

Lesson Plan
Lesson 8: Foundations of Similarity
Mathematics High School Math II

Unit Name: Unit 5: Similarity, Right Triangle Trigonometry, and Proof

Lesson Plan Number & Title: Lesson 8: Foundations of Similarity

Grade Level: High School Math II

Lesson Overview: Students are introduced to the mathematical understandings implied in the definition of similarity. They will develop the ability to explain the meaning of similarity for triangles in terms of transformations. Students will experimentally apply rigid transformations and dilations to determine if given triangles are similar. They will conjecture by measuring or using dynamic geometry software that there exists a series of rigid transformations and a dilation about the origin that maps one triangle to another if and only if all three pairs of corresponding angles are equal and all three pairs of corresponding sides are in proportion.

Students will develop conjectures about the properties of similar triangles that will serve as the basis for the theorems developed throughout the unit. Throughout this unit, two geometric figures are similar if there exists a sequence of rigid transformations followed by a dilation that maps one figure onto the other. The lesson is designed for a 45-minute class period.

Focus/Driving Question: What properties appear to be preserved by similarity transformations?

West Virginia College- and Career-Readiness Standards:

M.2HS.40 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

Manage the Lesson:

Students broaden their understanding of dilations to include the concept of similarity. Students also verify experimentally the properties of dilations given by a center and a scale factor. They apply rigid transformations and dilations to determine if given triangles are similar.

Note:

Completed student work from some or all of following investigations from Lesson 07 can be used to develop the concept of similarity: *7.07 Dilations – Segments A*, *7.08 Dilations – Segments B*, *7.09 Dilations – Segments AB*, *7.10 Dilations – Segments C*, *7.11 Dilations – Segments D*, *7.12 Dilations – Segments CD*, *7.13 Dilation Constructions A*, *7.14 Dilation Constructions B*, *7.15 Center of Dilation 1*, and *7.16 Center of Dilation 2*.

Academic Vocabulary Development:

In a variety of investigations and explorations, students strengthen their understanding of the vocabulary related to similar figures, and similar triangles in particular. Students develop the ability to use appropriate vocabulary in describing their understanding of similarity.

Launch/Introduction:

Display [8.01 Optical Illusions 2](#). As the first slide is presented, students are asked if the lines are straight and parallel. While the lines do appear to bow, the question itself may lead students to state that the lines are straight and parallel. Encourage students to verify this relationship by placing a straight edge, such as a meter stick, along the lines in the image. Introduce the idea from Lesson 07 that things may not always be as they appear.

As the second slide is presented, challenge students to read the COLOR of the words on the slide rather than the words themselves, as fast as they can. In the third slide, the two circular images are presented. One image appears to rotate clockwise while the second image appears to rotate counterclockwise. It may be necessary to allow a student to touch the one of the images on the slide and a second student to touch the second image on the slide to help convince the class that the circles are not rotating.

In a class discussion, use this opportunity to continue to stress that one cannot rely only on appearances. Highlight the need to create and then verify conjectures. As the unit progresses, both the idea of “proof” and the ability to prove will be developed.

Optical Illusions introduce several of the introductory lessons for this unit to highlight the need to verify conjectures rather than rely on initial appearances. Additional optical illusions can be found at sites provided in the Materials section of this template.

Investigate/Explore:

Open the [APPS MENU](#) and select Postulate 3 to show the class. Choose Dilating Figures and then Triangles. As a dilation point is chosen, a triangle and its image are displayed using the scale factor of 1.5. Ask students to examine the corresponding angles in the image and preimage. What appears to be the relationship of the corresponding angles? Do they also appear to be dilated by a factor of 1.5? i.e., if an angle in the preimage appears to be a 60 degree angle, does the angle in the image appear to be $1.5 * 60$, or 90 degrees? If not, what relationship appears? Have students explore other scale factors (both greater than 1 and less than 1) to eventually lead students to determine that the corresponding angles are congruent. Since Postulate 3 ensures that the lines are parallel, then using Theorem 3, the corresponding angles are congruent.

