are ≅.
2—Base ∠s are ≅.
3—Diagonals are ≅.

Rectangle
All the properties of parallelograms plus:
1—All ∠s are right ∠s.
2—Diagonals are ≅.

Rhombus
All the properties of parallelograms plus:
1—All sides are ≅.
2—Diagonals are ⊥.
3—Diagonals bisect opposite ∠s.

Square
All the properties of parallelograms, rectangles, and rhombuses

Discoveries with GeoGebra

Investigation 1:

1. Open GeoGebra. You may need to open a new window to display the coordinate axes.

2. Under Construction Tools, select the Segment between Two Points 

and click two points to create a line segment.

3. Next under Construction Tools, select the Perpendicular Bisector 

and click on each endpoint to create the perpendicular bisector of the segment.

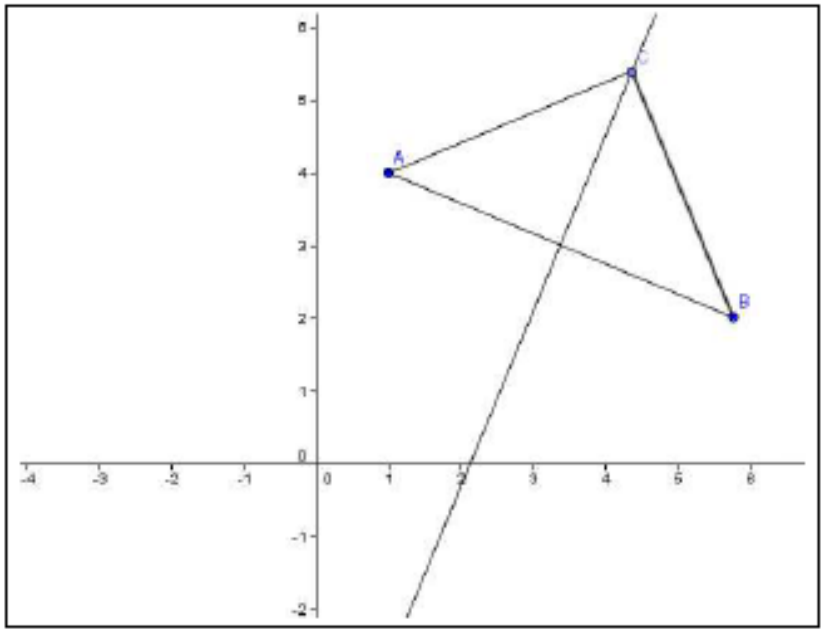
4. Under Construction Tools, select the New Point 

and then click on an arbitrary point on the perpendicular bisector.

5. Again under Construction Tools, select the Segment between Two Points 

and click to create the segments from this point to each of the original segment's endpoints.

6. The picture on the Graphics Screen should resemble this:



7. From the Algebra Screen, record the lengths of the two segments formed.

AC = _____ BC = _____

8. Select another arbitrary point, and again create segments to the endpoints of the original segment.

AD = _____ AD = _____

9. Make a conjecture about any point on the perpendicular bisector of a segment.

Investigation 2:

10. 1. Open GeoGebra. You will need to open a new window to display the coordinate axes. Also, under View, click on Algebra to see the grid.

11. Under Construction Tools, select the New Point and click at A(1, 1), B(5, 1), and C(3, 5) to create an isosceles triangle.

12. Next, click on the Angle Icon  and click on B, A, and C to determine the measure of angle A. Then click on C, B, and A to determine the measure of angle B.

13. Record the measure of angles A and B.

m∠A = _____ m∠B = _____

14. Create additional isosceles triangles and measure the base angles.

15. Make a conjecture about the base angles of any isosceles triangle.

T-Shirt Investigation

A graphics art class has been assigned a t-shirt design project. Each student is to create a design by drawing any quadrilateral, connecting the midpoints of the sides to form another quadrilateral, and coloring the regions.

What would be the most descriptive term for the inner quadrilateral?

Would the quadrilateral necessarily be a trapezoid, a parallelogram, a rhombus, a rectangle, a square?

Prove your conjecture formally.

