West Virginia math 4 life

Frameworks for Mathematics

Algebra I

West Virginia DEPARTMENT OF EDUCATION

Algebra I

In Algebra I, students use reasoning about structure to define and make sense of rational exponents and explore the algebraic structure of the rational and real number systems. They understand that numbers in real-world applications often have units attached to them—that is, the numbers are considered *quantities*. Students' work with numbers and operations throughout elementary and middle school has led them to an understanding of the structure of the number system; in Algebra I, students explore the structure of algebraic expressions and polynomials. They see that certain properties must persist when they work with expressions that are meant to represent numbers—which they now write in an abstract form involving variables. When two expressions with overlapping domains are set as equal to each other, resulting in an equation, there is an implied solution set (be it empty or non-empty), and students not only refine their techniques for solving equations and finding the solution set, but they can clearly explain the algebraic steps they used to do so.

Students began their exploration of linear equations in middle school, first by connecting proportional equations ($y = kx$, $k \neq 0$) to graphs, tables, and real-world contexts, and then moving toward an understanding of general linear equations ($y = mx + b$, $m \neq 0$) and their graphs. In Algebra I, students extend this knowledge to work with absolute value equations, linear inequalities, and systems of linear equations. After learning a more precise definition of *function* in this course, students examine this new idea in the familiar context of linear equations—for example, by seeing the solution of a linear equation as solving $f(x) = g(x)$ for two linear functions f and g.

Students continue to build their understanding of functions beyond linear ones by investigating tables, graphs, and equations that build on previous understandings of numbers and expressions. They make connections between different representations of the same function. They also learn to build functions in a modeling context and solve problems related to the resulting functions. Note that in Algebra I the focus is on linear, simple exponential, and quadratic equations.

Finally, students extend their prior experiences with data, using more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, students look at residuals to analyze the goodness of fit.

Modeling

Standards	Teacher Understandings	Resources	Student Understandings
Although the Modeling category does not include specific standards, the idea of using mathematics to model the world pervades all higher mathematics courses and should hold a significant place in instruction. Modeling is addressed first here to emphasize its importance in the higher mathematics curriculum.	Modeling at the higher mathematics level goes beyond the simple application of previously constructed mathematics and includes real-world problems. True modeling begins with students asking a question about the world around them, and mathematics is then constructed in the process of attempting to answer the question. Students may see when trying to answer their question that solving an equation arises as a necessity and that the equation often involves the specific instance of knowing the output value of a function at an unknown input value. Modeling problems have an element of being genuine problems, in the sense that students care about answering the question	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. It highlights some necessary foundational skills from previous grade levels. Math TREE Online Education Resources A curated set of aligned, internet	• When students are presented with a real-world situation and challenged to ask a question, new issues arise (e.g., which of the quantities present in this situation are known, and which are unknown?). • Students decide on a solution path that may need to be revised. They make use of tools such as calculators, dynamic geometry software, or spreadsheets. They try to use previously derived models (e.g., linear functions), but may find that a new formula or function will apply.

	under consideration. In modeling, mathematics is used as a tool to answer questions that students really want answered. Students examine a problem and formulate a *mathematical model* (an equation, table, graph, or the like), compute an answer or rewrite their expression to reveal new information, interpret and validate the results, and report out. This is a new approach for many teachers and may be challenging to implement, but the effort should show students that mathematics is relevant to their lives. From a pedagogical perspective, modeling gives a concrete basis from which to abstract the mathematics and often serves to motivate students to become independent learners. The important ideas surrounding rational functions, graphing, solving equations, and rates of change should be explored	resources for WV middle and high school math teachers. Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	

	through the lens of mathematical modeling. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.		