Strengthen this understanding by moving the dilation point so that one vertex of the image lies on top of the corresponding vertex of the preimage. Repeat this process for each of the vertices of the image.

In Lesson 7 student groups completed either *7.07 Dilations – Segments A* or *7.08 Dilations – Segments B* in which they explored the dilation of a triangle, or *7.10 Dilations – Segments C* or *7.11 Dilations – Segments D* in which they explored the dilation of a pentagon. Students will now investigate the relationship of the measure of the angles under dilation. Pair each student who explored the pentagon with a student who explored the triangle. Ask student groups to measure the angles of the image and preimage of their triangle and pentagon. In a classroom discussion, students should again conclude that under dilation the corresponding angles of an image and preimage are congruent.

Ask student groups to again reexamine their conclusions in either *7.09 Dilations – Segments AB* or *7.12 Dilations – Segments CD*. Note that under dilation, the lengths of the sides of segments of the image and preimage are proportional and the scale factor is equal to the constant of proportionality.

In [8.02 GeoGebra Investigation 1](#), students are guided to create an image and preimage under a dilation. Students are then guided to create a sequence of rigid transformations followed by a dilation that maps one figure onto the other. Students are then asked to create a triangle and its image and then create a sequence of rigid transformations followed by a dilation that will map one triangle onto the other. Finally, students repeat the process with the polygon of their choice. The intent of this GeoGebra Investigation I is to reinforce the definition of similar figures, with special emphasis on triangles.

In [8.03 Similar or Not Similar](#), students now use the definition of similarity in terms of similarity transformations to decide if two given polygons are similar. Distribute [8.03 Similar or Not Similar](#). Ask students to use patty paper and/or rulers to determine if the provided figures are similar or not. (The hearts and triangles are similar; the quadrilaterals and hexagons are not similar.)

Summarize/Debrief:

As a result of the investigations in this lesson, students will compile a list of the conjectures about the properties of similar triangles. Proving or disproving these conjectures will provide the impetus for the theorems developed in the next lesson.

Materials:

[8.01 Optical Illusions 2](#)

[8.02 GeoGebra Investigation 1](#)

[8.03 Similar or Not Similar](#)

Optical Illusions – Free Picture Illusions <http://www.123opticalillusions.com/>

Optical Illusions Pictures - <http://brainden.com/optical-illusions.htm>

Images of Optical Illusions -

<http://www.bing.com/images/search?q=optical+illusion+pictures&gpvt=optical+illusion+pictures&FORM=I GRE>

Career Connection:

