Unit Plan for Math II Unit 5

Unit Plan Title: Unit 5 Similarity, Right Triangle Trigonometry, and Proof

Grade Level: High School Math II

Unit Overview:

Throughout the unit, students will refine their reasoning skills as they justify their conjectures with convincing arguments and finally construct rigorous proofs using a variety of methods. The unit will encompass the postulates and theorems presented in the **APPS MENU** with a major focus on nonrigid transformations. Conjectures formed through investigations culminate in formally proving results about similarity. Applications of similarity, Pythagorean Theorem, and the trigonometric ratios expand students' understanding.

Prior to teaching this unit, it is imperative to explore the **APPS MENU** and download the APPS GUIDE which includes all the theorems, definitions, and postulates (available in both pdf. and doc. formats), as it will be utilized throughout the unit. These materials can be accessed at (note that upper and lower case are important in the address) http://wvctm.com/Math/Geometry/menu.html. These apps run only on modern browsers. In particular, they will not work correctly on versions of Internet Explorer before 9.0. If you have a version of Internet Explorer prior to 9.0, you have a few options. These options are explained in detail under Important Instructions on the webpage.

The definitions, postulates, theorems, and corollaries introduced in the **APPS MENU** become the structure for the entire unit. The structure of geometry dictates that the postulates, theorems, and corollaries must be presented in a logical order. Hence, the CSOs are addressed as appropriate to that order.

In traditional Euclidean geometry, we are interested in determining if two figures are congruent. That is, we care if the figures have the same size and shape but we do not care if they have the same position or orientation. But in transformational geometry we are trying to transform one figure so that it overlaps a second figure. This change of philosophy leads to several fundamental differences between the two disciplines. In Euclidean geometry, geometric figures are normally thought of as static. They are in a fixed position in space and don't move. In transformational geometry, the figures are dynamic. They not only can but must move so that one overlaps the other. In our presentation of the subject, we are restricting these movements to translations, rotations, reflections, and dilations. One significant advantage of transformational geometry is that it incorporates concepts from algebra, functional analysis, and set theory giving the student powerful tools not available in Euclidean geometry. Of course the advantage can also be a disadvantage in that the development requires that the student has mastered these tools.

Perhaps the most significant difference between transformational geometry and Euclidean geometry is the role played by functions. The translations, rotations, reflections, and dilations used in this unit are functions from the real plane to the real plane. The student must come to realize that these functions behave the same way as functions from the real line to the real line that they studied in Math 9 and used earlier in Math 10. In fact this geometric use of functions is an excellent way to allow them to glimpse the power, beauty, and utility of functional notation.

Since transformational geometry is a dynamic subject, the position and orientation of the figures takes on a much more central role than in Euclidean geometry. In fact many theorems reduce to the concept of whether two figures can be oriented so that one can be made to overlap the other. While it is tempting for a teacher experienced in Euclidean geometry but a novice in transformational geometry to minimize the role of orientation, this temptation must be resisted.

Another difference between the two approaches to geometry is the role of proof by contradiction. While this method of proof is certainly used in Euclidean geometry, its use is the exception and not the rule. In transformational geometry, the most common way of showing that a point transforms to where you "want it to be" is to assume that it does not and then get a contradiction. This contradiction usually results from the impossibility of orienting the figures correctly.

Unit Calendar:

Math II Unit 5 Similarity, Right Triangle Trigonometry, and Proof

West Virginia College- and Career-Readiness Standards:

Objectives Directly Taught or Learned Through Inquiry/Discovery	**Evidence of Student Mastery of Content**
Cluster: Prove geometric theorems. (Encourage multiple ways of writing proofs, such as narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.)	This cluster heading provides the impetus for this entire unit. In *1.06 Powerpoint on Converses, Inverses, and Contrapositives*, students will be informally assessed on identification of the various ways a conditional statement can be written. In *1.08 Conditional Comic*, students will generate their own conditional statements in a variety of forms, noting which statements are equivalent. In *2.01 Analyzing the Digit Place Game*, students will make decisions and justify their reasoning. *In 2.02 Sherlock Holmes Passage*, students will discuss the deductive process as a chain of conditional statements. In *2.03 Flow Proof Story*, students will order a series of statements to create a flow proof. In *2.05 Powerpoint Jeopardy*, students will demonstrate their understanding of justifications that can be utilized in proofs thus far.
M.2HS.39 Verify experimentally the properties of dilations given by a center and a scale factor. a. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	In *7.03 GeoGebra Investigation 1*, in *7.04 Geogebra Investigation 2,* and in *7.06 GeoGebra Investigation 3*, students explore line dilations. In *7.05 Justification of Line Task*, students verify their conjectures. In *7.13 Dilation Constructions A* and *7.14 Dilation Construction B*, students demonstrate their ability to construct a dilation given a scale factor. In *7.15 Center of Dilation 1* and in *7.16 Center of Dilation 2*, students determine the center of dilation.
M.2HS.40 Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all	In *8.02 Geogebra Investigation*, students create a similar image. In *8.03 Similar or Not Similar*, students perform a sequence of transformations to determine if the figures are similar.

