West Virginia math4life

Frameworks for Mathematics

Algebra II

West Virginia DEPARTMENT OF EDUCATION

Algebra II

In Algebra I, students use reasoning about structure to define and make sense of rational exponents and explore the algebraic structure of the rational and real number systems. They understand that numbers in real-world applications often have units attached to them—that is, the numbers are considered *quantities*. Students' work with numbers and operations throughout elementary and middle school has led them to an understanding of the structure of the number system; in Algebra I, students explore the structure of algebraic expressions and polynomials. They see that certain properties must persist when they work with expressions that are meant to represent numbers—which they now write in an abstract form involving variables. When two expressions with overlapping domains are set as equal to each other, resulting in an equation, there is an implied solution set (be it empty or non-empty), and students not only refine their techniques for solving equations and finding the solution set, but they can clearly explain the algebraic steps they used to do so.

Students begin their exploration of linear equations in middle school, first by connecting proportional equations ($y = kx$, $k \neq 0$) to graphs, tables, and real-world contexts, and then move toward an understanding of general linear equations ($y = mx + b$, $m \neq 0$) and their graphs. In Algebra I, students extend this knowledge to work with absolute value equations, linear inequalities, and systems of linear equations. After learning a more precise definition of *function* in this course, students examine this new idea in the familiar context of linear equations—for example, by seeing the solution of a linear equation as solving $f(x) = g(x)$ for two linear functions f and g.

Students continue to build their understanding of functions beyond linear ones by investigating tables, graphs, and equations that build on previous understandings of numbers and expressions. They make connections between different representations of the same function. They also learn to build functions in a modeling context and solve problems related to the resulting functions. Note that in Algebra I the focus is on linear, simple exponential, and quadratic equations.

Finally, students extend their prior experiences with data, using more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, students look at residuals to analyze the goodness of fit.

Modeling

Standards	Teacher Understandings	Resources	Student Understandings
Although the Modeling category does not include specific standards, the idea of using mathematics to model the world pervades all higher mathematics courses and should hold a significant place in instruction. Modeling is addressed first here to emphasize its importance in the higher mathematics curriculum.	Modeling at the higher mathematics level goes beyond the simple application of previously constructed mathematics and includes real-world problems. True modeling begins with students asking a question about the world around them, and mathematics is then constructed in the process of attempting to answer the question. Students may see when trying to answer their question that solving an equation arises as a necessity and that the equation often involves the specific instance of knowing the output value of a function at an unknown input value. Modeling problems have an element of being genuine problems, in the sense that students care about answering the question	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. It highlights some necessary foundational skills from previous grade levels. Math TREE Online Education Resources A curated set of aligned, internet	• When students are presented with a real-world situation and challenged to ask a question, all sorts of new issues arise (e.g., Which of the quantities present in this situation are known, and which are unknown?). • Students decide on a solution path that may need to be revised. They make use of tools such as calculators, dynamic geometry software, and spreadsheets. They try to use previously derived models (e.g., linear functions), but may find that a new formula or function will apply.

	under consideration. In modeling, mathematics is used as a tool to answer questions that students really want answered. Students examine a problem and formulate a *mathematical model* (an equation, table, graph, and so forth), compute an answer or rewrite their expression to reveal new information, interpret and validate the results, and report out. This is a new approach for many teachers and may be challenging to implement, but the effort should show students that mathematics is relevant to their lives. From a pedagogical perspective, modeling gives a concrete basis from which to abstract the mathematics and often serves to motivate students to become independent learners. The important ideas surrounding rational functions, graphing, solving equations, and rates of change should be explored	resources for WV middle and high school math teachers. Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	

	through the lens of mathematical modeling. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.		