Functions

Standards	Teacher Understandings	Resources	Student Understandings
LINEAR AND EXPONENTIAL RELATIONSHIPS **Understand the concept of a function and use function notation.** **M.A1HS.18** Recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear functions and exponential functions having integral domains. **M.A1HS.19** Use function notation, evaluate functions for inputs in their domains and interpret statements that use	Functions describe situations where one quantity determines another. For example, the return on $10,000 invested at an annualized percentage rate of 4.25% is a function of the length of time the money is invested. Because we continually form theories about dependencies between quantities in nature and society, functions are important tools in the construction of mathematical models. In school mathematics, functions usually have numerical inputs and outputs and are often defined by an algebraic expression. For example, the time in hours it takes for a car to drive 100 miles is a function of the car's speed in miles per hour, v; the rule $T(v) = \frac{100}{v}$ expresses this relationship algebraically	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. Math TREE Online Education Resources A curated set of aligned, internet resources for WV middle and high school math teachers.	• While the grade-eight standards call for students to work informally with functions, students in Algebra I begin to refine their understanding and use of the formal mathematical language of functions. Students develop an understanding of the concept of a function, interpret characteristics of functions in context, and represent functions in different ways. • Students make the connection between the graph of the equation $y = f(x)$ and the function itself and understand that the graph is a *representation* of a function. They connect the domain and range of a function to its graph. • Students work with linear, quadratic, and

function notation in terms of a context. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear functions and exponential functions having integral domains. **M.A1HS.20** Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. (e.g., The Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n)+ f(n-1) for n ≥ 1. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear functions and exponential functions having integral domains. Draw connection to M.A1HS.27, which requires students to write arithmetic and geometric sequences. Emphasize arithmetic and geometric sequences as	and defines a function whose name is T. The set of inputs to a function is called its *domain*. We often assume the domain to be all inputs for which the expression defining a function has a value, or for which the function makes sense in a given context. When describing relationships between quantities, the defining characteristic of a function is that the input value determines the output value, or equivalently, that the output value depends upon the input value (University of Arizona [UA] Progressions Documents 2013c, 2). A function can be described in various ways, such as by a graph (e.g., the trace of a seismograph); by a verbal rule, as in, "I'll give you a state, you give me the capital city"; by an assignment, such as the fact that each individual is given a unique Social Security Number; by	Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	exponential sequences and interpret the parameters of the expressions defining the terms of the sequence when they arise in context. • Students represent functions with graphs and identify key features in the graph. • Students develop fluency only with linear, exponential, and quadratic functions in Algebra I, which includes the ability to graph them by hand. • Students represent the same function algebraically in different forms and interpret these differences in terms of the graph or context. • Students see the effect of transformations on the graph of a function and understand why it appears that the effect on the graph is the opposite to the transformation on the

examples of linear and exponential functions. **Interpret functions that arise in applications in terms of a context.** **M.A1HS.21** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on linear and exponential functions. **M.A1HS.22** Relate the domain of a function to its graph and where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Instructional Note: Focus on linear and exponential functions.	an algebraic expression, such as $f(x) = a + bx$; or by a recursive rule, such as $f(n + 1) = f(n) + b$, $f(0) = a$. The graph of a function is often a useful way of visualizing the relationship that the function models, and manipulating a mathematical expression for a function can shed light on the function's properties. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and		variable. • Students find inverse functions in simple cases. • Students engage in modeling the world and investigating rates of change and patterns of growth. Two important families of functions characterized by laws of growth are linear functions, which grow at a constant rate, and exponential functions, which grow at a constant percent rate. • Students recognize and understand the defining characteristics of linear, quadratic, and exponential functions. • Students recognize situations that represent both linear and exponential functions and construct functions to describe the situations. • Students interpret the parameters in linear, exponential, and quadratic expressions

M.A1HS.23 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on linear functions and exponential functions whose domain is a subset of the integers. The Unit on Quadratic Functions and Modeling in this course and the Algebra II course address other types of functions. **Analyze functions using different representations.** **M.A1HS.24** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph exponential and logarithmic functions, showing intercepts and end behavior and trigonometric functions, showing period, midline and amplitude. Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example,	strengthen understandings throughout the school year.		and model physical problems with such functions.

compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100^{2n}$) **M.A1HS.25** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100^{2n}$) **Build a function that models a relationship between two quantities.** **M.A1HS.26** Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a			

decaying exponential, and relate these functions to the model.) Instructional Note: Limit to linear and exponential functions. **M.A1HS.27** Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Instructional Note: Limit to linear and exponential functions. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions. **Build new functions from existing functions.** **M.A1HS.28** Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function			

to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard. **Construct and compare linear, quadratic, and exponential models and solve problems.** **M.A1HS.29** Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another. **M.A1HS.30** Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a			

description of a relationship or two input-output pairs (include reading these from a table). Instructional Note: In constructing linear functions, draw on and consolidate previous work in Grade 8 on finding equations for lines and linear functions. **M.A1HS.31** Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Limit to comparisons between exponential and linear models. **Interpret expressions for functions in terms of the situation they model.** **M.A1HS.32** Interpret the parameters in a linear or exponential function in terms of a context. Instructional Note: Limit exponential functions to those of the form $f(x) = b^x + k$. **QUADRATIC FUNCTIONS AND MODELING** **Interpret functions that arise in applications in terms of a context.** **M.A1HS.51** For a function that models a relationship between two quantities, interpret key features of graphs and			

tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in the Unit on Linear and Exponential Relationships. **M.A1HS.52** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in the Unit on Linear and Exponential Relationships. **M.A1HS.53** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a			

specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in the Unit on Linear and Exponential Relationships. **Analyze functions using different representations.** **M.A1HS.54** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Instructional Note: Compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.			

M.A1HS.55 Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay. Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored. This standard extends the work begun in the Linear and Exponential Relationships unit on exponential functions with integer exponents. **M.A1HS.56** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).			

For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. Instructional Note: Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored. **Build a function that models a relationship between two quantities.** **M.A1HS.57** Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Instructional Note: Focus on situations that exhibit a quadratic relationship. **Build new functions from existing functions.** **M.A1HS.58**			

Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on quadratic functions, and consider including absolute value functions. **M.A1HS.59** Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or f(x) = (x+1)/(x-1) for $x \neq 1$. Instructional Note: Focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2$, $x > 0$. **Construct and compare linear, quadratic and exponential models and solve problems.** **M.A1HS.60** Observe using graphs and tables that a quantity increasing exponentially			

eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Compare linear and exponential growth to quadratic growth.			

Number and Quantity

Standards	Teacher Understandings	Resources	Student Understandings
RELATIONSHIPS BETWEEN QUANTITIES AND REASONING WITH EQUATIONS **Reason quantitatively and use units to solve problems.** **M.A1HS.1** Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. **M.A1HS.2** Define appropriate quantities for the purpose of descriptive modeling. Instructional Note: Working with quantities and the relationships	In the grade-eight standards, students encountered some examples of irrational numbers, such as π and $\sqrt{2}$ (or $\sqrt{n}$ where n is a non-square number). In Algebra I, students extend this understanding beyond the fact that there are numbers that are not rational; they begin to understand that the rational numbers form a closed system. Students have witnessed that with each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and	• Students make meaning of the representation of radicals with rational exponents. Students use basic properties of exponents to explain the meaning of rational exponents. • Students explain that the sum or product of two rational numbers is rational. Students see that rational numbers are closed under the operations of addition and multiplication. • Students use units to understand problems

between them provides grounding for work with expressions, equations, and functions. **M.A1HS.3** Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. **LINEAR AND EXPONENTIAL RELATIONSHIPS** **Extend the properties of exponents to rational exponents.** **M.A1HS.11** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. (e.g., We define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.) Instructional Note: Address this standard before discussing exponential functions with continuous domains. **M.A1HS.12** Rewrite expressions involving radicals and rational exponents using the properties of exponents. Instructional Note: Address this standard before	each new number system—whole numbers, rational numbers, and real numbers—the distributive law continues to hold, and the commutative and associative laws are still valid for both addition and multiplication. However, in Algebra I students go further along this path. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and	fluency, and application. Math TREE Online Education Resources A curated set of aligned, internet resources for WV middle and high school math teachers.	and make sense of the answers they deduce.

discussing exponential functions with continuous domains. **QUADRATIC FUNCTIONS AND MODELING** **Use properties of rational and irrational numbers.** **M.A1HS.50** Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. Instructional Note: Connect to physical situations (e.g., finding the perimeter of a square of area 2).	strengthen understandings throughout the school year.		