corresponding pairs of sides.	
M.2HS.41 Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	In *9.02 Triangle Similarity Criteria*, students make conjectures about shortcut for determining triangle similarity. The APPS MENU provides a demonstration of the proof of AA Similarity utilizing transformations.
M.2HS.42 Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. Implementation may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for M.2HS.C.3. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.	In Lesson 03, students will have their first encounter with the interactive APPS MENU found at http://wvctm.com/Math/Geometry/menu.html. The associated apps will be used throughout this unit to provide enlightenment, assist in discovery, and demonstrate proofs. In *3.01 Writing Justifications*, students will construct logical arguments. In *3.03 Powerpoint on Vertical Angles Theorem*, students have an opportunity to compare and contrast a paragraph proof, a two-column proof, and a paragraph proof utilizing transformations. In *3.04 Identifying Angles* and in *3.05 Angle Identification*, optional methods are provided as needed for students to increase their ability to identify angles formed by parallel lines. In *3.07 Comparing and Contrasting Methods of Proof*, students will construct their own observations about proof methods. In *4.01 Angles and Transversals*, students make conjectures about the relationship of angles formed by parallel lines. The APPS MENU will be utilized as they justify those conjectures. Students apply these theorems as they simulate Eratosthenes' method for determining the circumference of the earth.
M.2HS.43 Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of this standard may be extended to include concurrence of perpendicular bisectors and angle bisectors in preparation for the unit on Circles With and Without Coordinates.	In *4.04 Investigation of the Angles of a Triangle with GeoGebra*, students make conjectures about the triangle interior angle sum and utilize an app to formally justify their result. The APPS MENU is extremely helpful in building understanding of similarity and proportional reasoning. In *12.1 Medians* and in *12.2 Midsegments of a Triangle*, students create both traditional Euclidean and transformational proofs. The APPS MENU includes this proof.
M.2HS.44 Prove theorems about parallelograms. Theorems include: opposite sides are congruent,	In *5.01 Exploring Properties of Quadrilaterals*, students will formulate conjectures about properties

opposite angles are congruent, the diagonals of a parallelogram bisect each other and conversely, rectangles are parallelograms with congruent diagonals. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.	of a parallelogram. Students will access the APPS MENU as they justify their conjectures. In *5.02 Jigsaw with Properties of a Parallelogram*, students will apply these properties. In *5.04 Is It a Parallelogram?*,students are challenged to formally prove or disprove that a quadrilateral is a parallelogram. By using *6.02 Special Quadrilaterals* or other tools, students willmake conjectures about properties of special quadrilaterals. In *6.03 Quadrilateral Proofs*, students will deepen their understanding of these properties and of proof in general. In *6.08 T-Shirt Investigation*, students will complete a performance task to demonstrate their understanding.
M.2HS.45 Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally and conversely; the Pythagorean Theorem proved using triangle similarity.	In the problem Between the Lines, in *11.1 Pythagorean Theorem Skills*, in *11.2 Applying the Pythagorean Theorem*, and utilizing several websites, students apply the Pythagorean Theorem. The APPS MENU is extremely helpful in building understanding of similarity and proportional reasoning. In *12.3 Dividing a Segment with GeoGebra*, students divide a segment into proportional parts.
M.2HS.46 Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	In *9.01 Rectangle Task*, students demonstrate their understanding of similar figures. In *9.02 Triangle Similarity Criteria*, students make conjectures about shortcut for determining triangle similarity. In *9.04 SS Counterexample*, students determine that SS is not a method for proving triangles similar. In *10.01 Similarity Challenge*, in *10.03 Thumbs Up*, in *10.04 Me and My Shadow*, in *10.05 Mirror, Mirror on the Ground*, in *10.06 Diagram of a House*, and in *10.07 Similarity Problems*, students apply their understanding of similarity. In *11.3 Special Right Triangles* and *11.4 Applying Special Right Triangles*, students develop and apply special right triangle relationships.
M.2HS.47 Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	In *12.4 Triangle Medians*, in *12.5 Investigation of Medians*, and in *12.6 Triangle Medians Proof*, students develop and prove special segment relationships.
M.2HS.48 Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	In *13.2 Class Spreadsheet*, students develop the trigonometric ratios.
M.2HS.49 Explain and use the relationship between the sine and cosine of complementary angles.	In *13.3 Trigonometric Ratios of Complementary Angles*, students develop the relationship between the sine and cosine of a given angle.

M.2HS.50 Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	In *13.4 Trigonometry and Trains*, in *13.6 Indirect Measurement*, in *14.1 Transit Circle*, in *14.2 Rapid Transit*, and *14.4 Using Trigonometric Ratios*, students apply their understanding of the trigonometric ratios.
M.2HS.51 Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin(θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle. Instructional Note: Limit θ to angles between 0 and 90 degrees. Connect with the Pythagorean theorem and the distance formula. Extension of trigonometric functions to other angles through the unit circle is included in Mathematics III.	In *15.1 Distance Formula* and in the Walk in the Desert Problem, students create and apply the distance formula. In *15.2 The Trigonometric Ratios and the Pythagorean Theorem*, students utilize the Pythagorean Theorem and the trigonometric ratios to prove a Pythagorean Identity.