Functions

Standards	Teacher Understandings	Resources	Student Understandings
TRIGONOMETRIC FUNCTIONS **Extend the domain of trigonometric**	Work on functions began in Algebra I. In Algebra II,	**Educators' Guide** Organized by	• Students work with functions that model

functions using the unit circle. **M.A2HS.19** Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **M.A2HS.20** Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **Model periodic phenomena with trigonometric functions.** **M.A2HS.21** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **Prove and apply trigonometric identities.** **M.A2HS.22** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle. Instructional Note: An Algebra II course with an additional focus on trigonometry could include the standard "Prove the addition and	students encounter more sophisticated functions, such as polynomial functions of degree greater than 2, exponential functions having all real numbers as the domain, logarithmic functions, and extended trigonometric functions and their inverses. Several standards of the Functions category are repeated here, illustrating that the standards attempt to reach depth of understanding of the *concept* of a function. As stated in the University of Arizona (UA) For instance, students in Algebra II see quadratic, polynomial, and rational functions as belonging to the same system. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in	conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. Math TREE Online Education Resources A curated set of aligned, internet resources for WV middle and high school math teachers. Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate	data and choose an appropriate model function by considering the context that produced the data. • Students develop a more sophisticated ability to recognize rates of change, growth and decay, end behavior, roots, and other characteristics of functions. • Students use their expanding repertoire of families of functions to inform their choices for models. Students know how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. • Students use graphs to reason about rates of change of functions. • Students develop models for more complex situations than in previous courses, due to the expansion of the

subtraction formulas for sine, cosine, and tangent and use them to solve problems." This could be limited to acute angles in Algebra II. **MODELING WITH FUNCTIONS** **Interpret functions that arise in applications in terms of a context.** **M.A2HS.27** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Emphasize the selection of a model function based on behavior of data and context. **M.A2HS.28** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then	students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.	resources that can support instruction and identify skills most relevant to standards.	types of functions available to them. Modeling contexts provide a natural place for students to start building functions with simpler functions as components. • Students understand the concept of a *family of functions* and characterize such function families based on their properties. Students further explore these ideas with trigonometric functions. • Students worked with exponential models in Algebra I and continue this work in Algebra II. Students deduce that this function has an inverse, called the *logarithm to the base b*. • Students explore the properties of logarithms, connect these properties to those of exponents. • Students solve problems involving exponential functions and logarithms

the positive integers would be an appropriate domain for the function.) Note: Emphasize the selection of a model function based on behavior of data and context. **M.A2HS.29** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Note: Emphasize the selection of a model function based on behavior of data and context. **Analyze functions using different representations.** **M.A2HS.30** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. b. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. Instructional Note: Focus on applications and how key features			and express their answers using logarithm notation. • Students understand logarithms as functions that undo their corresponding exponential functions. • Students expand their understanding of the trigonometric functions first developed in Geometry. The graphs of the trigonometric functions are explored with attention to the connection between the unit-circle representation of the trigonometric functions and their properties—for example, to illustrate the periodicity of the functions, the relationship between the maximums and minimums of the sine and cosine graphs, zeros, and so forth. • Students use trigonometric functions to model periodic

relate to characteristics of a situation, making selection of a particular type of function model appropriate. **M.A2HS.31** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. **M.A2HS.32** Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. **Build a function that models a relationship between two quantities.** **M.A2HS.33** Write a function that describes a relationship between two quantities.			phenomena.

Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.) Instructional Note: Develop models for more complex or sophisticated situations than in previous courses. **Build new functions from existing functions.** **M.A2HS.34** Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Use transformations of functions to find models as students consider increasingly more complex situations. Observe the effect of multiple transformations on a single graph and the common effect of each transformation across function types.			

M.A2HS.35 Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. (e.g., $f(x) = 2x^3$ or f(x) = (x+1)/(x-1) for x ≠ 1.) Instructional Note: Use transformations of functions to find models as students consider increasingly more complex situations. Extend this standard to simple rational, simple radical, and simple exponential functions; connect this standard to M.A2HS.34. **Construct and compare linear, quadratic, and exponential models and solve problems.** **M.A2HS.36** For exponential models, express as a logarithm the solution to a bct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. Instructional Note: Consider extending this unit to include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that log xy = log x +log y.			