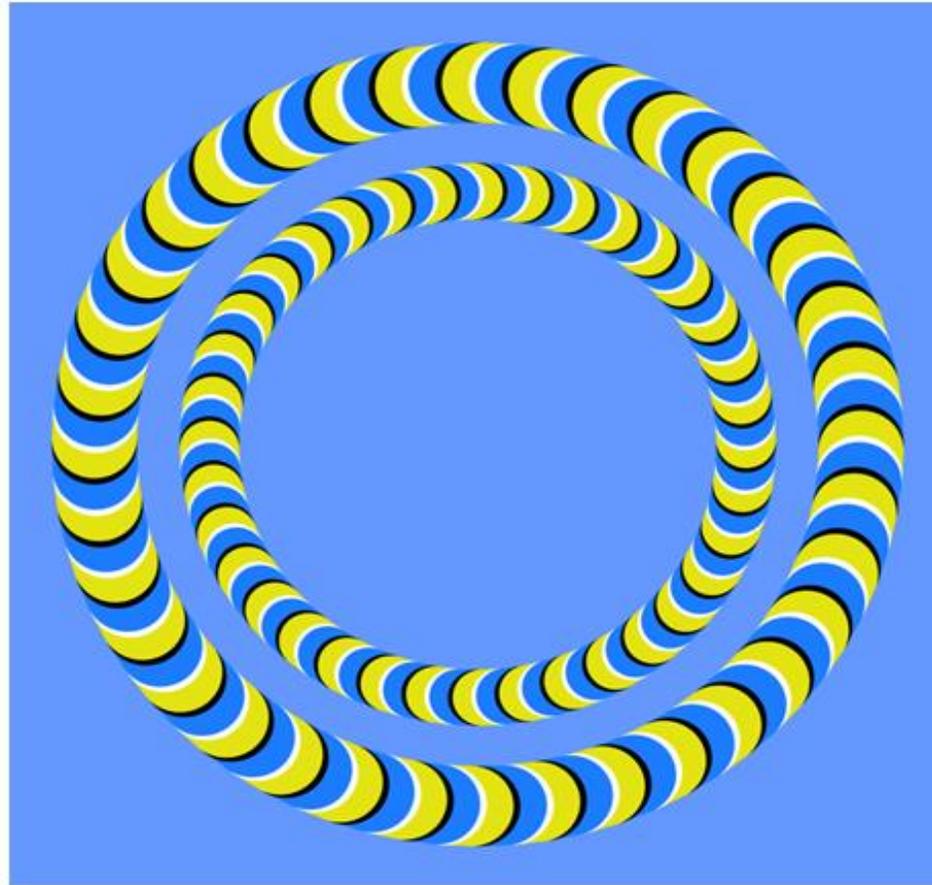
An understanding of similarity will prove to be a valuable asset in multiple career pathways. Any profession in which realistic models are an essential tool requires an understanding of dilations. Most construction-based careers, such as architects, contractors, and electricians, require the ability to create or interpret and understand scale models. Advances in aviation and aerospace are based on research and testing that is generally carried out with scale models. These occupations are related to the following Career Listings: Architecture and Construction; Arts, A/V Technology and Communications; Manufacturing; Science, Technology, Engineering and Mathematics.

