West Virginia math4life

Educators' Guide for Mathematics

Kindergarten

West Virginia DEPARTMENT OF EDUCATION

Kindergarten

Students in preschool and transitional kindergarten programs who have been exposed to important mathematical concepts—such as representing and comparing whole numbers, recognizing adding/removing objects as adding/subtracting and identifying and describing shapes—will be better prepared for kindergarten mathematics and for later learning.

Mathematics Instruction

In kindergarten, instructional time focuses on two critical areas: (1) representing and comparing whole numbers, initially with sets of objects; and (2) describing shapes and space. More learning time in kindergarten is devoted to developing and understanding of numbers and an understanding of addition and subtraction. Kindergarten students work toward fluency with addition and subtraction of whole numbers within 5.

West Virginia College- and Career-Readiness Standards for Mathematics

The West Virginia College- and Career-Readiness Standards for Mathematics (WVBE Policy 2520.2B) emphasize key content, skills, and practices at each grade level and support three major principles:

- Instruction is focused on grade-level standards.
- Instruction is attentive to learning across grades and to linking major topics within grades.
- Instruction develops conceptual understanding, procedural skill and fluency, and application.

Grade-level examples of these three major principles are indicated throughout this document.

Cluster headings can be viewed as the most effective way to communicate the focus and coherence of the standards. The instructional focus must be based on the depth of the ideas, the time needed to master the clusters, and their importance to future mathematics or the later demands of preparing for college and careers.

Teachers and administrators understand that the standards are not topics to be checked off after being covered in isolated units of instruction; rather, they provide content to be developed throughout the school year through rich instructional experiences presented in a coherent manner. West Virginia College- and Career-Readiness Standards for Mathematics are learning goals for students that must be mastered by the end of the kindergarten academic year in order for students to be prepared for the mathematics content at the first grade level.

Mathematical Fluency

Students demonstrate fluency of mathematical standards when they exhibit the following:

- Accuracy -- ability to produce an accurate answer
- Efficiency -- ability to choose an appropriate, expedient strategy for a specific computation problem
- Flexibility -- ability to use number relationships with ease in computation.

Connecting Mathematical Habits of Mind and Content

The Mathematical Habits of Mind (MHM) are developed throughout each grade and, together with the content standards, prescribe that students experience mathematics as a rigorous, coherent, useful, and logical subject. The Mathematical Habits of Mind represent a picture of what it looks like for students to understand and do mathematics in the classroom and should be integrated into every mathematics lesson for all students.

Although the description of the Mathematical Habits of Mind remains the same at all grade levels, the way these standards look as students engage with and master new and more advanced mathematical ideas does change. The following table presents examples of how the Mathematical Habits of Mind standards may be integrated into tasks appropriate for students in kindergarten.

Mathematical Habits of Mind—Explanation and Examples for Kindergarten

Mathematical Habits of Mind	Explanation and Examples
MHM1 Make sense of problems and persevere in solving them.	In kindergarten, students begin to build the understanding that doing mathematics involves solving problems and discussing how they solved them. Students explain to themselves the meaning of a problem and look for ways to solve it. Real-life experiences are used to support students' ability to connect mathematics to the world. To help students connect the language of mathematics to everyday life, ask students questions such as "How many students are absent?" or have them gather enough scissors for the students at their table. Students use concrete objects or pictures to help them conceptualize and solve problems. They may check their thinking by asking themselves, "Does this make sense?", or they may try another strategy.
MHM2 Reason abstractly and quantitatively.	Students begin to recognize that a number represents a specific quantity and connect the quantity to written symbols. Quantitative reasoning entails creating a representation of a problem while attending to the meanings of the quantities. For example, a student may write the numeral 11 to represent an amount of objects counted, select the correct number card 17 to follow 16 on a calendar, or build two piles of counters to compare the numbers 5 and 8. In addition, kindergarten students begin to draw pictures, manipulate objects, or use diagrams or charts to express quantitative ideas. Students need to be encouraged to answer questions such as "How do you know?"—which reinforces their reasoning and understanding and helps student develop mathematical language.
MHM3 Construct viable arguments and critique the reasoning of others.	Students construct arguments using actions and concrete materials, such as objects, pictures, or drawings. They begin to develop their mathematical communication skills as they participate in mathematical discussions involving questions such as "How did you get that?" and "Why is that true?" They explain their thinking to others and respond to others' thinking. They begin to develop the ability to reason and analyze situations as they consider questions such as "Are you sure that _________?", "Do you think that would happen all the time?", and "I wonder why _________?"

MHM4 Model with mathematics.	Students begin to represent problem situations in multiple ways—by using numbers, objects, words, or mathematical language, acting out the situation, making a chart or list, drawing pictures, creating equations, and so forth. For example, a student may use cubes or tiles to show the different number partners for 5, or place three objects on a 10-frame and then determine how many more are needed to "make a ten." Students rely on manipulatives (or other visual and concrete representations) while solving tasks and record an answer with a drawing or equation.
MHM5 Use appropriate tools strategically.	Students begin to consider tools available to them when solving a mathematical problem and decide when certain tools might be helpful. For instance, kindergartners may decide to use linking cubes to represent two quantities and then compare the two representations side by side, or later, make math drawings of the quantities. Students decide which tools may be helpful to use depending on the problem or task and explain why they use particular mathematical tools.
MHM6 Attend to precision.	Students begin to develop precise communication skills, calculations, and measurements. Students describe their own actions, strategies, and reasoning using grade-level-appropriate vocabulary. Opportunities to work with pictorial representations and concrete objects help students develop understanding and descriptive vocabulary. For example, students analyze and compare two- and three-dimensional shapes and sort objects based on appearance. While measuring objects, students make sure objects are directly beside each other. During tasks involving number sense, students check their work to ensure the accuracy and reasonableness of solutions. Students are encouraged to answer questions such as, "How do you know your answer is reasonable?"
MHM7 Look for and make use of structure.	Students begin to discern a pattern or structure in the number system. For instance, students recognize that 3 + 2 = 5 and 2 + 3 = 5. Students use counting strategies, such as counting on, counting all, or taking away, to build fluency with facts to 5. Students notice the written pattern in the "teen" numbers—that the numbers start with 1 (representing 1 ten) and end with the number of additional ones. Teachers might ask, "What do you notice when _________?"

MHM8 Look for and express regularity in repeated reasoning.	Students notice repetitive actions in counting, computations, and mathematical tasks. For example, the next number in a counting sequence is 1 more when counting by ones and 10 more when counting by tens (or 1 more group of 10). Students are encouraged to answer questions such as, "What would happen if _________?" and "There are 8 crayons in the box. Some are red and some are blue. How many of each could there be?" Kindergarten students realize 8 crayons could include 4 of each color (8 = 4 + 4), 5 of one color and 3 of another (8 = 5 + 3), and so on. For each solution, students repeatedly engage in the process of finding two numbers to join together to equal 8.

Adapted from Arizona Department of Education (ADE) 2010 and North Carolina Department of Public Instruction (NCDPI) 2013b.

Standards-Based Learning at Kindergarten

The following narrative is organized by the domains in the West Virginia College- and Career-Readiness Standards for Mathematics. It highlights some necessary foundational skills and provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind (**MHM**), and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application.

Domain: Counting and Cardinality

A critical area of instruction in kindergarten is counting, representing, and comparing numbers.

Counting and Cardinality
Know number names and the count sequence.
M.K.1 Count to 100 by ones and by tens.
M.K.2 Count forward beginning from a given number within the known sequence (instead of having to begin at 1).
M.K.3 Write numbers from 0 to 20. Represent a number of objects with a written numeral 0-20 (with 0 representing a count of no objects).