Mathematical Habits of Mind:

Mathematical Habits of Mind	**Evidence of Student Engagement in Mathematical Practices**
1. Make sense of problems and persevere in solving them. 2. Reason abstractly and quantitatively. 3. Construct viable arguments and critique the reasoning of others. 4. Model with mathematics. 5. Use appropriate tools strategically. 6. Attend to precision. 7. Look for and make use of structure. 8. Look for and express regularity in repeated reasoning.	Proficient students clarify the meaning of real world problems and identify entry points to their solution. They choose appropriate tools and make sense of quantities and relationships in problem situations. Students use assumptions and previously-established results to construct arguments and explore them. They justify conclusions, communicate using clear definitions, and respond to arguments, deciding if the arguments make sense. They ask clarifying questions. Students reflect on solutions to decide if outcomes make sense. They discern a pattern or structure and notice if calculations are repeated, while looking for both general methods and shortcuts. As they monitor and evaluate their progress, they will change course if necessary.

Focus/Driving Question:

How can the basic tools of symbolic logic including the converse, inverse, and contrapositive of a conditional statement be utilized in the construction of logical arguments to validate conjectures?
How can convincing arguments be formulated to support conjectures using conditional statements and valid reasons?
How can transformations provide a springboard for proving geometric theorems?
What relationships can be found among the angles formed when parallel lines are cut by a transversal, what are their connections to transformations, and how do they play a role in applications?
How can congruent triangles provide the means for discovering properties of quadrilaterals and deepening an understanding of proof?
What properties appear to be preserved by dilations?
Given the center of dilation and a scale factor, what relationships appear to exist between the image and preimage?

What properties appear to be preserved by similarity transformations?
How can conjectures be proven formally utilizing previously proven theorems?
How can the Similarity Theorems be utilized to determine inaccessible lengths and distances?
How can an understanding of similar triangles be used to prove the Pythagorean Theorem?
How can geometric theorems be applied algebraically?
How can geometric theorems be applied to easily partition a given length into any number of equal parts?
How are trigonometric ratios derived from the properties of similar triangles?
What connections exist between the Pythagorean Theorem, the Distance Formula, and the trigonometric ratios?

Student will Know:

Geometric terminology and notation

Properties of rigid and nonrigid transformations

A variety of approaches to constructing geometric proof

How to construct proofs to validate their conjectures

Postulates and theorems related to angles formed by parallel lines

Postulates and theorems related to triangles and dilations of triangles

Postulates and theorems related to the properties of parallelograms and other quadrilaterals

The basic trigonometric ratios

Student will Do:

Create rigorous proofs using multiple of formats (narrative paragraphs, diagrams, two-column) and dual approaches (traditional and transformational)

Prove theorems about lines, angles, parallelograms and other special quadrilaterals

Apply theorems and postulates in real-world situations

Select appropriate tools strategically in investigations and tasks

Progress from intuitive inducing of conclusions to formal deductions

Resources/Websites:

Meter sticks
Mirror (as large as available)
Patty paper
Trundle wheel or tape measures
Compasses
Tape measures or rulers

Clinometers (website directions are provided)
Grid paper
Protractors
Drinking straws, weights, string, tape (for making clinometers)
Rulers
Trundle wheel
Transit Circles: construct each using a board (22 cm by 22 cm), a wood strip (15.5 cm), 4 finishing nails, and 1 washer
Wooden stakes
See individual lessons for websites and handouts.

Assessment Plan:
Each lesson identifies opportunities for formative assessment. Students are asked to apply and demonstrate their current level of understanding through a variety of performance tasks. Throughout the unit, students have numerous opportunities to demonstrate their understanding using GeoGebra as the investigative tool. Student discourse as they explore the **APPS MENU** is vital to assessing their understanding of this unit. Traditional and transformational approaches to proof are constructed throughout the unit. From their first experience with the Vertical Angles Theorem to the Midsegment Theorem the power of a transformation approaches is highlighted.

Students play the Digits Place Game and analyze their choices, justifying their reasoning. A Jeopardy Game provides an opportunity for students to demonstrate their background knowledge built in Math I. In Writing Justifications, students begin for formalize their reasoning. Using a graphic organizer, students compare and contrast methods of proof, noting advantages of transformational approaches. In *5.04 Is It a Parallelogram?* students generate counterexamples or justify formally if a quadrilateral is a parallelogram. In *6.03 Quadrilateral Proofs*, students construct proofs to argue from a given hypothesis to the desired conclusion. In the *6.08 T-Shirt Investigation*, students grapple with a task that allows them to select their own tools and their own strategies for drawing conclusions.

Students make conjectures based on numerous dilation constructions and justify their conclusions. In *7.17 Check Your Understanding*, students apply their ability to determine when triangles are similar. In *10.04 Me and My Shadow* and *10.05 Mirror, Mirror on the Ground*, students will apply their understanding of similar triangles to determine inaccessible heights and distances. Students finally apply similarity to the development of the trigonometric ratios.

Major Projects: (Group) or (Individual)

5.04 Is It a Parallelogram? (individual)
Students generate counterexamples or justify formally if a quadrilateral is a parallelogram.

6.08 T-Shirt Task (individual)
Students grapple with a task that allows them to select their own tools and their own strategies for drawing conclusions.

9.01 Rectangle Task (individual)

Students determine which rectangles are similar and support their arguments.