Algebra

Standards	Teacher Understandings	Resources	Student Understandings
RELATIONSHIPS BETWEEN QUANTITIES AND REASONING WITH EQUATIONS **Interpret the structure of expressions.** **M.A1HS.4** Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients.	In the Algebra conceptual category, students extend the work with expressions that they started in the middle-grades standards. They create and solve equations in context, utilizing the power of	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical	• Students continue to develop their skills in reading more complicated expressions with comprehension, such as those that involve multiple variables and exponents,

b. Interpret complicated expressions by viewing one or more of their parts as a single entity. (e.g., Interpret P(1 + r)n as the product of P and a factor not depending on P. Instructional Note: Limit to linear expressions and to exponential expressions with integer exponents. **Create equations that describe numbers or relationships.** **M.A1HS.5** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs. **M.A1HS.6** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Limit to linear and exponential equations, and, in the case of exponential equations,	variable expressions to model real-world problems and solve them with attention to units and the meaning of the answers they obtain. They continue to graph equations, understanding the resulting picture as a representation of the points satisfying the equation. This conceptual category accounts for a large portion of the Algebra I course and, along with the Functions category, represents the main body of content. The Algebra conceptual category in higher mathematics is very closely related to the Functions conceptual category (UA Progressions Documents 2013b, 2): • An expression in one variable can be viewed as defining a function: the act of evaluating the expression is an act of producing the function's output given the input.	Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. Math TREE Online Education Resources A curated set of aligned, internet resources for WV middle and high school math teachers. Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	through the analysis of their underlying structure. • Students factor second-degree polynomials by making use of such special forms and by using factoring techniques based on properties of operations. • Students purposefully transform expressions into equivalent forms that are suitable for the purpose at hand. • Students use different forms of the same expression to reveal important characteristics of the expression. • Students explore the set of polynomials in x as a system in its own right, subject to certain operations and properties. Students draw parallels between the set of integers—wherein integers can be added, subtracted, and multiplied according to certain properties—and

limit to situations requiring evaluation of exponential functions at integer inputs. **M.A1HS.7** Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: Limit to linear equations and inequalities. **M.A1HS.8** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Limit to formulas with a linear focus. **Understand solving equations as a process of reasoning and explain the reasoning.** **M.A1HS.9** Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a	• An equation in two variables can sometimes be viewed as defining a function, if one of the variables is designated as the input variable and the other as the output variable, and if there is just one output for each input. This is the case if the expression is of the form $y = (\text{expression in } x)$ or if it can be put into that form by solving for y. • The concept of equivalent expressions can be understood in terms of functions: if two expressions are equivalent, they define the same function. • The solutions to an equation in one variable can be understood as the input values that yield the same output in the two functions defined by the expressions on each side of the equation. This insight allows for the method of finding approximate solutions		the set of all polynomials with real coefficients. • Students add linear or quadratic polynomials and multiply linear polynomials to obtain quadratic polynomials. • Students create equations to solve problems, correctly graph the equations on coordinate axes, and interpret solutions in a modeling context. • Students solve equations *and use properties to explain* the steps as resulting from previous true equations. In this way, the idea of *proof*, while not explicitly named, is given a prominent role in the solving of equations, and the reasoning and justification process is not simply relegated to a future mathematics course. • Students solve linear equations and inequalities in one

solution method. Instructional Note: Students should focus on and master linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Algebra II. **Solve equations and inequalities in one variable.** **M.A1HS.10** Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Instructional Note: Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$. **Solve systems of equations.** **M.A1HS.13** Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	by graphing functions defined by each side and finding the points where the graphs intersect. Thus, in light of understanding functions, the main content of the Algebra category (solving equations, working with expressions, and so forth) has a very important purpose. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content		variable, including equations and inequalities with absolute values and equations with coefficients represented by letters. • Students learn various techniques for solving quadratic equations and the relationships between those techniques. • Students solve simple systems of equations consisting of a linear equation and a quadratic equation in two variables both algebraically and graphically. • Students extend their work with exponents to working with quadratic functions and equations that have real roots. To extend their understanding of these quadratic expressions and the functions they define, students investigate properties of quadratics and their

M.A1HS.14 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Instructional Note: Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to standards in Geometry which require students to prove the slope criteria for parallel lines. **Represent and solve equations and inequalities graphically.** **M.A1HS.15** Recognize that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Instructional Note: Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.	standards and to revisit and practice skills and strengthen understandings throughout the school year.		graphs in the Functions conceptual category. • Students develop an understanding of the relationship between the algebraic representation of an equation and its graph plotted in the coordinate plane and understand geometric interpretations of solutions to equations and inequalities. In Algebra I, students work only with linear, exponential, quadratic, step, piecewise, and absolute value functions.