Several learning progressions originate in knowing number names and the count sequence. One of the first major concepts in a student's mathematical development is *cardinality*. Cardinality can be explained as knowing that the number word spoken tells the quantity and that the number on which a person ends when counting represents the entire amount counted. The idea is that numbers mean *amount*, and no matter how you arrange and rearrange the items, the amount is the same. Students can generally say the counting words up to a given number before they can use these numbers to count objects or to tell the number of objects.

Kindergarten students are introduced to the counting sequence (**M.K.1–M.K.2**). When counting orally by ones, students begin to understand that the next number in the sequence is one more. Similarly, when counting by tens, the next number in the sequence is "10 more."

Examples: Counting Sequences for Forward Counting to 100 by Ones	**M.K.1**
• The "ones" (1–10) • The "teens" (10, 11, 12, 13, 14, 15, 16, 17, 18, 19) • "Crossing the decade" (15, 16, 17, 18, 19, 20, 21, 22, 23, 24, or, similarly, 26–34, 35–44, and so forth) Students often have trouble with counting forward sequences through the teen numbers and numbers that cross the decade. Focusing on short counting sequences may be helpful. Adapted from Kansas Association of Teachers of Mathematics (KATM) 2012, Kindergarten Flipbook.	

Initially students will think of counting as a string of words; they repeat the number sequence, but they are not able to stop at a given number. For example, when students are directed to count to 17, they will begin counting, but they keep counting when they come to 17. They are unable to stop. Teachers may have students practice counting to a given number. The teacher listens to hear if students stop at the given number. This activity may be like a game, so students can practice counting to a given number.

A kindergarten teacher should assess students' ability to say the number sequence through ten. This is the first stepping stone toward fluency in mathematics. If a student is unable to say the number sequence to ten, the teacher should provide multiple opportunities to hear the number sequence. Gradually students transition to using counting as a tool to describe amounts in their world. Counting can be reinforced throughout the school day.

Examples	**M.K.1**
• Count the number of chairs of students who are absent. • Count the number of stairs, shoes, and so forth. • Count groups of 10, such as "fingers in the classroom" (10 fingers per student) [**MHM6, MHM7, MHM8**].	

Kindergarten students also count forward—beginning from a given number—instead of starting at 1. Counting forward (or "counting on") may be confusing for young students, because it conflicts with the initial strategy they learned about counting from the beginning. Activities or games that require students to add on to a previous count to reach a targeted number may encourage development of this concept (adapted from KATM 2012, Kindergarten Flipbook).

Kindergarten students learn to write numbers from 0 to 20 (**M.K.3**) and represent a number of objects with a written numeral in the 0–20 range (using numerals as symbols for quantities). They understand that 0 represents a count of no objects. Students need multiple opportunities to count objects and recognize that a number represents a specific quantity. As this understanding develops, students begin to read and write numerals. The emphasis is placed first on quantity and then on connecting quantities to the written symbols.

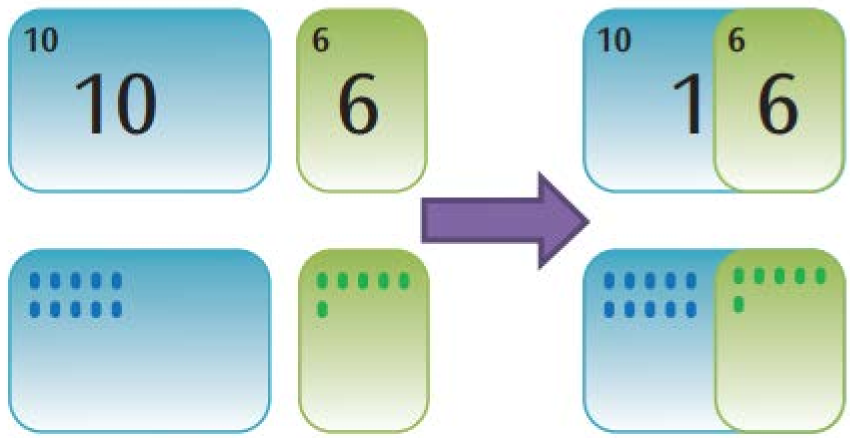
Example: A Learning Sequence for Understanding Numbers
A specific learning sequence might consist of these steps: 1. Count up to 20 objects in many settings and situations over several weeks. 2. Start to recognize, identify, and read the written numerals, and match the numerals to given sets of objects. 3. Write the numerals to represent counted objects. Adapted from ADE 2010.

As students connect quantities and written numerals, they also develop mathematical practices such as reasoning abstractly and quantitatively (**MHM2**). They use precise vocabulary to express how they know that their count is accurate (**MHM6**). They also use the structures and patterns of the number system and apply this understanding to counting (**MHM7, MHM8**) [adapted from ADE 2010].

Common Misconceptions

- Some students might not see zero (0) as a number. Ask students to write 0 and say "zero" to represent the number of items left when all items have been taken away. Avoid using the word none to represent this situation. Counting numbers begin with the number 1 and are used to count objects. The number 0 is a member of the set of whole numbers. Counting numbers are a subset of the set of whole numbers.
- Teen numbers can also be confusing for young students. To help avoid confusion, these numbers are taught as a bundle of 10 ones and some extra ones. This approach supports a foundation for understanding both the place-value concept and symbols that represent each teen number. Layered place-value cards may help students understand the difficult teen numbers; see figure K-1.

Figure K-1. Layering Place-Value Cards to Illustrate Teen Numbers



Adapted from KATM 2012, Kindergarten Flipbook.

Counting and Cardinality

Count to tell the number of objects.

M.K.4

Understand the relationship between numbers and quantities; connect counting to cardinality.

a. When counting objects, say the number names in the standard order, pairing each object with one and only one number name and each number name with one and only one object.
b. Understand that the last number name said tells the number of objects counted and the number of objects is the same regardless of their arrangement or the order in which they were counted.
c. Understand that each successive number name refers to a quantity that is one larger.

M.K.5

Count to answer questions (e.g., “How many?”) about as many as 20 things arranged in a line, a rectangular array, a circle, or as many as 10 things in a scattered configuration; given a number from 1–20, count out that many objects.

In kindergarten, students develop an understanding of the relationship between numbers and quantities and connect counting to cardinality (**M.K.4**). Learning to count is a complex mental and physical activity that requires staying connected to the objects that are being counted. Children must understand that the count sequence has meaning when counting objects: that the last count word indicates the amount or the cardinality of the set (Van de Walle 2007). Kindergarten students use their understanding of the relationship between numbers and quantities to count a set of objects and see sets and numerals in relationship to one another, rather than as isolated concepts.

There are numerous opportunities for students to manipulate concrete objects or visual representations (e.g., dot cards, 10-frames) and connect number names with their quantities, which can help students master the concept of counting (adapted from NCDPI 2013b).

As students learn to count a group of objects, they first must be able to rote count (the standard-order principle). Next, then learn to pair each word said with one object (**M.K.4a**). This is usually facilitated by an indicating act (such as touching, pointing to, or moving objects) that keeps each word said paired to only one object (the one-to-one-correspondence principle). Students learn that the last number named tells the number of objects counted (the cardinality principle), that the number of objects is the same regardless of their arrangement or the order in which they were counted (the order-irrelevance principle), and that the above principles apply to any collection, whether tangible or not, and that the objects counted may have similar or different attributes (the abstraction principle). They also understand that each successive number name refers to a quantity that is 1 larger (**M.K.4b–c**) [adapted from UA Progressions Documents 2011a].

To develop their understanding of the relationship between numbers and quantities, students might count objects, placing one more object in the group at a time.

Example	**M.K.4**
Using cubes, students count an existing group and then place another cube in the set to continue counting. Students continue placing one more cube in the set at a time and then identify the new total number of cubes. Students see that the counting sequence results in a quantity that increases by one each time another cube is placed in the group. Students may need to recount from one, but the goal is for students to count on from the existing number of cubes—a conceptual start for the grade-one skill of counting to 120, starting at any number less than 120.	