Lesson Reflection:

The activities throughout this lesson have been designed to develop an intuitive understanding of similarity that will culminate in the proofs of the next lesson.

In lesson 1, teachers were provided with a guide to aid them in reflecting upon the lesson as they seek to improve their practice. Certainly, it may not be feasible to formally complete such a reflection after every lesson, but hopefully the questions can generate some ideas for contemplation.

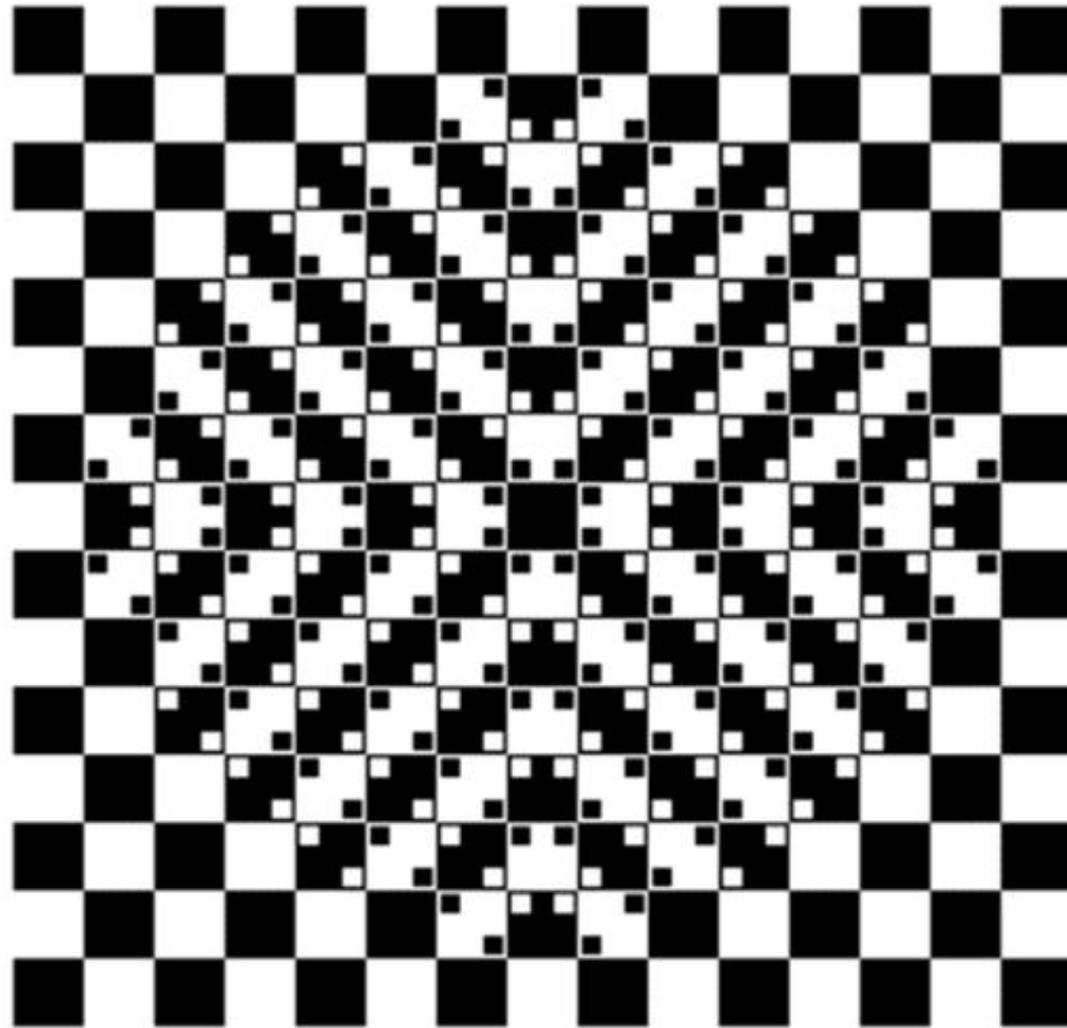
Are these circles rotating?