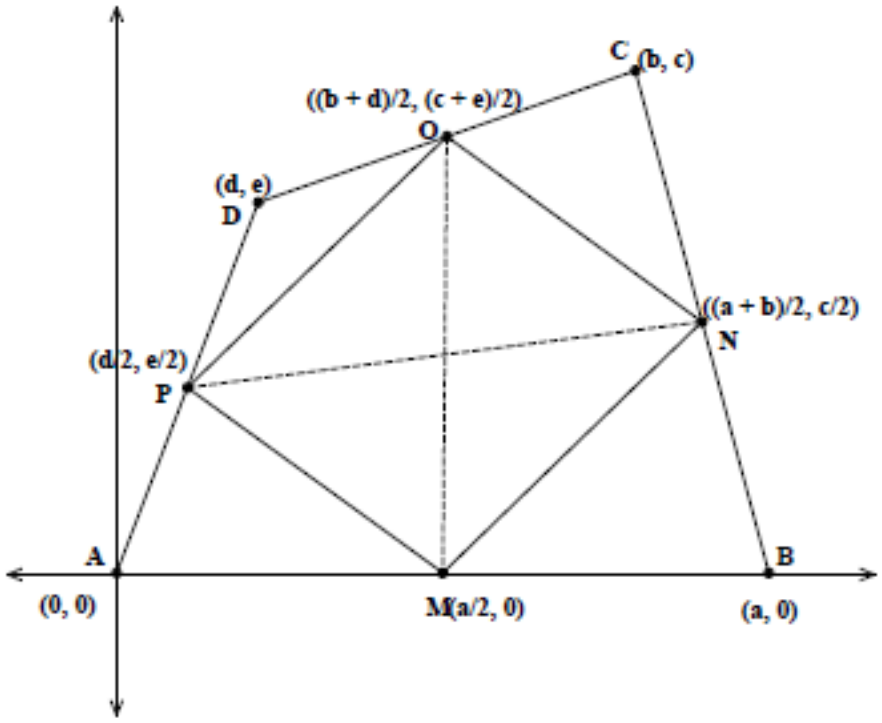
T-Shirt Investigation Answer

A graphics art class has been assigned a t-shirt design project. Each student is to create a design by drawing any quadrilateral, connecting the midpoints of the sides to form another quadrilateral, and coloring the regions.

What would be the most descriptive term for the inner quadrilateral?

Would the quadrilateral necessarily be a trapezoid, a parallelogram, a rhombus, a rectangle, a square?

Prove your conjecture formally.



Proof: Draw a quadrilateral on the coordinate plane. Label the vertices and the midpoints of each of the sides. Quadrilateral MNOP is formed by connecting the midpoints of the sides of the original quadrilateral ABCD. Since the midpoint of $\overline{MO}$ is $(\frac{a+b+d}{4}, \frac{c+e}{4})$ and the midpoint of $\overline{PN}$ is also $(\frac{a+b+d}{4}, \frac{c+e}{4})$, the diagonals bisect each other which implies that MNOP is a parallelogram.

Geometrica Fights Back

Mystery of the Guilty Quadrilateral

Once upon a time and just a few days ago in a far, far away land known as Geometrica an unspeakable crime occurred. On a dark and dreary night as the Circular family lay sleeping in their soft, round beds and dreaming of their favorite dessert, (π, of course), a criminal circled the bed of their youngest and kidnapped her. Their neighbor, Mrs. Equi Angular said that she and her husband, Mr. Tri Angular, heard an awful scream that surely was audible for at least a mile radius. The Angulars sprang from their bed to see what was the matter, and saw a strange four-sided figure leaping from the Circular's upstairs window. Well, the Angulars gave a description of the terrible beast and so did many other Geometrica residents. There was never a ransom note received, but the Circulars truly believe that there is still hope to apprehend this criminal and retrieve their little circle. Therefore, Detective Pentagonal of Geometrica's Most Wanted has asked for your assistance in solving this crime. Below you will find descriptions that tipsters have given the authorities. Your job is to list the suspects from your line-up of twelve that meet each set of criteria.