Number and Quantity

Standards	Teacher Understandings	Resources	Student Understandings
POLYNOMIAL, RATIONAL, AND RADICAL RELATIONSHIPS **Perform arithmetic operations with complex numbers.** **M.A2HS.1** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real. **M.A2HS.2** Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers. **Use complex numbers in polynomial identities and equations.** **M.A2HS.3** Solve quadratic equations with real coefficients that have complex solutions. Instructional Note: Limit to polynomials with real coefficients. **M.A2HS.4** Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$. Instructional Note: Limit to polynomials with real coefficients.	In Algebra I, students worked with examples of quadratic functions and solved quadratic equations, encountering situations in which a resulting equation did not have a solution that is a real number. In Algebra II, students complete their extension of the concept of *number* to include complex numbers. Students begin to work with complex numbers and apply their understanding of properties of operations and exponents and radicals to solve equations which do not have a solution that is a real number. They also apply their understanding of properties of operations and exponents and radicals to solve equations. The extended number system forms yet another system that behaves according to familiar rules	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. Math TREE Online Education Resources A curated set of aligned, internet resources for WV middle and high school math teachers.	• Students work with examples of quadratic functions and solve quadratic equations, encountering situations in which a resulting equation does not have a solution that is a real number. • Students expand their extension of the concept of *number* to include complex numbers so that an equation that does not have a solution that is a real number can be solved. • By exploring polynomials that can be factored with real and complex roots, students develop an understanding of the Fundamental Theorem of Algebra.

M.A2HS.5 Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Instructional Note: Limit to polynomials with real coefficients.	and properties. By exploring examples of polynomials that can be factored with real and complex roots, students develop an understanding of the Fundamental Theorem of Algebra; they can show that the theorem is true for quadratic polynomials by an application of the quadratic formula and an understanding of the relationship between roots of a quadratic equation and the linear factors of the quadratic polynomial. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content		

	plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.		

Algebra

Standards	Teacher Understandings	Resources	Student Understandings
Perform arithmetic operations on polynomials. **M.A2HS.9** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Instructional Note: Extend beyond the quadratic polynomials found in Algebra I. **Understand the relationship between zeros and factors of polynomials.** **M.A2HS.10** Know and apply the Remainder	Along with the standards in the number and quantity conceptual category, the Algebra conceptual category standards develop the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and	• Students continue to pay attention to the meaning of expressions in context and interpret the parts of an expression by "chunking," that is, viewing parts of an expression as a single entity. Students' facility in using special cases of polynomial factoring allows them to fully factor more complicated polynomials. • Students examine the sums of examples of

Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). **M.A2HS.11** Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial. **Use polynomial identities to solve problems.** **M.A2HS.12** Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. Instructional Note: This cluster has many possibilities for optional enrichment, such as relating the example in M.A2HS.10 to the solution of the system $u^2 + v^2 = 1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x + y)^{n+1} = (x + y)(x + y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction. **M.A2HS.13** Know and apply the Binomial Theorem	multiplication of polynomials with multiplication of multi-digit integers and division of polynomials with long division of integers. Rational numbers extend the arithmetic of integers by allowing division by all numbers except zero; similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of this section is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will	fluency, and application. [Math TREE Online Education Resources] A curated set of aligned, internet resources for WV middle and high school math teachers. [Quantile Teacher Assistant] This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	finite geometric series and look for patterns to justify why the equation for the sum holds. • Students continue to develop their understanding of the set of polynomials as a system analogous to the set of integers that exhibits particular properties, and they explore the relationship between the factorization of polynomials and the roots of a polynomial. • Students use the zeros of a polynomial to create a rough sketch of its graph and connect the results to their understanding of polynomials as functions. • Students explore rational functions as a system analogous to that of rational numbers and see rational functions as useful for describing many real-world situations. • Students work with all available types of

for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.[2] Instructional Note: This cluster has many possibilities for optional enrichment, such as relating the example in M.A2HS.10 to the solution of the system $u^2 + v^2 = 1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x + y)^{n+1} = (x + y)(x + y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction. **Rewrite rational expressions.** **M.A2HS.14** Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. Instructional Note: The limitations on rational functions apply to the rational expressions. **M.A2HS.15** Understand that rational expressions form a system analogous to the	leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.		functions to create equations. Although the functions will often be linear, exponential, or quadratic, the types of problems draw from more complex situations. • Students extend their equation-solving skills to those involving rational expressions and radical equations and make sense of extraneous solutions that may arise. Students also understand that some equations can be solved only approximately with the tools they possess.

rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. Instructional Note: This standard requires the general division algorithm for polynomials. **Understand solving equations as a process of reasoning and explain the reasoning.** **M.A2HS.16** Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. Instructional Note: Extend to simple rational and radical equations. **Represent and solve equations and inequalities graphically.** **M.A2HS.17** Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Instructional Note: Include cases where f(x) and/or g(x) are linear,			

polynomial, rational, absolute value, exponential, and logarithmic functions. Instructional Note: Include combinations of linear, polynomial, rational, radical, absolute value, and exponential functions. **MODELING WITH FUNCTIONS** **Create equations that describe numbers or relationships.** **M.A2HS.23** Create equations and inequalities in one variable and use them to solve problems. Instructional Note: Include equations arising from linear and quadratic functions, and simple rational and exponential functions. **M.A2HS.24** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: While functions will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. (e.g., Finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line).			

M.A2HS.25 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: While functions will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line. **M.A2HS.26** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) While functions will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line			

allows one to find the distance from a point to a line. This example applies to earlier instances of this standard, not to the current course.			

Geometry

No traditional Algebra II course would be complete without an examination of planar curves represented by the general equation $ax^2 + by^2 + cx + dy + e = 0$. In Algebra II, students use "completing the square" (a skill learned in Algebra I) to decide if the equation represents a circle or parabola. They graph the shapes and relate the graph to the equation. The study of ellipses and hyperbolas is reserved for a later course.

Statistics and Probability

Standards	Teacher Understandings	Resources	Student Understandings
INFERENCES AND CONCLUSIONS FROM DATA **Summarize, represent, and interpret data on a single count or measurement variable.** **M.A2HS.37** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such	Students in Algebra II move beyond analysis of data to make sound statistical decisions based on probability models. The reasoning process is as follows: develop a statistical question in the form of a hypothesis (supposition) about a population parameter, choose a probability model for	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the	• Students build on their understanding of data distributions to help see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). • Students revisit prior courses different ways of collecting data and using

a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. Instructional Note: While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution. **Understand and evaluate random processes underlying statistical experiments.** **M.A2HS.38** Understand statistics as a process for making inferences about population parameters based on a random sample from that population. Instructional Note: Include comparing theoretical and empirical results to evaluate the effectiveness of a treatment. **M.A2HS.39** Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. (e.g., A model says a	collecting data relevant to that parameter, collect data, and compare the results seen in the data with what is expected under the hypothesis. If the observed results are far from what is expected and have a low probability of occurring under the hypothesis, then that hypothesis is called into question. In other words, the evidence against the hypothesis is weighed by probability. By investigating simple examples of simulations of experiments and observing outcomes of the data, students gain an understanding of what it means for a model to fit a particular data set. This includes comparing theoretical and empirical results to evaluate the effectiveness of a treatment. **Content by Cluster** Teachers must provide students opportunity to master each content	importance of developing conceptual understanding, procedural skill and fluency, and application. Math TREE Online Education Resources A curated set of aligned, internet resources for WV middle and high school math teachers. Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	graphical displays and summary statistics with a focus on how the way in which data are collected determines the scope and nature of the conclusions that can be drawn from those data. • Students informally develop an understanding of the concept of *statistical significance* through simulation as meaning a result that is unlikely to have occurred solely through random selection in sampling or random assignment in an experiment. • Students' ability to apply probability models to make and analyze decisions is extended to more complex probability models, including situations such as those involving quality control or diagnostic test that yield both false-positive and false-negative results.

spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?) **Make inferences and justify conclusions from sample surveys, experiments, and observational studies.** **M.A2HS.40** Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment. **M.A2HS.41** Use data from a sample survey to	standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.		

estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment. Focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness. **M.A2HS.42** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and			

use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment. Focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness. **M.A2HS.43** Evaluate reports based on data. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to			

have occurred solely as a result of random selection in sampling or random assignment in an experiment. **Use probability to evaluate outcomes of decisions.** **M.A2HS.44** Use probabilities to make fair decisions (e.g., drawing by lots or using a random number generator). Instructional Note: Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results. **M.A2HS.45** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game). Instructional Note: Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results.			

Steven L. Paine, Ed.D.
West Virginia Superintendent of Schools