To count accurately, students rely on:

- Knowing patterns and arbitrary parts of the number–word sequence;

- Assigning one number word to one object (one-to-one correspondence);
- Keeping track of objects that have already been counted (adapted from ADE 2010 and GaDOE 2011).

Students answer questions such as “How many are there?” by counting objects in a set and understanding that the last number stated represents the total amount of objects (cardinality, **M.K.5**). Over time, students realize that the same set counted several different times will be the same amount each time. Counting objects arranged in a line is easiest; with more practice, students learn to count objects in more difficult arrangements, such as rectangular arrays, circles, and scattered configurations.

Scattered arrangements are the most challenging for students; therefore, kindergarten students count only up to 10 objects if arranged this way. Given a number from 1 to 20, kindergarten students also count out that many objects. This is also more difficult for students than simply counting the total number of objects because, as students count, they need to remember the number of objects to be counted out (adapted from UA Progressions Documents 2011a and NCDPI 2013b).

Examples of Counting Strategies	**M.K.4a–b**
There are numerous counting strategies that students may use, depending on how objects are arranged. Here are a few examples: • Move objects as each object is counted. • Line up objects to count. • Touch objects in a scattered arrangement as each object is counted. • Count objects in a scattered arrangement by visually scanning each object without touching. Adapted from KATM 2012, Kindergarten Flipbook.	

Instructional Focus
As students use various counting strategies when they participate in counting activities, they reinforce their understanding of the relationship between numbers and quantities and support mathematical practices such as modeling with mathematics (**MHM4**), the use of precise language (**MHM6**), and repeated reasoning to find a solution (**MHM8**).

Students quickly come to perceive the number of items in small groups, such as recognizing dot arrangements in different patterns without counting the objects. This is known as *perceptual subitizing*, a fundamental skill in the development of students' understanding of numbers. Perceptual subitizing develops into *conceptual subitizing*—recognizing a collection of objects as a composite of subparts and as a whole (e.g., seeing a five-dot domino and thinking 1 and 4 or seeing a set with two subsets of 2 and saying 4) [adapted from UA Progressions Documents 2011a]. Particularly important is the 5 + n pattern, in which one row of 5 circles has 1, 2, 3, 4, or 5 dots below to show 6, 7, 8, 9, and 10; see figure K-2. These rows are separated more than the individual dots to ensure students see the group of 5 and the extra dots.

Figure K-2. The 5+*n* pattern

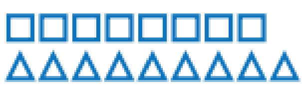
Source: UA Progressions Documents 2011a.

Subitizing supports the development of addition and subtraction strategies, such as counting on and composing and decomposing numbers. Students need practice to develop competency in perceptual subitizing.

Example	**M.K.5**
The teacher might place different amounts of beans on a mat (beginning with amounts of 4 or fewer) and then ask students to say how many beans they see. As students become proficient, dot cards can also be utilized to develop fluency. For example, the teacher can show a large dot card to students, and students then take the number counters they think they need to cover the dots on the card. Then one child places his or her counters on the dots while the rest of the class counts and checks. Eventually, the teacher briefly shows one large dot card and puts it down quickly. Then students try to recognize the number of dots without counting.	

Counting and Cardinality
Compare Numbers.
M.K.6 Identify whether the number of objects in one group is greater than, less than, or equal to the number of objects in another group (e.g., by using matching and counting strategies). **M.K.7** Compare two numbers between 1 and 10 presented as written numerals.

In kindergarten, students compare the number of objects in one group (with up to 10 objects) to the number of objects in another group (**M.K.6**). Students need a strong sense of the relationship between quantities and numerals to compare groups and answer related questions accurately. They may use matching strategies or counting strategies to determine whether one group is greater than, less than, or equal to the number of objects in another group.

Example: More Triangles or More Squares?		**M.K.6 (MHM1, MHM2)**
Student 1 I lined up 1 square with 1 triangle. Since there is 1 extra triangle, there are more triangles than squares. 	**Student 2** I counted the squares and got 8. Then I counted the triangles and got 9. Since 9 is bigger than 8, there are more triangles than squares.	**Student 3** I put them in a pile. I then took away objects. Every time I took a square, I also took a triangle. When I had taken almost all of the shapes away, there was still a triangle left. That means that there are more triangles than squares.

Adapted from KATM 2012, Kindergarten Flipbook.

Matching and Counting Strategies for Comparing Groups of Objects
• **Matching.** Students use one-to-one correspondence, repeatedly matching one object from one set with one object from the other set to determine which set has more objects. • **Counting.** Students count the objects in each set and then identify which set has more, fewer, or an equal number of objects.

- **Observation.** Students may use observations to compare two quantities. For example, by looking at two sets of objects, they may be able to tell which set has more or fewer without counting.
- **Benchmark Numbers.** Introduce the use of 0, 5, and 10 as benchmark numbers to help students further develop their sense of quantity as well as their ability to compare numbers. Benchmarks of 5 and 10 are especially useful with the 5 + *n* patterns.

Adapted from KATM 2012, Kindergarten Flipbook.

An important level of understanding is reached when students can compare two numbers from 1 to 10 represented as written numerals, without counting (**M.K.7**). Students demonstrate their understanding of numbers when they can justify their answers (**MHM3**).

Example	**M.K.7**
When a student gives an answer, the teacher may ask a probing question, such as “How do you know?”, to elicit student thinking and reasoning (**MHM3, MHM8**). Students might justify their answer (e.g., 7 greater than 5) by demonstrating a one-to-one match, counting again, or using similar approaches that help to explain or verify the answer (adapted from KATM 2012, Kindergarten Flipbook).	

Instructional Focus
Comparing numbers and groups in kindergarten will progress to comparing addition and subtraction situations in grade one. For example, “Which is more?” or “Which is fewer?” will progress to “How many more?” or “How many fewer?”

Domain: Operations and Algebraic Thinking

Kindergarten students develop an understanding of addition as putting together and subtraction as taking apart or taking from. Students work with numbers within ten, and they work toward fluency with these operations for numbers within 5.

Operations and Algebraic Thinking
Understand addition as putting together and adding to, and understand subtraction as taking apart and taking from.
M.K.8
Represent addition and subtraction with objects, fingers, mental images, drawings, sounds (e.g., claps), and acting out situations, verbal explanations, expressions, or equations.

M.K.9
Solve addition and subtraction word problems and add and subtract within 10 by using objects or drawings to represent the problem.

M.K.10
Decompose numbers less than or equal to 10 into pairs in more than one way by using objects or drawings, and record each decomposition by a drawing or equation (e.g., 5 = 2 + 3 and 5 = 4 + 1).

M.K.11
For any number from 1 to 9, find the number that makes 10 when added to the given number by using objects or drawings, and record the answer with a drawing or equation.

M.K.12
Fluently add and subtract within 5.

Kindergarten students develop their understanding of addition and subtraction by making sense of word problems (**MHM1, MHM2**). Students experience a variety of addition situations that involve putting together and adding to and a variety of subtraction situations that involve taking apart and taking from (**M.K.8–9**). Students use objects (such as two-color counters, clothespins on hangers, connecting cubes, 5-frames, and stickers), fingers, mental images, sounds, drawings, verbal explanations and acting out the situation to represent these operations (**MHM1, MHM2, MHM4, MHM5**) [adapted from KATM 2012, Kindergarten Flipbook].

Students use both mathematical and non-mathematical language to explain their interpretation of a problem and the solution. Initially, students work with numbers within 5, which helps them move from perceptual subitizing to conceptual subitizing, in which they say the addends and the total (e.g., 2 and 1 make 3). Students will generally use fingers to keep track of addends and parts of addends and develop rapid visual and kinesthetic recognition of numbers up to 5 on their fingers. Eventually, students will expand their work in addition and subtraction from within 5 to within 10.