Unit Reflection:

The formative assessment process requires teachers to reflect on the performance of each student during the unit of study by asking these questions: Have all students mastered the Next Generation standards targeted for this unit of study? Is it necessary to re-teach a concept to some members of the class while others benefit from an exercise that enriches or extends their learning during the unit? Teachers should also cause students to reflect upon their learning during the unit of study by having them reflect on questions such as: What have I learned? Are there concepts or skills I believe I need to continue to work with? We often neglect reflection, this very important stage in the learning process. By taking time to reflect upon where our students are in their learning, we can design the next unit of study to better meet their identified needs.

Teachers are provided with a *1.09 Teacher Reflection Sheet* to guide their thinking as they seek to improve their practice. Certainly, it may not be feasible to formally complete such a reflection after every lesson, but hopefully the questions can generate some ideas for contemplation.

Career Connections:

To be well-equipped for the career options of this new century, understanding of transformational geometry will prove essential. All professions in which realistic models are an essential tool require an understanding of dilations. Most construction-based careers, such as architects, contractors, and electricians, require the ability to create or interpret and understand scale models. Advances in aviation and aerospace are based on research and testing that is generally carried out with scale models. These occupations are related to the following Career Listings: Architecture and Construction; Arts, A/V Technology and Communications; Manufacturing; Science, Technology, Engineering and Mathematics.

Unit Plan Outline (Lesson Plans link):

Lesson 1: Getting Ready for Proof
Lesson 2: Getting Ready for Proof #2
Lesson 3: Developing Formal Proof and Definitions
Lesson 4: Angles formed by Parallel Lines—Multiple Applications
Lesson 5: Deepening Understanding of Proof through Exploration of Parallelograms
Lesson 6: Deepening Understanding of Proof through Exploration of Special Quadrilaterals
Lesson 7: Exploring Properties of Dilations
Lesson 8: Foundations of Similarity
Lesson 9: Justifying the Similarity Theorems
Lesson 10: Applications of Similarity
Lesson 11: Pythagorean Theorem and Its Converse; Special Right Triangles
Lesson 12: Medians and Proportionality
Lesson 13: Trigonometric Ratios
Lesson 14: Using Trigonometric Ratios
Lesson 15: Pythagorean Identity

Note: Italicized documents can be found in the corresponding lesson plans.

Planning Calendar
Unit Title: Math II Unit 5 Similarity, Right Triangle Trigonometry and Proof

Day 1	Day 2	Day 3	Day 4
Lesson 1: Getting Ready for Proof **FQ**: How can the basic tools of symbolic logic including the converse, inverse, and contrapositive of a conditional statement be utilized in the construction of logical arguments to validate conjectures? **WVCCRS**: Cluster: Prove geometric theorems. (Encourage multiple ways of writing proofs, such as narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.) **MHM2, MHM3, MHM7** Students will develop the foundations of logical reasoning as a prelude to validating their conjectures using rigorous methods of proof. This lesson is the first of two lessons that will provide the scaffolding for the understanding of formal proof. Up to this point, the work of geometry has been primarily one of conjecture based upon investigations. Methods of rigorous proof will be addressed later in the unit in the context of specific theorems. A variety of approaches to understanding these theorems will afford accessibility for all students, with increased demand placed upon the STEM student who will be progressing to Math III STEM. This lesson is designed for 45 – 90 minutes depending on the sophistication of the understanding of the students. Students have considered the idea of a converse of a statement in Math 8.	**Lesson 1: Getting Ready for Proof** **FQ**: How can the basic tools of symbolic logic including the converse, inverse, and contrapositive of a conditional statement be utilized in the construction of logical arguments to validate conjectures? **WVCCRS**: Cluster: Prove geometric theorems. (Encourage multiple ways of writing proofs, such as narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.) **MHM2, MHM3, MHM7** Students will develop the foundations of logical reasoning as a prelude to validating their conjectures using rigorous methods of proof. This lesson is the first of two lessons that will provide the scaffolding for the understanding of formal proof. Up to this point, the work of geometry has been primarily one of conjecture based upon investigations. Methods of rigorous proof will be addressed later in the unit in the context of specific theorems. A variety of approaches to understanding these theorems will afford accessibility for all students, with increased demand placed upon the STEM student who will be progressing to Math III STEM. This lesson is designed for 45 – 90 minutes depending on the sophistication of the understanding of the students. Students have considered the idea of a converse of a statement in Math 8.	**Lesson 2: Getting Ready for Proof #2** **FQ**: How does a mapping apply to the entire geometric shape? **WVCCRS**: Cluster: Prove geometric theorems. (Encourage multiple ways of writing proofs, such as narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.) **MHM2, MHM3, MHM7** Students will continue to hone their reasoning skills, to justify conjectures with convincing arguments, and to assemble and connect the language of geometry developed in Math I for use in constructing more formal proofs. This lesson is the second of two lessons that will provide the scaffolding for the understanding of formal proof. Up to this point, the work of geometry has been primarily one of conjecture based upon investigations. Methods of rigorous proof will be addressed later in the unit in the context of specific theorems. A variety of approaches to understanding these theorems will afford accessibility for all students, with increased demand placed upon the STEM student who will be progressing to Math III STEM. This lesson is designed for 45 – 90 minutes depending on the sophistication of the understanding of the students.	**Lesson 2: Getting Ready for Proof #2** **FQ**: How does a mapping apply to the entire geometric shape? **WVCCRS**: Cluster: Prove geometric theorems. (Encourage multiple ways of writing proofs, such as narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.) **MHM2, MHM3, MHM7** Students will continue to hone their reasoning skills, to justify conjectures with convincing arguments, and to assemble and connect the language of geometry developed in Math I for use in constructing more formal proofs. This lesson is the second of two lessons that will provide the scaffolding for the understanding of formal proof. Up to this point, the work of geometry has been primarily one of conjecture based upon investigations. Methods of rigorous proof will be addressed later in the unit in the context of specific theorems. A variety of approaches to understanding these theorems will afford accessibility for all students, with increased demand placed upon the STEM student who will be progressing to Math III STEM. This lesson is designed for 45 – 90 minutes depending on the sophistication of the understanding of the students.