M.A1HS.16 Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations). Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential and logarithmic functions. Instructional Note: Focus on cases where f(x) and g(x) are linear or exponential. **M.A1HS.17** Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes. **EXPRESSIONS AND EQUATIONS** **Interpret the structure of equations.** **M.A1HS.41** Interpret expressions that represent a quantity in terms of its context.			

a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret P(1 + r)n as the product of P and a factor not depending on P. Instructional Note: Exponents are extended from the integer exponents found in the unit on Relationships between Quantities and Reasoning with Equations to rational exponents focusing on those that represent square or cube roots. Instructional Note: Focus on quadratic and exponential expressions. **M.A1HS.42** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. Instructional Note: Focus on quadratic and exponential expressions. **Write expressions in equivalent forms to solve problems.** **M.A1HS.43** Choose and produce an equivalent form of an expression to reveal and			

explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. Instructional Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal. **Perform arithmetic operations on polynomials.** **M.A1HS.44** Recognize that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction,			

and multiplication; add, subtract, and multiply polynomials. Instructional Note: Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x. **Create equations that describe numbers or relationships.** **M.A1HS.45** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Extend work on linear and exponential equations in the Relationships between Quantities and Reasoning with Equations unit to quadratic equations. **M.A1HS.46** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Extend work on linear and exponential equations in the Relationships between Quantities and Reasoning with Equations unit to quadratic equations.			

M.A1HS.47 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R. Instructional Note: Extend work on linear and exponential equations in the Relationships between Quantities and Reasoning with Equations unit to quadratic equations. Extend this standard to formulas involving squared variables. **Solve equations and inequalities in one variable.** **M.A1HS.48** Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions			

and write them as a ± bi for real numbers a and b. Instructional Note: Students should learn of the existence of the complex number system, but will not solve quadratics with complex solutions until Algebra II.			

Statistics and Probability

Standards	Teacher Understandings	Resources	Student Understandings
DESCRIPTIVE STATISTICS **Summarize, represent, and interpret data on a single count or measurement variable.** **M.A1HS.33** Represent data with plots on the real number line (dot plots, histograms, and box plots). **M.A1HS.34** Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Instructional Note: In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of	In Algebra I, students build on their understanding of key ideas for describing distributions—shape, center, and spread—presented in the standards for grades six through eight. This enhanced understanding allows them to give more precise answers to deeper questions, often involving comparisons of data sets. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application.	• Students use shape and the questions(s) to be answered to decide on the median or mean as the more appropriate measure of center and to justify their choice through statistical reasoning. • Students take a deeper look at bivariate data, using their knowledge of proportions to describe categorical associations and using their knowledge of functions to fit models to quantitative data.

the data distribution, such as the shape of the distribution or the existence of extreme data points. **M.A1HS.35** Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Instructional Note: In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points. **Summarize, represent, and interpret data on two categorical and quantitative variables.** **M.A1HS.36** Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data. **M.A1HS.37** Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.	grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.	Math TREE Online Education Resources A curated set of aligned, internet resources for WV middle and high school math teachers. Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	• Students extend their knowledge of scatter plots to fit mathematical models that capture key elements of the relationship between two variables and to explain what the model tells us about the relationship. • Students examine *residuals* to reveal additional information about the behavior of the data. • Students compute *correlation coefficients* and interpret the value of the coefficient. Students see situations where correlation and causation are mistakenly interchanged, and they are not careful to closely examine the story that data and computed statistics are trying to tell.

a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. Instructional Note: Focus should be on situations for which linear models are appropriate. c. Fit a linear function for scatter plots that suggest a linear association. Instructional Note: Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals. **Interpret linear models.** **M.A1HS.38** Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Instructional Note: Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the			

correlation coefficient as a measure of how well the data fit the relationship. **M.A1HS.39** Compute (using technology) and interpret the correlation coefficient of a linear fit. Instructional Note: Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship. **M.A1HS.40** Distinguish between correlation and causation. Instructional Note: The important distinction between a statistical relationship and a cause-and-effect relationship is the focus.			

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