Students are introduced to expressions and equations using appropriate symbols, including +, —, and =. Teachers may write expressions (e.g., 3 – 1) or equations (e.g., $3 - 1 = \square$ or 3 = 1 + 2) that represent operations and problems with real-world contexts to reinforce students' understanding of these concepts. Teachers should emphasize that an equal sign (=) means "has the same value as." Students should see these equations and be encouraged to write them; however, they are not required to write equations. In kindergarten, the use of formal vocabulary for both addition and subtraction (such as *minuend*, *subtrahend*, and *addend*) is not necessary.

For English learners, phonologically identical words (e.g., *sum* and *some*, *whole* and *hole*) may be challenging; thus it is better to use the word *total* instead of *sum* for all students in kindergarten and grade one. Using the word *partners* instead of *addends* is also a helpful conceptual support for children in these grades. To support English learners, these words are explicitly taught as they are introduced (adapted from UA Progressions Documents 2011a). For more information, refer to the Universal Access chapter.

Instructional Focus
When students represent addition and subtraction, this also supports mathematical practices as they use objects or pictures to represent quantities (**M.K.8**), reason quantitatively to make sense of quantities and develop a clear representation of the problem (**MHM2**), mathematize a real-world situation (**MHM4**), and use tools appropriately to model the problem (**MHM5**). Math drawings also facilitate student reflection and discussion and help young students justify answers (**MHM3**).

Word problems with real-life applications provide students with a context to develop their understanding of addition and subtraction (**M.K.9**). Kindergarten students learn that addition is *putting together* and *adding to* and subtraction is *taking apart* and *taking from*. Kindergartners use objects or math drawings (with simple shapes such as circles) to model word problems (adapted from ADE 2010).

The most common types of addition and subtraction problems for kindergarten students are displayed with dark shading in the following table. Students add and subtract within 10 to solve these types of problems.

Types of Addition and Subtraction Problems (Kindergarten)

Type of Problem	Result Unknown	Change Unknown	Start Unknown
Add to	Two bunnies sat on the grass. Three more bunnies hopped there. How many bunnies are on the grass now? $2 + 3 = \square$	Two bunnies were sitting on the grass. Some more bunnies hopped over to the first two bunnies. Then there were 5 bunnies sitting on the grass. $2 + \square = 5$	Some bunnies were sitting on the grass. Three more bunnies hopped there. Then there were 5 bunnies. How many bunnies were on the grass before? $\square + 3 = 5$

Take from	Five apples were on the table. I ate 2 apples. How many apples are on the table now? $5 - 2 = \square$	Five apples were on the table. I ate some apples. Then there were 3 apples. How many apples did I eat? $5 - \square = 3$	Some apples were on the table. I ate 2 apples. Then there were 3 apples. How many apples were on the table before? $\square - 2 = 3$

	Total Unknown	**Addend Unknown**	**Both Addends Unknown**
Put together/ Take apart	Three red apples and 2 green apples are on the table. How many apples are on the table? $3 + 2 = \square$	Five apples are on the table. Three are red, and the rest are green. How many apples are green? $3 + \square = 5$, $5 - 3 = \square$	Grandma has 5 flowers. How many can she put in her red vase and how many in her blue vase? $5 = 0 + 5$, $5 = 5 + 0$ $5 = 1 + 4$, $5 = 4 + 1$ $5 = 2 + 3$, $5 = 3 + 2$

	Difference Unknown	Bigger Unknown	Smaller Unknown
Compare	(“How many more?” version): Lucy has 2 apples. Julie has 5 apples. How many more apples does Julie have than Lucy?	(Version with *more*): Julie has 3 more apples than Lucy. Lucy has 2 apples. How many apples does Julie have?	(Version with *more*): Julie has 3 more apples than Lucy. Julie has 5 apples. How many apples does Lucy have?
	(“How many fewer?” version): Lucy has 2 apples. Julie has 5 apples. How many fewer apples does Lucy have than Julie?	(Version with *fewer*): Lucy has 3 fewer apples than Julie. Lucy has 2 apples. How many apples does Julie have?	(Version with *fewer*): Lucy has 3 fewer apples than Julie. Julie has five apples. How many apples does Lucy have?
	$2 + \square = 5$, $5 - 2 = \square$	$2 + 3 = \square$, $3 + 2 = \square$	$5 - 3 = \square$, $\square + 3 = 5$

Note: Kindergarten students solve problem types with the darkest shading; students in grades one and two solve problems of all subtypes. Unshaded problems are the most difficult; first-grade students work with these problems but do not master them until grade two (adapted from NGA/CCSSO 2010d and UA Progressions Documents 2011a).

To solve word problems, students learn to apply various computational methods, as summarized in the table below. Kindergarten students generally use Level 1 methods, moving on to Level 2 and Level 3 methods in later grades.

Methods Used for Solving Single-Digit Addition and Subtraction Problems

Level 1: Direct Modeling by Counting All or Taking Away

Represent the situation or numerical problem with groups of objects, a drawing, or fingers. Model the situation by composing two addend groups or decomposing a total group. Count the resulting total or addend.

Level 2: Counting On

Embed an addend within the total (the addend is perceived simultaneously as an addend and as part of the total). Count this total, but abbreviate the counting by omitting the count of this addend; instead, begin with the number word of this addend. The count is tracked and monitored in some way (e.g., with fingers, objects, mental images of objects, body motions, or other count words).

For addition, the count is stopped when the amount of the remaining addend has been counted. The last number word is the total. For subtraction, the count is stopped when the total occurs in the count. The tracking method indicates the difference (seen as the unknown addend).

Level 3: Converting to an Easier Equivalent Problem
Decompose an addend and compose a part with another addend.

Adapted from UA Progressions Documents 2011a.

Students learn that a set of objects may be broken into two sets in multiple ways. For example, a set of 5 objects may be separated into two sets—3 and 2 or 4 and 1 (**M.K.10**). Thus, when breaking apart a set (decomposing), students develop the understanding that a smaller set of objects exists within that larger set. Students require numerous experiences with decomposing sets of objects and recording with pictures and numbers, and the teacher should make connections between the drawings and symbols ($5 = 4 + 1$, $5 = 3 + 2$, $5 = 2 + 3$, $5 = 1 + 4$, and $5 = 5 + 0$), showing the total on the left and the two addends on the right. Students can find patterns in all of the decompositions of a given number and eventually summarize these patterns for several numbers. Experience with decomposing also emphasizes that the equal sign (=) means "has the value as."

Students may use objects such as beads, cubes, two-color counters, or square tiles to show different number pairs for a given number. For example, for the number 5, students may split a set of 5 objects into 1 and 4, 2 and 3, and 5 and 0. Students may also use drawings to show different number pairs for a given number (**MHM1, MHM2, MHM4**).

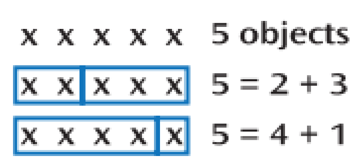
Example: Decomposing 5 **M.K.10**

Students may draw 5 objects, showing how to decompose in several ways.

They may write equations involving 5 and its decompositions, such as:

$$5 = 4 + 1$$
$$3 + 2 = 5$$
$$2 + 3 = 4 + 1$$

Students can systematically list all the possible number partners for a given number. For example, they may list all number partners for 5 ($0 + 5$, $1 + 4$, $2 + 3$, $3 + 2$, $4 + 1$, and $5 + 0$) and describe the pattern in the addends—that is, each number is one less or one more than the previous addend.



Adapted from KATM 2012, Kindergarten Flipbook.