Day 5	Day 6	Day 7	Day 8
Lesson 3: Developing Formal Proof and Definitions **FQ**: What properties are preserved by translation? **WVCCRS**: M.2HS.42 **MHM2, MHM3, MHM7** Students will compare and contrast a two-column traditional proof with a paragraph proof that utilizes a transformational method. They will also develop definitions necessary to further the rigor of the proof process. This lesson is pivotal in comparing methods of proof and in demonstrating a proof that is based on transformations. It will introduce definitions that are not typical of traditional geometry courses that most teachers have taught previously. Definitions that are necessary for later proofs that utilize transformations are introduced in this lesson. For example, the role of orientation of points to lines plays a more significant role in transformational geometry than in a more traditional geometry course. There is always a tension between maintaining rigor and yet allowing accessibility to content. With this in mind, the definitions and theorems in this unit are supported with a series of apps that will clarify statements that may at first reading appear wordy and even cumbersome, but are necessary to preserve the integrity of the proofs. The apps, particularly for inclusion students, will provide a visual representation of a definition or theorem and thus augment the meaning of what may have been difficult to comprehend for the student. Teachers are encouraged to utilize this tool to deepen student understanding. This lesson is designed for approximately 90 minutes, but may need more time as it is fundamental to ensuing lessons.	**Lesson 3: Developing Formal Proof and Definitions** **FQ**: What properties are preserved by translation? **WVCCRS**: M.2HS.42 **MHM2, MHM3, MHM7** Students will compare and contrast a two-column traditional proof with a paragraph proof that utilizes a transformational method. They will also develop definitions necessary to further the rigor of the proof process. This lesson is pivotal in comparing methods of proof and in demonstrating a proof that is based on transformations. It will introduce definitions that are not typical of traditional geometry courses that most teachers have taught previously. Definitions that are necessary for later proofs that utilize transformations are introduced in this lesson. For example, the role of orientation of points to lines plays a more significant role in transformational geometry than in a more traditional geometry course. There is always a tension between maintaining rigor and yet allowing accessibility to content. With this in mind, the definitions and theorems in this unit are supported with a series of apps that will clarify statements that may at first reading appear wordy and even cumbersome, but are necessary to preserve the integrity of the proofs. The apps, particularly for inclusion students, will provide a visual representation of a definition or theorem and thus augment the meaning of what may have been difficult to comprehend for the student. Teachers are encouraged to utilize this tool to deepen student understanding. This lesson is designed for approximately 90 minutes, but may need more time as it is fundamental to ensuing lessons.	**Lesson 4: Angles formed by Parallel Lines—Multiple Applications** **FQ**: What relationships can be found among the angles formed when parallel lines are cut by a transversal, what are their connections to transformations, and how do they play a role in applications? **WVCCRS**: M.2HS.42, M.2HS.43 **MHM1, MHM2, MHM3, MHM4, MHM5, MHM7** Students will prove theorems about congruent angles formed by parallel lines and will apply those ideas to prove theorems about triangle relationships. They will continue to develop proofs in a variety of ways and begin to notice the advantages of transformational methods in creating proofs. Students will broaden their understanding of angles as they recognize that certain angles are either equal or supplementary by virtue of their relationship. The definitions developed in the previous lesson will provide the impetus for this investigation of the relationship of angles formed by parallel lines. Five theorems will be developed in this lesson, all of which are related to parallel lines. Connections will continue to be made to transformations as students, particularly through the use of the apps, explore transformational methods of proving these theorems. The teacher is encouraged to both examine the transformational methods and compare them to more traditional approaches to deepen student understanding. This lesson is designed for approximately 120 minutes, but time may vary depending on the background of the students.	**Lesson 4: Angles formed by Parallel Lines—Multiple Applications** **FQ**: What relationships can be found among the angles formed when parallel lines are cut by a transversal, what are their connections to transformations, and how do they play a role in applications? **WVCCRS:** M.2HS.42, M.2HS.43 **MHM1, MHM2, MHM3, MHM4, MHM5, MHM7** Students will prove theorems about congruent angles formed by parallel lines and will apply those ideas to prove theorems about triangle relationships. They will continue to develop proofs in a variety of ways and begin to notice the advantages of transformational methods in creating proofs. Students will broaden their understanding of angles as they recognize that certain angles are either equal or supplementary by virtue of their relationship. The definitions developed in the previous lesson will provide the impetus for this investigation of the relationship of angles formed by parallel lines. Five theorems will be developed in this lesson, all of which are related to parallel lines. Connections will continue to be made to transformations as students, particularly through the use of the apps, explore transformational methods of proving these theorems. The teacher is encouraged to both examine the transformational methods and compare them to more traditional approaches to deepen student understanding. This lesson is designed for approximately 120 minutes, but time may vary depending on the background of the students.

