

Frameworks for Mathematics *Grade 3*





West Virginia Board of Education 2018-2019

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Grade 3

In grade three, students continue to build upon their mathematical foundation as they focus on the operations of multiplication and division and the concept of fractions as numbers. Instructional time focuses on four critical areas: (1) developing understanding of multiplication and division, as well as strategies for multiplication and division within 100; (2) developing understanding of fractions, especially unit fractions (fractions with a numerator of 1); (3) developing understanding of the structure of rectangular arrays and of area; and (4) describing and analyzing two-dimensional shapes. Students also work toward fluency with addition and subtraction within 1000 and multiplication and division within 100. By the end of grade three, students know all products of two one-digit numbers from memory.

Operations and Algebraic Thinking

Standards	Teacher Understandings	Resources	Student Understandings
Represent and solve problems involving multiplication and division. M.3.1 Interpret products of whole numbers, e.g., interpret 5 × 7 as the total number of objects in 5 groups of 7 objects each (e.g., describe context in which a total number of objects can be expressed as 5 × 7). M.3.2 Interpret whole-number quotients of whole numbers, e.g., interpret 56 ÷ 8 as the number of objects in each share when 56 objects are partitioned equally into 8 shares, or as a number of shares when 56 objects are partitioned into equal shares of 8 objects each (e.g.,	It is important for teachers to understand that neglecting any grade-level standards will leave gaps in students' skills and understandings. This will leave students unprepared for the challenges they face in later grades. Students use the Mathematical Habits of Mind to interact with the grade level content standards. The teacher needs to craft instructional tasks that connect the Mathematical Habits of	The following provides resources for teachers and students: Math TREE Online Education Resources A curated set of aligned internet resources for WV elementary teachers Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and	 Students understand that multiplication may be used to determine the total number of objects when the specified groups have a specified number of members. Students know how to use a variety of visual models to represent word problems related to each of the multiplicative structures. Students understand multiplication and



describe a context in which a number of shares or a number of groups can be expressed as $56 \div 8$).

M.3.3

Use multiplication and division within 100 to solve word problems in situations involving equal groups, arrays and measurement quantities (e.g., by using drawings and equations with a symbol for the unknown number to represent the problem).

M.3.4 Determine the unknown whole number in a multiplication or division equation relating three whole numbers (e.g., determine the unknown number that makes the equation true in each of the equations $8 \times ? = 48, 5 = ? \div 3, 6 \times 6 = ?$).

Understand properties of multiplication and the relationship between multiplication and division.

M.3.5

Apply properties of operations as strategies to multiply and divide (e.g., If $6 \times 4 = 24$ is known, then $4 \times 6 = 24$ is also known: Commutative Property of Multiplication. $3 \times 5 \times 2$ can be found by $3 \times 5 = 15$, then $15 \times 2 = 30$, or by $5 \times 2 = 10$, then $3 \times 10 = 30$: Associative Property of Multiplication. Knowing that $8 \times 5 = 40$ and $8 \times 2 = 16$, one can find 8×7 as $8 \times (5 + 2) = (8 \times 5) + (8 \times 2)$

Mind to the content standards.

The standards in the Operations and Algebraic Thinking Domain build on the understanding of addition and subtraction developed in grade two. These standards also connect to grade two Geometry standards where students partition a rectangle into rows and columns of the same size.

Grade three students may develop an incomplete understanding of multiplication and division. This will impact their work in later grades as students begin working with numbers other than whole numbers. It is beneficial to begin work with multiplication and division of numbers using contextual problems.

Fluency of multiplication facts are developed using number patterns and number relationships. This fluency is an end of grade identify skills most relevant to standards.

Illustrative Mathematics

http://www.illustrative mathmatics.org

This website provides teachers with learning tasks that develop the WV College- and Career- Readiness Standards for Mathematics and support that extends the teacher's content knowledge of mathematics.

Graham Fletcher Site G Fletchy

http://www.gfletchy.co

This website includes learning progression videos related to counting and 3-Act tasks that may be connected to the WV College- and Career-Readiness Standards for Mathematics.

Inside Mathematics

- division are inverse operations.
- Students understand and use properties of operations to solve multiplication and division problems:
 - Commutative Property of Multiplication
 - Associative Property of Multiplication
 - DistributiveProperty ofMultiplication
 - MultiplicativeIdentity Property
 - MultiplicationProperty of Zero
- Students will fluently multiply with 100 using strategies related number relationships.

Common Misconceptions

- Students may overgeneralize the properties of multiplication to division.
- Students will place the divisors and the



= 40 + 16 = 56: Distributive Property. Instructional Note: Students need not use formal terms for these properties.