Example: Demonstrating Conceptual Understanding, Application, and Connection to the Mathematical Habits of Mind **M.K.10**

Shake and Spill

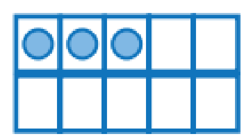
Students use 5 two-color counters (e.g., red on one side and yellow on the other) and a cup (optional). The students put the counters in the cup, shake it, and spill them onto a table. The students determine how many of each color is showing and record the sum by using drawings or equations. The students should “shake and spill” several times to show different pairs of numbers that sum to 5.

Working with equations with one number on the left and an operation on the right (e.g., 5 = 2 + 3) to record groups of 5 things decomposed as groups of 2 and 3 things (**M.K.10**) helps students to understand that equations indicate quantities on both sides of the equal sign have the same value (**MHM7**). Understanding the meaning of mathematical symbols allows students to develop precision in their communication about mathematics (**MHM6**). The equation can also be reversed so that an operation is on the left and the number is on the right (e.g., 2 + 3 = 5). Such equations model “add to” situations (adapted from UA Progressions Documents 2011a).

Number pairs that total 10 are foundational for students’ ability to work fluently within base-ten numbers and operations. In kindergarten, students find the number that makes 10 when added to the given number for any number from 1 to 9. Students use objects or drawings and record their answers with a drawing or equation (**M.K.11**). Students use different models, such as 10-frames, cubes, and two-color counters to help them visualize these number pairs for 10 (**MHM1, MHM2, MHM4**).

Examples: Tools and Strategies for Making a Ten **M.K.11**

A student places 3 objects on a 10-frame and then determines how many more are needed to “make a ten.” Students may use electronic versions of 10-frames to develop this skill (**MHM5**).



A student snaps 10 cubes together to make a pretend train.

- The student breaks (decomposes) the train into two parts. He or she identifies how many cubes are in each part and records the associated equation (10 = ______+______).
- The student breaks the train into two parts. He or she counts how many cubes are in one part and determines how many are in the other part without directly counting that part. Then the student records the associated equation (if the counted part has 4 cubes, the equation would be 10 = 4 +______).
- The student covers up part of the train, without counting the covered part. He or she counts the cubes that are showing and determines how many are covered up. Then the student records the associated equation (if the counted part has 7 cubes, the equation would be 10 = 7 +______) [**MHM8**].
- The student tosses 10 two-color counters on the table and records how many of each color are facing up (**MHM8**).

Adapted from KATM 2012, Kindergarten Flipbook.

Later in the year, students solve addition and subtraction equations for numbers within 5 (for example, $2 + 1 = \square$ or $3 - 1 = \square$) while still connecting these equations to situations verbally or with drawings. Experience with decompositions of numbers and with “add to” and “take from” situations enables students to develop fluency in adding and subtracting within 5 (**M.K.12**).

FLUENCY

In the standards for kindergarten through grade six, there are individual content standards that set expectations for fluency in computation (e.g., “Fluently add and subtract within 5” [**M.K.12**]). Such standards are culminations of progressions of learning that often span several grades and involve conceptual understanding, thoughtful practice, and extra support where necessary.

The word *fluent* is used in the standards to mean “reasonably fast and accurate” and the ability to use certain facts and procedures with enough facility that using them does not slow down or derail the problem solver as he or she works on more complex problems. Procedural fluency requires skill in carrying out procedures flexibly, accurately, efficiently, and appropriately. Developing fluency in each grade may involve a mixture of simply knowing some answers, knowing some answers from patterns, and knowing some answers from the use of strategies.

Adapted from UA Progressions Documents 2011a.

Below are several strategies that kindergarten students may use to attain fluency with addition and subtraction within 5:

- Visualizing the small numbers involved
- Counting on (e.g., for 3 + 2, students will say "3," then count on two more, "4, 5," and finish by saying the solution is "5")
- Counting back (e.g., for 4 – 1, students will say "4," then count back one, "3," and state that the solution is "3")
- Counting up to subtract (e.g., for 5 – 3 , students will say "3," then count up until they get to 5, keeping track of how many they counted up, stating that the solution is "2")
- Using doubles (e.g., for 2 + 3, students may say, "I know that 2 + 2 is 4, and 1 more is 5")
- Using the commutative property (e.g., students may say, "I know that 2 + 1 = 3, so 1 + 2 = 3") [Students may articulate this property without using the formal terminology.]
- Using fact families (e.g., students may say, "I know that 2 + 3 = 5, so 5 – 3 = 2") [adapted from KATM 2012, Kindergarten Flipbook]

Example: Demonstrate Conceptual Understanding of Addition	M.K.8
Draw Me a Picture Provide students with a dry erase board, marker, and eraser. Show the students an equation using addends whose sum will be 10 or less. (5 + 1 =) Direct students to use their white board to draw a simple picture that represents this equation and identify the total by writing the numeral that represents the total.	

Domain: Number and Operations in Base Ten

Number and Operations in Base Ten
Work with numbers 11-19 to gain foundations for place value.
M.K.13 Compose and decompose numbers from 11 to 19 into ten ones and some further ones by using objects or drawings, and record each composition or decomposition by a drawing or equation (e.g., 18 = 10 + 8); understand that these numbers are composed of ten ones (one ten) and one, two, three, four, five, six, seven, eight, or nine ones.

Kindergarten teachers help their students lay the foundation for understanding the base-ten system by drawing special attention to the number 10. Students compose and decompose numbers from 11 to 19 into 10 ones and some further ones. Students use objects or drawings and record each composition or decomposition with a drawing or equation (e.g., 16 = 9 + 7) [**M.K.13**].

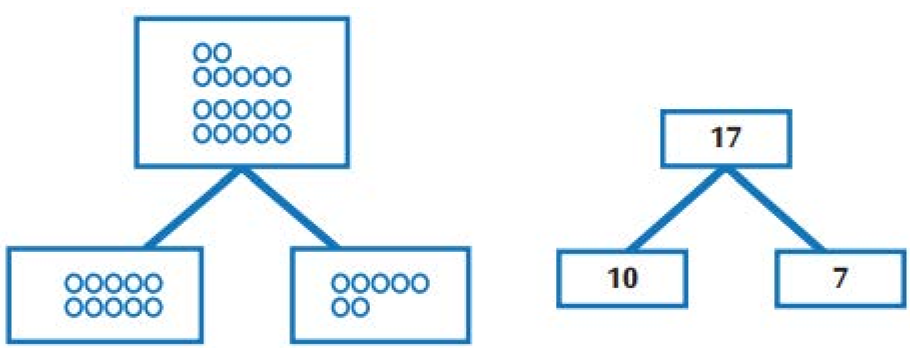
Students describe, explore, and explain how the counting numbers from 11 through 19 are composed of 10 ones and some more ones. For example, when focusing on the number 14, students count out 14 objects using one-to-one correspondence and then use those objects to compose one group of 10 ones and 4 additional ones. Students connect the representation to the symbol “14” and recognize the written pattern in these numbers—that the numbers start with 1 (represents 1 ten) and end with the number of additional ones (**MHM1, MHM2, MHM4, MHM5, MHM6, MHM7, MHM8**) [adapted from UA Progressions Documents 2012b].

