West Virginia math4life

Frameworks for Mathematics

Geometry

West Virginia DEPARTMENT OF EDUCATION

Geometry

Although there are many types of geometry, school mathematics is devoted primarily to plane Euclidean geometry, studied both synthetically (without coordinates) and analytically (with coordinates). In the higher mathematics courses, students begin to formalize their geometry experiences from elementary and middle school, using definitions that are more precise and developing careful proofs. The standards for grades seven and eight call for students to see two-dimensional shapes as part of a generic plane (i.e., the *Euclidean plane*) and to explore transformations of this plane as a way to determine whether two shapes are congruent or similar. These concepts are formalized in the Geometry course, and students use transformations to prove geometric theorems. The definition of *congruence* in terms of rigid motions provides a broad understanding of this means of proof, and students explore the consequences of this definition in terms of congruence criteria and proofs of geometric theorems.

Students investigate triangles and decide when they are similar—and with this newfound knowledge and their prior understanding of proportional relationships, they define trigonometric ratios and solve problems by using right triangles. They investigate circles and prove theorems about them. Connecting to their prior experience with the coordinate plane, they prove geometric theorems by using coordinates and describe shapes with equations. Students extend their knowledge of area and volume formulas to those for circles, cylinders, and other rounded shapes. Finally, continuing the development of statistics and probability, students investigate probability concepts in precise terms, including the independence of events and conditional probability.

Modeling

Standards	Teacher Understandings	Resources	Student Understandings
Although the Modeling category does not include specific standards, the idea of using mathematics to model the world pervades all higher mathematics courses and should hold a significant place in instruction. Modeling is addressed first here to emphasize its importance in the higher mathematics curriculum.	Modeling at the higher mathematics level goes beyond the simple application of previously constructed mathematics and includes real-world problems. True modeling begins with students asking a question about the world around them, and	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the **Mathematical Habits**	• When students are presented with a real-world situation and challenged to ask a question, all sorts of new issues arise (e.g., Which of the quantities present in this situation are known, and which are unknown?).

	mathematics is then constructed in the process of attempting to answer the question. Students may see when trying to answer their question that solving an equation arises as a necessity and that the equation often involves the specific instance of knowing the output value of a function at an unknown input value. Modeling problems have an element of being genuine problems, in the sense that students care about answering the question under consideration. In modeling, mathematics is used as a tool to answer questions that students really want answered. Students examine a problem and formulate a *mathematical model* (an equation, table, graph, or the like), compute an answer or rewrite their expression to reveal new information, interpret and validate the results, and report out. This	**of Mind**, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. It highlights some necessary foundational skills from previous grade levels. Math TREE Online Education Resources A curated set of aligned internet resources for WV middle and high school math teachers. Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills	• Students decide on a solution path that may need to be revised. They make use of tools such as calculators, dynamic geometry software, or spreadsheets. They try to use previously derived models (e.g., linear functions), but may find that a new formula or function will apply.

	is a new approach for many teachers and may be challenging to implement, but the effort should show students that mathematics is relevant to their lives. From a pedagogical perspective, modeling gives a concrete basis from which to abstract the mathematics and often serves to motivate students to become independent learners. The important ideas surrounding rational functions, graphing, solving equations, and rates of change should be explored through the lens of mathematical modeling. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade level content standards will leave gaps in students' skills and understandings and will	most relevant to standards.	

	leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.		

Geometry

Standards	Teacher Understandings	Resources	Student Understandings
CONGRUENCE, PROOF, AND CONSTRUCTIONS **Experiment with transformations in the plane.** **M.GHS.1** Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc. **M.GHS.2** Represent transformations in the plane using, for example, transparencies and geometry software; describe	A large portion of instruction in the traditional Geometry course is formed by the standards of the geometry conceptual category. Here, students develop the ideas of congruence and similarity through transformations. They prove theorems, both with and without the use of coordinates. They explore right-triangle trigonometry, as well as circles and parabolas. Standard **MHM#3,** "Construct viable arguments	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the **Mathematical Habits of Mind**, and demonstrate the importance of developing conceptual	• Students replace the imprecise definition that shapes are congruent when they "have the same size and shape" with a more mathematically precise one: Two shapes are congruent if there is a sequence of rigid motions in the plane that takes one shape exactly onto the other. • Students build more precise definitions for

transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle). **M.GHS.3** Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).	and critique the reasoning of others," plays a predominant role throughout the Geometry course. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.	understanding, procedural skill and fluency, and application. Math TREE Online Education Resources A curated set of aligned internet resources for WV middle and high school math teachers. Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	the rigid motions (rotation, reflection, and translation) based on previously defined and understood terms such as point, line, between, angle, circle, perpendicular, and so forth. • Students show, using rigid motions, that congruent triangles have congruent corresponding parts and that, conversely, if the corresponding parts of two triangles are congruent, then there is a sequence of rigid motions that takes one triangle to the other. • Students justify the typical triangle congruence criteria such as ASA, SAS, and SSS. • Students prove geometric theorems. • Students develop a more precise mathematical definition of similarity. • Students explore the properties of dilations in

M.GHS.4 Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle). **M.GHS.5** Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, for example, graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a			detail and develop an understanding of the notion of scale factor. • Students explore the consequences of two triangles being similar: that they have congruent angles and that their side lengths are in the same proportion. • Students understand the trigonometric functions as relationships completely determined by angles. They further their understanding of these functions by investigating relationships between sine, cosine, and tangent; by exploring the relationship between the sine and cosine of complementary angles; and by applying their knowledge of right triangles to real-world situations. • Students advance their knowledge of right-triangle trigonometry by applying trigonometric

specified center through a specified angle) **Understand congruence in terms of rigid motions.** **M.GHS.6** Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems. **M.GHS.7** Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students			ratios in non-right triangles. • Students use reasoning about similarity and trigonometric identities to derive the Laws of Sines and Cosines only in acute triangles, and use these and other relationships to solve problems. • Students develop a definition of radian measure. • Students derive the formula for the circle and parabola. • Students continue to use coordinates to prove geometric theorems with algebraic technique, including the slope criteria for parallel and perpendicular lines. • Students develop the reasoning required to make sense of a proof and to communicate the essence of the proof to a peer. • Students understand and use volume and area

reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems. **M.GHS.8** Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems. **Prove geometric theorems.** **M.GHS.9** Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a			formulas for curved objects.

perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. **M.GHS.10** Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of this standard may			

be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for M.GHS.36. **M.GHS.11** Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. **Make geometric constructions.** **M.GHS.12** Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular			

lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them. **M.GHS.13** Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them. **SIMILARITY, PROOF, AND TRIGONOMETRY** **Understand similarity in terms of similarity transformations.** **M.GHS.14** Verify experimentally the properties of			

dilations given by a center and a scale factor. a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **M.GHS.15** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **M.GHS.16** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. **Prove theorems involving similarity.** **M.GHS.17** Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other			

two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity. **M.GHS.18** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **Define trigonometric ratios and solve problems involving right triangles.** **M.GHS.19** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **M.GHS.20** Explain and use the relationship between the sine and cosine of complementary angles. **M.GHS.21** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **Apply trigonometry to general triangles.** **M.GHS.22** Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an			

auxiliary line from a vertex perpendicular to the opposite side. **M.GHS.23** Prove the Laws of Sines and Cosines and use them to solve problems. Instructional Note: With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles. **M.GHS.24** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles. Instructional Note: With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles. **EXTENDING TO THREE DIMENSIONS** **Explain volume formulas and use them to solve problems.** **M.GHS.25** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. Instructional Note: Informal arguments for area and			

volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. Similarly, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k. **M.GHS.26** Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. Instructional Note: Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. Similarly, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k. **Visualize the relation between two dimensional and three-dimensional objects.** **M.GHS.27** Identify the shapes of two-dimensional cross-sections of three-dimensional			