Day 9	Day 10	Day 11	Day 12
Lesson 5: Deepening Understanding of Proof through Exploration of Parallelograms **FQ**: How can congruent triangles provide the means for discovering properties of quadrilaterals and deepening an understanding of proof? **WVCCRS**: M.2HS.44 **MHM1, MHM2, MHM3, MHM4, MHM5, MHM7** Students will continue to develop an understanding of proof while they investigate how congruent triangles can provide information about the sides, angles, and diagonals of a quadrilateral. They will be challenged to discover properties of quadrilaterals and to justify their reasoning using a variety of methods of proof. Using congruent triangles, students will explore the relationships of the sides, angles, and diagonals of a parallelogram. They will develop conjectures and write convincing proofs in a variety of formats. Four theorems will be developed in this lesson, all of which are related to quadrilaterals. The proofs will primarily utilize tools already available from Math I. The primary goal is to increase students' comfort level in multiple ways of writing proofs. This lesson is designed for approximately 60 to 90 minutes, but time may vary depending on the background of the students.	**Lesson 5: Deepening Understanding of Proof through Exploration of Parallelograms** **FQ**: How can congruent triangles provide the means for discovering properties of quadrilaterals and deepening an understanding of proof? **WVCCRS**: M.2HS.44 **MHM1, MHM2, MHM3, MHM4, MHM5, MHM7** Students will continue to develop an understanding of proof while they investigate how congruent triangles can provide information about the sides, angles, and diagonals of a quadrilateral. They will be challenged to discover properties of quadrilaterals and to justify their reasoning using a variety of methods of proof. Using congruent triangles, students will explore the relationships of the sides, angles, and diagonals of a parallelogram. They will develop conjectures and write convincing proofs in a variety of formats. Four theorems will be developed in this lesson, all of which are related to quadrilaterals. The proofs will primarily utilize tools already available from Math I. The primary goal is to increase students' comfort level in multiple ways of writing proofs. This lesson is designed for approximately 60 to 90 minutes, but time may vary depending on the background of the students.	**Lesson 6: Deepening Understanding of Proof through Exploration of Special Quadrilaterals** **FQ**: How can congruent triangles provide the means for discovering properties of special quadrilaterals, isosceles triangles and perpendicular bisectors while deepening an understanding of proof? **WVCCRS**: M.2HS.44 **MHM1, MHM2, MHM3, MHM4, MHM5, MHM7** Students will continue to develop an understanding of proof while they investigate how congruent triangles can provide information about the sides, angles, and diagonals of special quadrilaterals. They will also consider relationships in isosceles triangles and perpendicular bisectors. They will be challenged to discover properties of quadrilaterals, isosceles triangles, and perpendicular bisectors and to justify their reasoning using a variety of methods of proof. Using congruent triangles, students will explore the relationships of the sides, angles, and diagonals of special quadrilaterals and also investigate isosceles triangles and perpendicular bisectors. They will develop conjectures and write convincing proofs in a variety of formats. The primary goal is to increase students' comfort level in multiple ways of writing proofs. This lesson is designed for approximately 60 to 90 minutes, but time may vary depending on the background of the students.	**Lesson 6: Deepening Understanding of Proof through Exploration of Special Quadrilaterals** **FQ**: How can congruent triangles provide the means for discovering properties of special quadrilaterals, isosceles triangles and perpendicular bisectors while deepening an understanding of proof? **WVCCRS**: M.2HS.44 **MHM1, MHM2, MHM3, MHM4, MHM5, MHM7** Students will continue to develop an understanding of proof while they investigate how congruent triangles can provide information about the sides, angles, and diagonals of special quadrilaterals. They will also consider relationships in isosceles triangles and perpendicular bisectors. They will be challenged to discover properties of quadrilaterals, isosceles triangles, and perpendicular bisectors and to justify their reasoning using a variety of methods of proof. Using congruent triangles, students will explore the relationships of the sides, angles, and diagonals of special quadrilaterals and also investigate isosceles triangles and perpendicular bisectors. They will develop conjectures and write convincing proofs in a variety of formats. The primary goal is to increase students' comfort level in multiple ways of writing proofs. This lesson is designed for approximately 60 to 90 minutes, but time may vary depending on the background of the students.