Students may have difficulty understanding that as a singular word, *ten* means “10 ones.” For many students, understanding that a group of 10 ones can be replaced by a single word and that they both represent 10 (as 10 ones or as one ten) is confusing. Students learn that this set of numbers (11–19) does not follow a consistent pattern in the verbal counting sequence. For example:

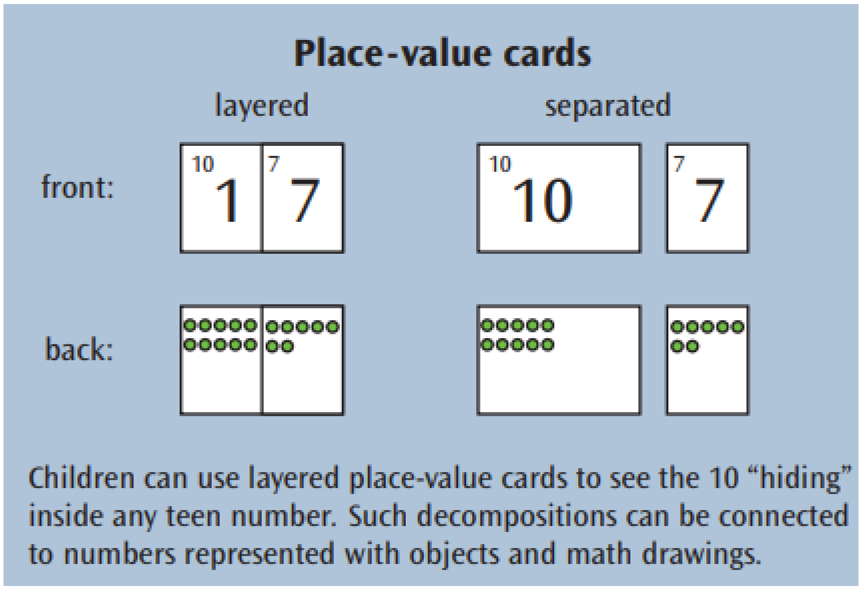
- *Eleven* and *twelve* are special number words.
- *Teen* means 1 ten plus ones.
- The verbal counting sequence for teen numbers is backwards—we say the ones digit before the tens digit. For example, 27 reads tens to ones (twenty-seven), but 17 reads ones to tens (seven-teen). To develop student understanding of written teen numbers, students read numbers as well as describe quantities. For example, for the number 17, students read “seventeen,” decompose the number as “1 group of 10 ones and 7 additional ones,” and record their understanding as 17 = 10 + 7 or use math drawings. This clarifies the pattern for them. Kindergarten students see addition and subtraction equations. Student writing of equations in kindergarten is encouraged, but it is not required (adapted from ADE 2010).

Examples: Understanding Teen Numbers	**M.K.13**
Math drawings and other activities can help students develop place-value understanding of teen numbers.	

Using 10-frames and number-bond diagrams



Using layered place-value cards



Source: UA Progressions Documents 2012b.

Domain: Measurement and Data

Measurement and Data
Describe and compare measurable attributes.
M.K.14 Describe measurable attributes of objects, such as length or weight and describe several measurable attributes of a single object.
M.K.15 Directly compare two objects with a measurable attribute in common, to see which object has "more of" or "less of" the attribute, and describe the difference.

Students recognize and distinguish measurable attributes (e.g., length, area, volume) from non-measurable attributes (e.g., big or bigger) [**M.K.14**]. Initially, many students will not be able to differentiate between these two types of attributes. Students will say one object is "bigger" than another without clarifying that it is longer, greater in area or volume, and so forth.

For students to describe attributes such as length and weight accurately, they need multiple opportunities to explore these attributes informally. Teachers encourage students' conversations to extend from describing objects as *big*, *small*, *long*, *tall*, or *high* to naming, discussing, and demonstrating with gestures the appropriate attribute (e.g., length, area, volume, or weight).

For example, a student might describe the measurable attributes of an empty can or milk carton by talking about how tall, wide, and heavy the can is, or how much liquid will fit inside the container. All of these are measurable attributes. By contrast, non-measurable attributes include designs, words, colors, or pictures on the can. As students discuss these situations and compare objects using different attributes, they learn to distinguish, label, and describe several measurable attributes of a single object (**MHM4, MHM5, MHM6, MHM7**).

Students directly compare two objects with a measurable attribute in common, to see which object has "more" or "less of" the attribute and describe the difference (**M.K.15**). For example, students directly compare the heights of two children and describe one child as taller or shorter. Language plays an important role in this standard, as students describe the similarities and differences of measurable attributes of objects with terms such as shorter than, taller than, lighter than, the same as, and so forth (**MHM2, MHM4, MHM6, MHM7**).

When making direct comparisons for length, students must attend to the "starting point" of each object (e.g., the ends need to be lined up at the same point) or students need to compensate when the starting points are not lined up. Students develop an understanding of conservation of length (if an object is moved, its length does not change), an important concept when comparing the lengths of two objects (adapted from ADE 2010 and UA Progressions Documents 2012a).

With practice, students become increasingly competent at direct comparison—comparing the amount of an attribute in two objects without measurement. For example, when comparing the volume of two different boxes, ask students to discuss and justify their answers to these questions: *Which box will hold more? Which box will hold less? Will the two boxes hold the same amount? How could you find out?* Students can decide to fill one box with dried beans and then pour the beans into the other box to determine the answers to these questions (adapted from KATM 2012, Kindergarten Flipbook).

The following table presents a sample classroom activity that connects the Standards for Mathematical Content and Standards for Mathematical Practice.

Connecting to the Mathematical Habits of Mind—Kindergarten

Standards Addressed	Explanation and Example
Connections to Mathematical Habits of Mind **MHM3** **Construct viable arguments and critique the reasoning of others.** As students complete this task, they will share their thinking with their partner. They will be able to tell how they know an object is longer or shorter than the strip of paper. **MHM5** **Use appropriate tools strategically.** The students will use the strip of paper as the measuring tool to compare lengths and the balance to compare mass. They will begin to develop knowledge of which tools are appropriate to measure the identified attribute. For example, the students will not use the balance to measure length. **MHM6** **Attend to precision.** As students complete the task, they will need to place the strip and the objects in line with each other in order to determine if the object is longer or shorter. **Mathematics Content Standards** **M.K.15** Directly compare two objects with a measurable attribute in common, to see which object has "more of" or "less of" the attribute and describe the differences.	**Task: Longer or Shorter** For this task, students will work with a partner. **Materials:** Strips of paper approximately 5 inches long, (Does not need to be exact.), large sheet of paper (12" x 18") one side labeled longer and one side labeled shorter. (May color code words if this will assist students. **Option 1** Give one strip of paper to each pair of students. Direct students to find classroom objects that are longer or shorter than the strip of paper and place them on the graphic organizer. The students justify their placement to their partner. The teacher observes students as they place their strip of paper to measure the objects. The teacher also listens to the students as they justify their placement. The teacher may ask, "How did you decide where to place the object?, Can you show me how you measured this object?" **Option 2** The teacher may provide each pair of students with a collection of classroom objects. The students will use the strip of paper to measure each object and decide if it is longer or shorter. Extension Task: The teacher places a balance with an object that has mass in the learning center and a collection of classroom objects. The graphic

	organizer will use the words *heavier* and *lighter*. The students will use the balance to determine if the objects are heavier or lighter than the identified object.

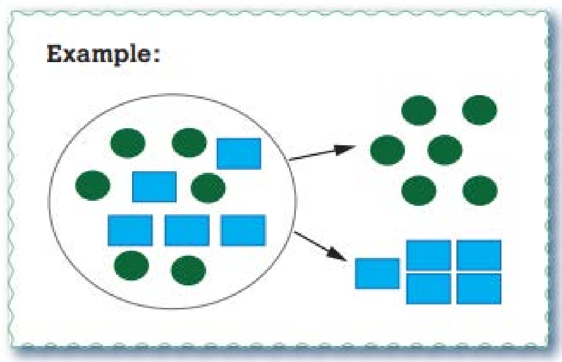
Measurement and Data

Classify objects and count the number of objects in each category.

M.K.16

Classify objects into given categories, count the numbers of objects in each category, and sort the categories by count. Category counts should be limited to less than or equal to 10. (e.g., Identify coins and sort them into groups of 5s or 10s.)

Kindergarten students connect counting and ordering skills and understandings to help them classify objects or people into given categories, count the number of objects in each category, and sort the categories by count (**M.K.16**).