objects, and identify three-dimensional objects generated by rotations of two-dimensional objects. **Apply geometric concepts in modeling situations.** **M.GHS.28** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). Instructional Note: Focus on situations that require relating two- and three-dimensional objects, determining and using volume, and the trigonometry of general triangles. **CONNECTING ALGEBRA AND GEOMETRY THROUGH COORDINATES** **Use coordinates to prove simple geometric theorems algebraically.** **M.GHS.29** Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2). **M.GHS.30** Prove the slope criteria for parallel and perpendicular lines and uses them to			

solve geometric problems. (e.g., Find the equation of a line parallel or perpendicular to a given line that passes through a given point.) Instructional Note: Relate work on parallel lines to work in High School Algebra I involving systems of equations having no solution or infinitely many solutions. **M.GHS.31** Find the point on a directed line segment between two given points that partitions the segment in a given ratio. **M.GHS.32** Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. This standard provides practice with the distance formula and its connection with the Pythagorean theorem. **Translate between the geometric description and the equation for a conic section.** **M.GHS.33** Derive the equation of a parabola given a focus and directrix. Instructional Note: The directrix should be parallel to a coordinate axis.			

CIRCLES WITH AND WITHOUT COORDINATES **Understand and apply theorems about circles.** **M.GHS.34** Prove that all circles are similar. **M.GHS.35** Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. **M.GHS.36** Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle. **M.GHS.37** Construct a tangent line from a point outside a given circle to the circle. **Find arc lengths and areas of sectors of circles.** **M.GHS.38** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and			

define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Instructional Note: Emphasize the similarity of all circles. Reason that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course. **Translate between the geometric description and the equation for a conic section.** **M.GHS.39** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. **Use coordinates to prove simple geometric theorems algebraically.** **M.GHS.40** Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing			

the point (0, 2).) Instructional Note: Include simple proofs involving circles. **Apply geometric concepts in modeling situations.** **M.GHS.41** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). Instructional Note: Focus on situations in which the analysis of circles is required. **MODELING WITH GEOMETRY** **Visualize relationships between two dimensional and three-dimensional objects and apply geometric concepts in modeling situations.** **M.GHS.53** Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). **M.GHS.54** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot). **M.GHS.55** Apply geometric methods to solve design problems (e.g., designing an			

object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).			

Statistics and Probability

Standards	Teacher Understandings	Resources	Student Understandings
APPLICATIONS OF PROBABILITY **Understand independence and conditional probability and use them to interpret data.** **M.GHS.42** Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not"). **M.GHS.43** Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this	In grades seven and eight, students learn some basic concepts related to probability, including chance processes, probability models, and sample spaces. In higher mathematics, the relative-frequency approach to probability is extended to conditional probability and independence, rules of probability and their use in finding probabilities of compound events, and the use of probability distributions to solve problems involving expected	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the **Mathematical Habits of Mind**, and demonstrate the importance of developing conceptual understanding, procedural skill and	• Students develop an understanding of conditional probability, experiencing two types of problems: those in which the uniform probabilities attached to outcomes lead to independence of the outcomes, and those in which they do not. • Students explore finding probabilities of compound events by using the Addition Rule and the general Multiplication Rule.

characterization to determine if they are independent. **M.GHS.44** Recognize the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. Instructional Note: Build on work with two-way tables from Algebra I to develop understanding of conditional probability and independence. **M.GHS.45** Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other	value (University of Arizona [UA] Progressions Documents 2012d). Building on probability concepts that began in the middle grades, students in the Geometry course use the language of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students should make use of geometric probability models wherever possible and use probability to make informed decisions (National Governors Association Center for Best Practices, Council of Chief State School Officers [NGA/CCSSO]. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting	fluency, and application. Math TREE Online Education Resources A curated set of aligned internet resources for WV middle and high school math teachers. Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	• Students use probability models and probability experiments to make decisions.

subjects and compare the results. Instructional Note: Build on work with two-way tables from Algebra I to develop understanding of conditional probability and independence. **M.GHS.46** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.	grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.		

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