Day 13	Day 14	Day 15	Day 16
Lesson 7: Exploring Properties of Dilations **FQ**: What properties appear to be preserved by dilations? Given the center of dilation and a scale factor, what relationships appear to exist between the image and preimage? **WVCCRS**: M.2HS.39 **MHM1, MHM2, MHM3, MHM5, MHM6, MHM8** Students are reintroduced to the idea of dilation in the investigation and explore the relationships that exist between the figures formed. Through investigations, they develop conjectures about the properties of dilations. Students also verify experimentally the properties of dilations given by a center and a scale factor. Students develop a variety of skills as they work with dilations. Given the preimage and the image, students are able to determine both the point of dilation and the scale factor. Given the point of dilation, scale factor and preimage, students are able to create the image. The lesson is designed for one or two 90-minute periods. Depending upon the background of the students, some of the investigations may be combined or eliminated and some investigations may be assigned as homework.	**Lesson 7: Exploring Properties of Dilations** **FQ**: What properties appear to be preserved by dilations? Given the center of dilation and a scale factor, what relationships appear to exist between the image and preimage? **WVCCRS**: M.2HS.39 **MHM1, MHM2, MHM3, MHM5, MHM6, MHM8** Students are reintroduced to the idea of dilation in the investigation and explore the relationships that exist between the figures formed. Through investigations, they develop conjectures about the properties of dilations. Students also verify experimentally the properties of dilations given by a center and a scale factor. Students develop a variety of skills as they work with dilations. Given the preimage and the image, students are able to determine both the point of dilation and the scale factor. Given the point of dilation, scale factor and preimage, students are able to create the image. The lesson is designed for one or two 90-minute periods. Depending upon the background of the students, some of the investigations may be combined or eliminated and some investigations may be assigned as homework.	**Lesson 8: Foundations of Similarity** **FQ**: What properties appear to be preserved by similarity transformations? **WVCCRS**: M.2HS.40 **MHM1, MHM2, MHM3, MHM5, MHM6, MHM7** Students are introduced to the mathematical understandings implied in the definition of similarity. They will develop the ability to explain the meaning of similarity for triangles in terms of transformations. Students will experimentally apply rigid transformations and dilations to determine if given triangles are similar. They will conjecture by measuring or using dynamic geometry software that there exists a series of rigid transformations and a dilation about the origin that maps one triangle to another if and only if all three pairs of corresponding angles are equal and all three pairs of corresponding sides are in proportion. The lesson is designed for a 45-minute class period.	**Lesson 8: Foundations of Similarity** **FQ**: What properties appear to be preserved by similarity transformations? **WVCCRS**: M.2HS.40 **MHM1, MHM2, MHM3, MHM5, MHM6, MHM7** Students are introduced to the mathematical understandings implied in the definition of similarity. They will develop the ability to explain the meaning of similarity for triangles in terms of transformations. Students will experimentally apply rigid transformations and dilations to determine if given triangles are similar. They will conjecture by measuring or using dynamic geometry software that there exists a series of rigid transformations and a dilation about the origin that maps one triangle to another if and only if all three pairs of corresponding angles are equal and all three pairs of corresponding sides are in proportion. This lesson is designed for a 45-minute class period.

Day 17	Day 18	Day 19	Day 20
Lesson 9: Justifying the Similarity Theorems **FQ**: How can conjectures be proven formally utilizing previously proven theorems? **WVCCRS**: M.2HS.45, M.2HS.46 **MHM2, MHM3, MHM5 MHM6, MHM7** In the previous lesson, dilations have motivated a study of similarity. Through a variety of investigations, students have explored dilations and the special relationships that exist between corresponding angles and sides in similar shapes, establishing a foundational understanding of similarity. In this lesson students will continue to hone their skills in justifying their thinking by formally proving the results that they conjecture during the investigations in the lesson. Their toolbox now contains diverse strategies for developing a formal geometric proof so it is important that students be encouraged to write convincing arguments that show the logic that led to their conclusions. This lesson is designed for approximately 90 to 120 minutes, but time may vary depending on the background of the students.	**Lesson 9: Justifying the Similarity Theorems** **FQ**: How can conjectures be proven formally utilizing previously proven theorems? **WVCCRS**: M.2HS.45, M.2HS.46 **MHM2, MHM3, MHM5 MHM6, MHM7** In the previous lesson, dilations have motivated a study of similarity. Through a variety of investigations, students have explored dilations and the special relationships that exist between corresponding angles and sides in similar shapes, establishing a foundational understanding of similarity. In this lesson students will continue to hone their skills in justifying their thinking by formally proving the results that they conjecture during the investigations in the lesson. Their toolbox now contains diverse strategies for developing a formal geometric proof so it is important that students be encouraged to write convincing arguments that show the logic that led to their conclusions. This lesson is designed for approximately 90 to 120 minutes, but time may vary depending on the background of the students.	**Lesson 10: Applications of Similarity** **FQ**: How can the Similarity Theorems be utilized to determine inaccessible lengths and distances? **WVCCRS**:, M.2HS.46 **MHM1, MHM2, MHM3 MHM5, MHM6** Having established the conditions under which triangles are similar by formal proof, students will now apply these results in multiple contexts. It is important for students to realize that finally their knowledge of similarity will be helpful as they analyze and solve new applications. Students will be working in small groups in an outdoor setting to solve some of the application problems in this lesson. This lesson is designed for approximately 90 minutes.	**Lesson 10: Applications of Similarity** **FQ**: How can the Similarity Theorems be utilized to determine inaccessible lengths and distances? **WVCCRS**:, M.2HS.46 **MHM1, MHM2, MHM3 MHM5, MHM6** Having established the conditions under which triangles are similar by formal proof, students will now apply these results in multiple contexts. It is important for students to realize that finally their knowledge of similarity will be helpful as they analyze and solve new applications. Students will be working in small groups in an outdoor setting to solve some of the application problems in this lesson. This lesson is designed for approximately 90 minutes.