Students identify similarities and differences between objects (e.g., size, color, shape) and use these attributes to sort a collection of objects (**MHM2, MHM6, MHM7**).

When the objects are sorted, students count the objects in each set and then order each of the sets by the amount in each set.

For example, when given a collection of buttons, students separate buttons into different piles based on color. Next, they count the number of buttons in each pile (e.g., blue [5], green [4], orange [3], and purple [4]). Finally, they organize the groups by the quantity in each group—for example, from the smallest group (orange) to the largest group (blue), and groups with the same number (green and purple) are placed together.

Students explain their thinking. Teachers may use prompts such as these to ask students to explain their thought processes:

- Explain how you sorted the objects.
- Explain how you labeled each set with a category.
- Answer a variety of counting questions (such as “How many ________?”).

- Compare the sorted groups using words such as *most, least, same,* and *different* (adapted from KATM 2012, Kindergarten Flipbook).

Instructional Focus
As kindergartners classify objects, they build a foundation for collecting data and creating and analyzing graphical representations in later grades. Also, as students count the number of objects in each category and order the categories by count, they reinforce important skills and understanding in comparing numbers, which are part of the major work at this grade in the Counting and Cardinality domain (**M.K.4–7**). Students can also reinforce mathematical practices as they make sense of problems by counting and recounting (**MHM1**) and explaining their process and reasoning (**MHM3**).

Domain: Geometry

Geometry
Identify and describe shapes (squares, circles, triangles, rectangles, hexagons, cubes, cones, cylinders, and spheres).
M.K.17 Describe objects in the environment using names of shapes and describe the relative positions of these objects using terms such as above, below, beside, in front of, behind and next to.
M.K.18 Correctly name shapes regardless of their orientations or overall size.
M.K.19 Through the use of real-life objects, identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid").

Students use positional words to describe objects in the environment (**M.K.17**). Examples of positional words include *in* and *out*, *inside* and *outside*, *down* and *up*, *above* and *below*, *over* and *under*, *before* and *after*, *top* and *bottom*, *front* and *back*, *right* and *left*, *on* and *off*, *begin* and *end*, and *near* and *far*.

Students develop spatial sense by connecting geometric shapes to their everyday lives. Students need opportunities to identify and name two- and three-dimensional shapes in and outside of the classroom and describe relative positions by answering questions such as these:

- Which way is the cafeteria? (*The cafeteria is to the right.*)
- Which shape is near the rectangle? (*The circle is near the rectangle.*)
- Where is the green ball? (*The green ball is on top of the cupboard.*)
- What types of shapes do you see on the floor of the basketball court? (*I see a rectangle and a circle on the basketball court.*)

Students begin to name and describe three-dimensional shapes with mathematical vocabulary, using words such as *sphere*, *cube*, *cylinder*, and *cone*, and answer related questions (**MHM6, MHM7**). Examples for standard **M.K.17** include the following:

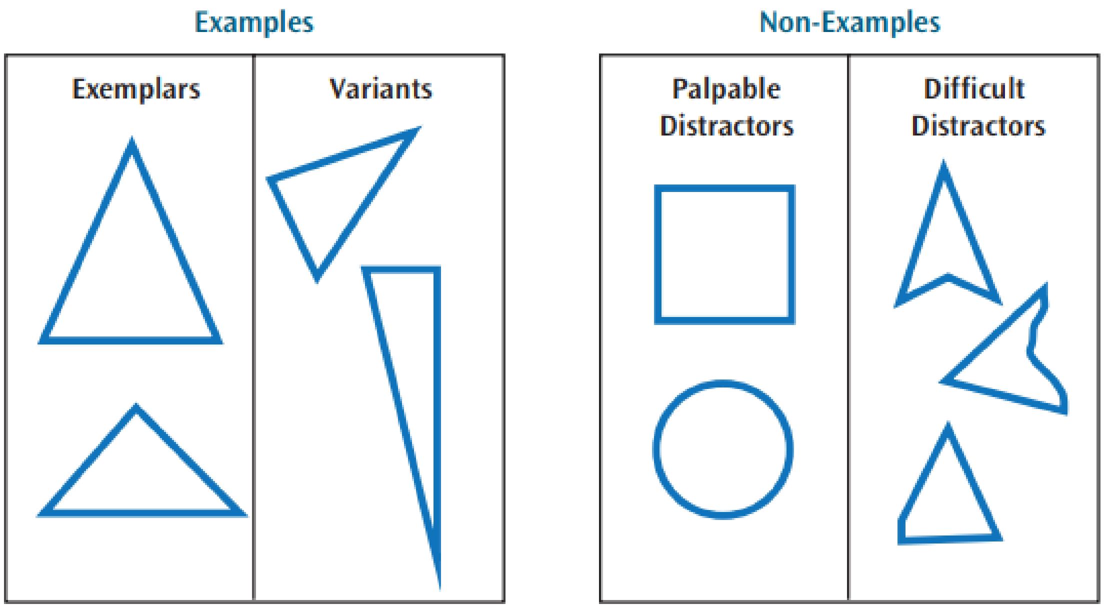
- Ask students to find rectangles in the classroom and describe the relative positions of the rectangles they see. (*Possible answer*: The rectangle [a poster] is over the sphere [globe]).
- The teacher holds up objects—such as an ice-cream cone, a number cube, or a ball—and asks students to identify each shape.
- The teacher places an object next to, behind, above, below, beside, or in front of another object and asks positional questions such as "Where is the object?" (adapted from ADE 2010; KATM 2012, Kindergarten Flipbook; and UA Progressions Documents 2012c).

Kindergarten students work with a variety of shapes that have different sizes. They learn to match two-dimensional shapes even when the shapes have different orientations (**M.K.18**). Students name shapes that occur in everyday situations, such as circles, triangles, and squares, and distinguish them from non-examples of these categories.

Students develop an intuitive image of each shape category. Figure K-3 includes examples and non-examples of triangles, as described below:

- Examples
  - Exemplars—typical visual prototypes of the shape category
  - Variants—other examples of the shape category
- Non-Examples
  - Palpable distractors—non-examples with little or no overall resemblance to the exemplars
  - Difficult distractors—visually similar to examples, but lack at least one defining attribute

Figure K-3. Examples and Non-Examples of Triangles



Adapted from UA Progressions Documents 2012c.

Common Misconceptions

- Most kindergarten students are unable to recognize an "upside-down triangle" as a triangle, because of its orientation. However, students should be exposed to many types of triangles, in many different orientations, to eliminate the misconception that a triangle is always vertex-up and equilateral.
- A square with a vertex pointing downward is often referred to as a "diamond." This needless introduction of a new shape name should be avoided, as it only serves to confuse the fact that such a shape is still a square, though its orientation is atypical.
- Students often mistakenly believe that a rectangle must have "two long sides and two short sides." A rectangle is a quadrilateral with four right angles, and its definition does not include a reference to the length of its sides.

Below are several strategies to help kindergarten students learn about shapes (**MHM6, MHM7**):

- Students form visual templates or refer to models for shape categories (e.g., Children recognize a shape as a rectangle because it looks like a door).
- Students see examples of triangles that have sides with three different lengths and then contrast triangles with non-triangles.

- The teacher hands out pairs of paper shapes in different sizes. Each student is given one shape. Then students need to find the partner who has the same shape.
- The teacher brings in a variety of spheres (a tennis ball, basketball, globe, table-tennis ball, and so forth) to demonstrate that size does not change the name of a shape (adapted from ADE 2010; KATM 2012, Kindergarten Flipbook; and UA Progressions Documents 2012c).