State the colors of the following words as fast as you can.

RED
ORANGE
BLUE
PURPLE
GREEN

Are these straight and parallel?

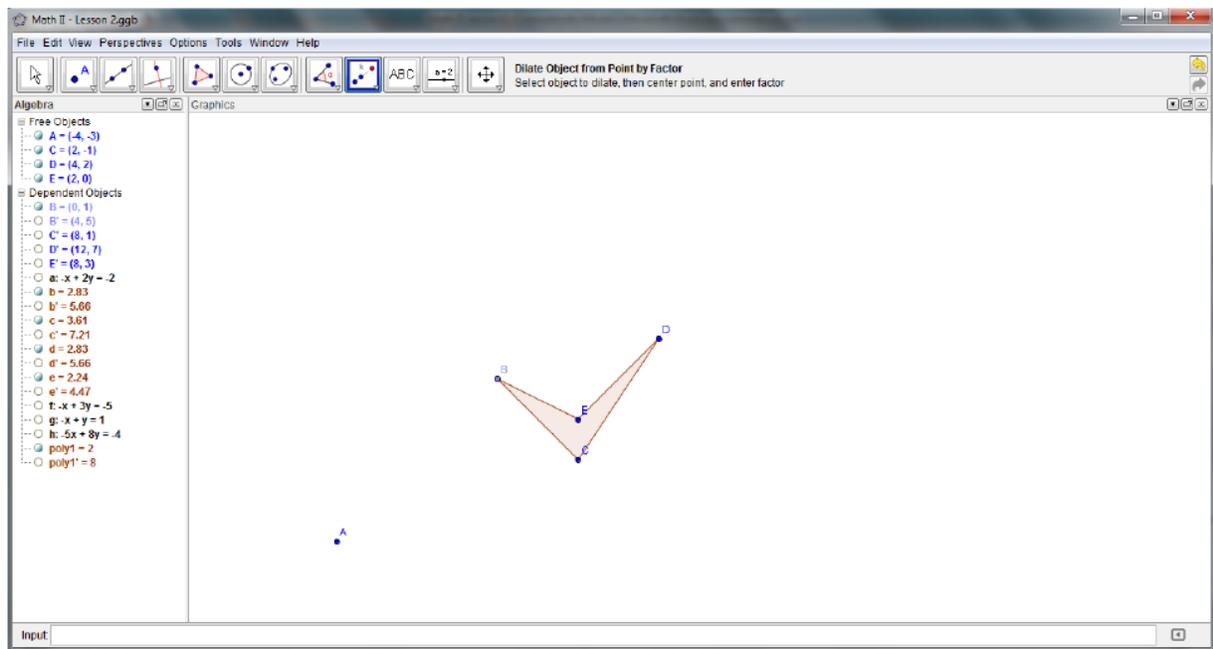


GeoGebra Investigation 1: Dilations

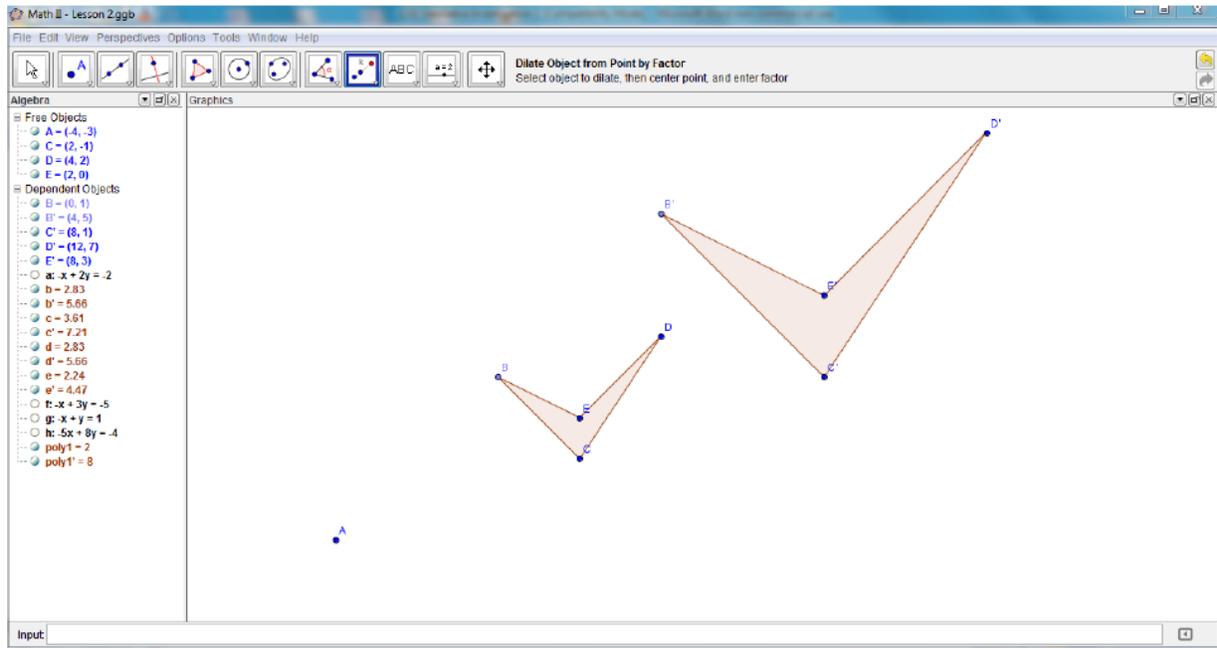
Create a sequence of rigid transformations followed by a dilation that maps one figure onto another.

Directed Investigation

1. Choose the 2nd option button in the toolbar and select "New Point" to create point A.
2. Locate the Input bar. (It will be found either directly above or directly below the Algebra and Graphics screens.) Enter (-4,3). This will be the dilation point.
3. Repeat the process to enter points (1,0) and (2, -1) and (4, 2) and (2, 0). As these points are entered, point (1,0) will be named B, point (2, -1) will be named C, (4, 2) will be named D, and (2, 0) will be named E.
4. Choose the 5th option button in the toolbar and select "Polygon." Select points B, C, D, and E in that order to create quadrilateral BCDE.