Day 21	Day 22	Day 23	Day 24
Lesson 11: Pythagorean Theorem and Its Converse; Special Right Triangles **FQ**: How can an understanding of similar triangles be used to prove the Pythagorean Theorem? **WVCCRS**: M.2HS.43, M.2HS.45, M.2HS.46 **MHM1, MHM2, MHM3, MHM5, MHM6, MHM7, MHM8** Students continue to build and apply a formal understanding of similarity. They combine their understanding of similarity and proportional reasoning to prove the Pythagorean Theorem. Through exploration and investigation, students apply their understanding of similar triangles and proportional reasoning to prove the Pythagorean Theorem. This lesson is designed for approximately 90 to 120 minutes, but time may vary depending on the background of the students.	**Lesson 11: Pythagorean Theorem and Its Converse; Special Right Triangles** **FQ**: How can an understanding of similar triangles be used to prove the Pythagorean Theorem? **WVCCRS**: M.2HS.43, M.2HS.45, M.2HS.46 **MHM1, MHM2, MHM3, MHM5, MHM6, MHM7, MHM8** Students continue to build and apply a formal understanding of similarity. They combine their understanding of similarity and proportional reasoning to prove the Pythagorean Theorem. Through exploration and investigation, students apply their understanding of similar triangles and proportional reasoning to prove the Pythagorean Theorem. This lesson is designed for approximately 90 to 120 minutes, but time may vary depending on the background of the students.	**Lesson 12: Medians and Proportionality** **FQ**: How can geometric theorems be applied algebraically? How can geometric theorems be applied to easily partition a given length into any number of equal parts? **WVCCRS**: M.2HS.47 **MHM1, MHM2, MHM3, MHM5, MHM6, MHM7, MHM8** Students develop their ability to use coordinates to apply geometric theorems. Through exploration and investigation, students apply their understanding of geometric theorems to divide a line segment into a given ratio. This lesson is designed for approximately 90 to 120 minutes, but time may vary depending on the background of the students.	**Lesson 12: Medians and Proportionality** **FQ**: How can geometric theorems be applied algebraically? How can geometric theorems be applied to easily partition a given length into any number of equal parts? **WVCCRS**: M.2HS.47 **MHM1, MHM2, MHM3, MHM5, MHM6, MHM7, MHM8** Students develop their ability to use coordinates to apply geometric theorems. Through exploration and investigation, students apply their understanding of geometric theorems to divide a line segment into a given ratio. This lesson is designed for approximately 90 to 120 minutes, but time may vary depending on the background of the students.

Day 25	Day 26	Day 27	Day 28
Lesson 13: Introduction to the Trigonometric Ratios **FQ**: How are trigonometric ratios derived from the properties of similar triangles? **WVCCRS**: M.2HS.46, M.2HS.48, M.2HS.49, M.2HS.50 **MHM1, MHM2, MHM3, MHM5, MHM6, MHM7, MHM8** Students expand their understanding of similar triangles to develop an understanding of the trigonometric ratios. Students develop an understanding of the relationship between the sine and cosine of complementary angles. Students apply their understanding of the trigonometric ratios and the Pythagorean Theorem to solve problems in which right triangles can be found. Students apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. This lesson is designed for approximately 90 to 120 minutes, but time may vary depending on the background of the students.	**Lesson 13: Introduction to the Trigonometric Ratios** **FQ**: How are trigonometric ratios derived from the properties of similar triangles? **WVCCRS**: M.2HS.46, M.2HS.48, M.2HS.49, M.2HS.50 **MHM1, MHM2, MHM3, MHM5, MHM6, MHM7, MHM8** Students expand their understanding of similar triangles to develop an understanding of the trigonometric ratios. Students develop an understanding of the relationship between the sine and cosine of complementary angles. Students apply their understanding of the trigonometric ratios and the Pythagorean Theorem to solve problems in which right triangles can be found. Students apply similarity in right triangles to understand right triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. This lesson is designed for approximately 90 to 120 minutes, but time may vary depending on the background of the students.	**Lesson 14: Using Trigonometric Ratios** **FQ**: How are trigonometric ratios derived from the properties of similar triangles? **WVCCRS**: M.2HS.46, M.2HS.48, M.2HS.49, M.2HS.50 **MHM1, MHM2, MHM3, MHM5, MHM6, MHM7, MHM8** In a variety of problem situations, students develop a deeper understanding of the trigonometric functions and their applications. This lesson is designed for approximately 45 minutes.	**Lesson 14: Using Trigonometric Ratios** **FQ**: What connections exist between the Pythagorean Theorem, the Distance Formula, and the trigonometric ratios? **WVCCRS**: M.2HS.46, M.2HS.48, M.2HS.49, M.2HS.50, M.2HS.51 **MHM1, MHM2, MHM3, MHM5, MHM6, MHM7, MHM8** The activities throughout this lesson challenge students to develop a deeper understanding of the Pythagorean Theorem. In previous courses, students have seen the Distance Formula as a restatement of the Pythagorean Theorem. Students advance and apply this understanding to develop the Pythagorean Identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and realize that this, too, is a restatement of the Pythagorean Theorem. This lesson is designed for approximately 45 minutes.