1. Four-sided figure and convex
   Suspects:_____
2. Four-sided figure with two pairs of opposite sides parallel
   Suspects:_____
3. Four-sided figure with four right angles
   Suspects:_____
4. Four-sided figure with all sides congruent
   Suspects:_____
5. Four-sided figure whose diagonals bisect each other
   Suspects:_____
6. Four-sided figure whose opposite angles are congruent
   Suspects:_____
7. Four-sided figure with only one set of parallel sides
   Suspects:_____
8. Four-sided figure whose consecutive angles are supplementary
   Suspects:_____
9. Four-sided figure whose diagonals are congruent
   Suspects:_____
10. Four-sided figure whose diagonals are perpendicular
    Suspects:_____
11. Four-sided figure with all sides congruent and four right angles
    Suspects:_____

12. Four-sided figure with all sides congruent and perpendicular diagonals
Suspects:_____

13. Four-sided figure with all sides congruent and congruent diagonals
Suspects:_____

14. Four-sided figure whose opposite sides are congruent
Suspects:_____

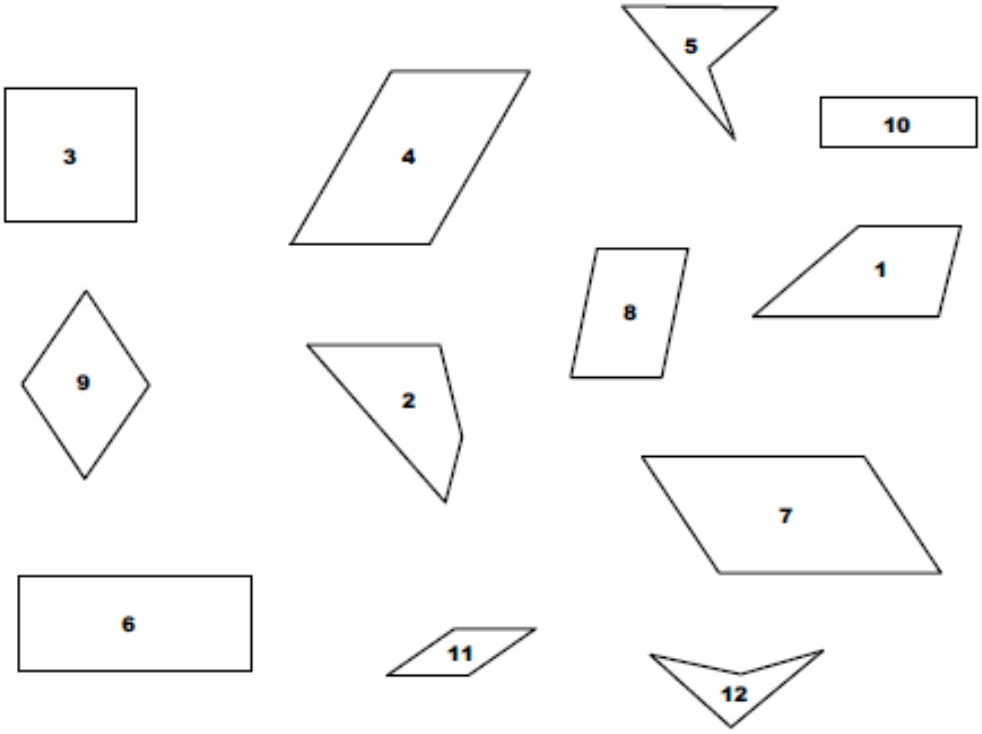
15. Four-sided figure and non-convex
Suspects:_____

16. Four-sided figure with no parallel sides
Suspects:_____

17. Four-sided figure with no congruent sides
Suspects:_____

18. Suspect might be a parallelogram.
Suspects:_____

Who kidnapped the Circular's child? The guilty Quad is the one whose number appears the most in the above list.
Suspect number _____ is the criminal.



Answers to Geometrica Fights Back

1. 1, 2, 3, 4, 6, 7, 8, 9, 10, 11
2. 3, 4, 6, 7, 8, 9, 10, 11
3. 3, 6, 10
4. 3, 9, 11
5. 3, 4, 6, 7, 8, 9, 10, 11
6. 3, 4, 6, 7, 8, 9, 10, 11
7. 1
8. 3, 4, 6, 7, 8, 9, 10, 11
9. 3, 6, 10
10. 3, 9, 11
11. 3
12. 3, 9, 11
13. 3
14. 3, 4, 6, 7, 8, 9, 10, 11
15. 5, 12
16. 2, 5, 12
17. 2, 5
18. 3, 4, 6, 7, 8, 9, 10, 11

The kidnapper is number 3, the square.