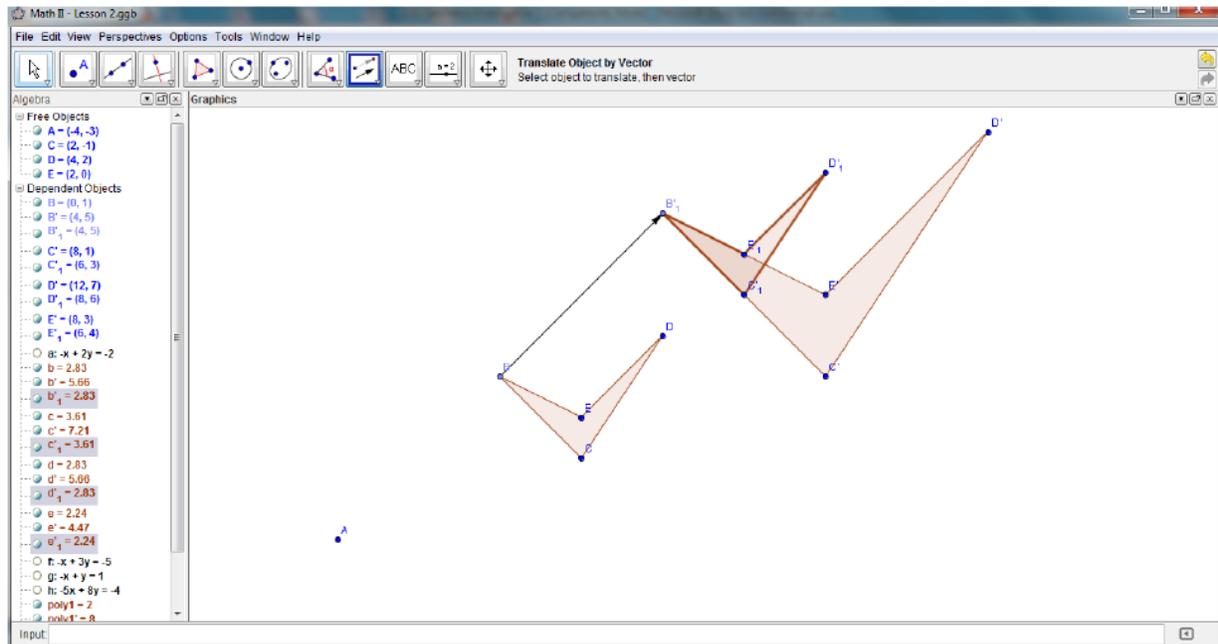


5. Choose the 9nd option button in the toolbar and select "Dilate Object from Point by Factor." Select quadrilateral BCDE, dilation point A, and scale factor 2. This will create the image B'C'D'E'.



6. Now create a sequence of rigid transformations followed by a dilation that maps the preimage BCDE onto the image B'C'D'E'. Choose the 9nd option button in the toolbar and select "Translate Object by Vector." Select quadrilateral BCDE and create a vector that maps B onto B'. This will translate quadrilateral BCDE onto quadrilateral B'C'D'E' with SB lying atop (or mapped to) SB'.

7. Choose the 9th option button in the toolbar and select "Dilate Object from Point by Factor." Select quadrilateral BCDE, dilation point B', and scale factor 2. This will dilate BCDE and now the preimage lies atop (or has been mapped to) the image B'C'D'E'.



Independent Investigation

8. Using a process similar to above, create dilation point A and triangle BCD.
9. Dilate triangle BCD with a scale factor of _____ to create triangle B'C'D'. (Record the scale factor here for use later in the investigation.)
10. Create a sequence of rigid transformations followed by a dilation that will map triangle BCD onto triangle B'C'D'.
11. Repeat the process for a polygon of your choice.

Similar or Not Similar

Two figures are similar if there exists a sequence of rigid transformations followed by a dilation that maps one figure onto the other.

Determine if the following figures are similar. Use patty paper and rulers as needed.