Students identify shapes as two-dimensional (lying in a plane, "flat") or three-dimensional ("solid") [**M.K.19**] and differentiate between two-dimensional and three-dimensional shapes (**MHM6, MHM7**). For example:

- Students name a picture of a shape as two-dimensional because it is flat and can be measured in only **two** ways (by its length and width).
- Students name an object as three-dimensional because it is not flat (it is a solid object or shape) and can be measured by length, width, and height (or depth) [adapted from ADE 2010].

Geometry
Analyze, compare, create and compose shapes. **M.K.20** Analyze and compare two- and three-dimensional shapes, in different sizes and orientations, using informal language to describe their similarities, differences, parts (e.g., number of sides and vertices/"corners"), and other attributes (e.g., having sides of equal length). Instructional Note: Student focus should include real-world shapes. **M.K.21** Model shapes in the world by building shapes from components (e.g., sticks and clay balls) and drawing shapes. **M.K.22** Compose simple shapes to form larger shapes (e.g., "Can these two triangles, with full sides touching, join to make a rectangle?").

Kindergarten students connect their work with identifying and classifying simple shapes (refer to standards **M.K.17–19**) to help them compare shapes and manipulate two or more shapes to create a new shape. This understanding also builds foundations for students to "reason with shapes and their attributes" in grade one (refer to standards **M.1.19–21**).

Students describe similarities and differences between and among shapes using informal language (**M.K.20**). These experiences help young students begin to understand how three-dimensional shapes are composed of two-dimensional shapes—for example, the base and the top of a cylinder is a circle, the face of a cube is a square. In early explorations of geometric properties, students discover how categories of shapes are subsumed within other categories. For instance, they will recognize that a square is a special type of rectangle.

Example: Sorting Shapes	**M.K.20**
Students are provided a pile of shapes including triangles, squares, hexagons, and rectangles. Direct the students to sort the shapes based upon a common attribute other than color (which is a non-measurable attribute). After students have sorted the shapes, ask "How did you determine your groups?, What attribute are you using to sort your shapes?, and Can you sort them another way?" After students demonstrate an understanding of the shape attributes, distribute one shape to each student. The teacher directs students with three sides to one area of the classroom, four sides to one area of the classroom, and six sides to one area of the classroom. Repeat with other attributes.	

Students work with various triangles, rectangles, and hexagons with sides that are not all congruent. Initially, students describe shapes using everyday language and then expand their vocabulary to include geometric terms such as *sides* and *vertices* (or *corners*). Opportunities to work with pictorial representations and concrete objects, as well as technology, will help students develop their understanding and descriptive vocabulary for both two- and three-dimensional shapes (**MHM4, MHM6, MHM7**).

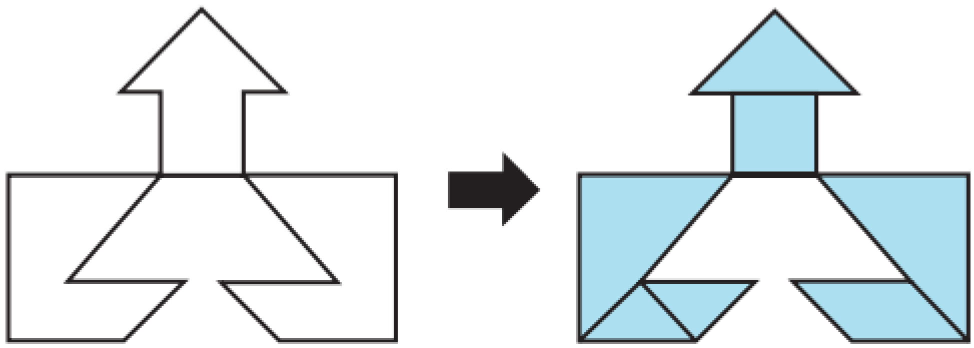
In kindergarten, students model shapes they observe in everyday life by building shapes from various components (e.g., clay, glue, tape, sticks, paper, straws) and by drawing shapes (**M.K.21**). Two-dimensional shapes are flat, and three-dimensional shapes are not flat (and can be "solid"), so students *draw* or *create* two-dimensional shapes and *build* three-dimensional shapes (**MHM1, MHM4, MHM7**).

Students compose simple shapes to form larger shapes and answer questions such as, "Can you join these two triangles with full sides touching to make a rectangle?" (**M.K.22**). Composing shapes is an important concept in kindergarten. Students move from identifying and classifying simple shapes to manipulating two or more shapes to create a new shape. Students rotate, flip, and arrange puzzle pieces, and they move shapes to make a design or picture. Finally, students manipulate simple shapes to make a new shape (**MHM1, MHM3, MHM4, MHM7**) [adapted from KATM 2012, Kindergarten Flipbook].

Puzzles provide opportunities for students to apply spatial relationships and develop problem-solving skills in an entertaining and meaningful way. Pattern blocks and tangrams are often utilized when students work with two-dimensional shapes.

Example: Exploring Shapes with Tangrams	**M.K.22**

Students make a schoolhouse using tangrams. The teacher models how to place the pieces and discusses how it is necessary to turn over, rotate, or slide pieces to complete a puzzle.



Each student or pair of students is then provided with a set of tangrams and a simple puzzle, such as the outlined version of the schoolhouse above. Students use their pieces to complete the puzzle.

Adapted from National Council of Teachers of Mathematics Illuminations 2013d.

Examples of interactive tangram puzzles are available at the National Council of Teachers of Mathematics Web site (http://www.nctm.org/standards/content.aspx?id=25012 [accessed July 31, 2014]).

Essential Learning for the Next Grade

In kindergarten through grade five, the focus is on the addition, subtraction, multiplication, and division of whole numbers, fractions, and decimals, with a balance of concepts, skills, and problem-solving. Arithmetic is viewed as an important set of skills and also as an analytical subject that, done thoughtfully, prepares students for algebra. Measurement and geometry develop alongside number and operations and are tied specifically to arithmetic along the way.

In kindergarten through grade two, students focus on addition, subtraction, and measurement using whole numbers. To be prepared for grade-one mathematics, students should be able to demonstrate that they have acquired specific mathematical concepts and procedural skills by the end of kindergarten and have met the fluency expectations. For kindergartners, the expected fluencies are to add and subtract within 5 (**M.K.12**). Addition and subtraction are introduced in kindergarten, and these fluencies and the conceptual understandings that support them are foundational for work in later grades.

It is particularly important for kindergarten students to attain the concepts, skills, and understandings necessary to know the number names and the count sequence (**M.K.1–M.K.3**); count to tell the number of objects (**M.K.4–M.K.5**); compare numbers (**M.K.6–M.K.7**); understand addition as putting together and adding to; and understand subtraction as taking apart and taking from (**M.K.8–M.K.12**). Also, working with numbers to gain foundations for place value (**M.K.13**) is essential to understanding the base-ten number system.

Counting and Cardinality

In kindergarten, students learn to count. Students connect counting to *cardinality*—knowing that the number word tells the quantity and that the number on which a person ends when counting represents the entire amount counted. Until this concept is developed, counting is merely a routine procedure done when a number is needed, and students will not understand how to apply numbers to solve problems.

By the end of kindergarten, important number concepts and skills for students include counting by ones and tens to 100 (rote counting); continuing a counting sequence when beginning from a number greater than 1 (counting on); counting objects to 20; writing numbers to 20; understanding one-to-one correspondence; identifying a quantity using both numerals and words; representing numbers with numerals (and pictures and words); understanding numbers and the relationships between numbers and quantities; and understanding the concepts of *more* and *less*. Counting to 100 and representing numbers with numerals (0 to 20) will prepare students to read and write numbers to 120 in grade one.

Addition and Subtraction

By the end of kindergarten, students are expected to add and subtract within 10 and solve addition and subtraction word problems using objects or drawings to represent the problems. Students are also expected to be fluent with addition and subtraction within 5. Fluency with addition and subtraction will prepare students to add within 100 in grade one. Addition and subtraction constitute a major instructional focus for kindergarten through grade two.

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