M.3.6

Understand division as an unknownfactor problem (e.g., find 32 ÷ 8 by finding the number that makes 32 when multiplied by 8).

Multiply and divide within 100.

M.3.7

Learn multiplication tables (facts) with speed and memory in order to fluently multiply and divide within 100, using strategies such as the relationship between multiplication and division (e.g., knowing that $8 \times 5 = 40$, one knows that $40 \div 5 = 8$) or properties of operations by the end of Grade 3.

Solve problems involving the four operations, and identify and explain patterns in arithmetic.

M.3.8

Solve two-step word problems using the four operations, represent these problems using equations with a letter standing for the unknown quantity. Assess the reasonableness of answers using mental computation and estimation strategies including rounding. Instructional Note: This

three expectation.
Students need multiple opportunities over time to practice these strategies in order to develop fluency.
Games are an effective tool to use for students to practice the strategies.

http://insidemathematics.org

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NCTM Illuminations

https://illuminations.nctm.org/ Illuminations is a

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- dividend in the opposite locations when setting up a problem.
- Students may use the words "goes into" which carries little meaning in division. Students may say "6 goes into 24" instead of "24 divided by 6"



standard is limited to problems posed		
with whole numbers and having whole		
number answers; students should know	1	
how to perform operations in the		
conventional order when there are no		
parentheses to specify a particular		
order (Order of Operations).		
M.3.9		
Identify arithmetic patterns (including		
patterns in the addition table or		
multiplication table) and explain those		
using properties of operations (e.g.,		
observe that 4 times a number is		
always even and explain why 4 times a		
number can be decomposed into two		
equal addends).		

Number and Operations in Base Ten

Standards	Teacher Understandings	Resources	Student Understandings
Use place value understanding and properties of operations to perform multi-digit arithmetic.	It is important for teachers to understand that neglecting any grade-level standards will leave gaps in	The following is a list of resources for teachers and students:	Students use place value to round numbers to the nearest 10 or hundred.
M.3.10 Use place value understanding to round whole numbers to the nearest 10 or 100.	students' skills and understandings. This will leave students unprepared for the challenges they face in later grades.	Math TREE Online Education Resources A curated set of aligned internet resources for WV elementary	 Students use place value understanding to perform multi-digit arithmetic. Students are able to
M.3.11 Fluently add and subtract within 1000 using strategies and algorithms based on place value, properties of	Students use the Mathematical Habits of Mind to interact with the	Quantile Teacher Assistant	compose/decompose numbers in order to perform



operations, and/or the relationship between addition and subtraction.

M.3.12

Multiply one-digit whole numbers by multiples of 10 in the range 10-90 (e.g., 9×80 , 5×60) using strategies based on place value and properties of operations.

grade level content standards. The teacher needs to craft instructional tasks that connect the Mathematical Habits of Mind to the content standards.

Students in grade three begin to solve addition and subtraction problems using numbers within 1000..
Students continue to solve problems using strategies based on place value and relationships. The teacher may have students who are transitioning to use the standard algorithm; however, this is not a grade level expectation until grade four.

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This website includes learning progression videos related to counting and 3-Act tasks that may be connected to the WV addition/subtraction computation.

Common Misconceptions

when students subtract multi-digit numbers that require them to decompose a hundred into 10 tens or a ten into 10 ones, they will simply subtract the small digit from the larger digit without considering the understanding of the problem.



College- and Career-Readiness Standards for Mathematics. **Inside Mathematics** http://insidemathemat ics.org Inside Mathematics is a nationally recognized multimedia website for educators around the world. This site includes videos, learning tasks, and performance assessment tasks. **NCTM Illuminations** https://illuminations.n ctm.org/ Illuminations is a project designed by NCTM. The site includes lessons, activities and computer applets. **Math Coach's Corner Donna Boucher** http://www.mathcoach scorner.com This site is a blog by an elementary mathematics coach. Her blog includes



	mathematical background on concepts; as well as, mathematical tasks.	

Number and Operations Fractions

Standards	Teacher Understandings	Resources	Student Understandings
Develop understanding of fractions as numbers. M.3.13 Understand a fraction 1/b as the quantity formed by 1 part when a whole is partitioned into b equal parts; understand a fraction a/b as the quantity formed by a parts of size 1/b. Instructional Note: Fractions in this	It is important for teachers to understand that neglecting any grade-level standards will leave gaps in students' skills and understandings. This will leave students unprepared for the challenges they face in later grades.	The following is a list of resources for teachers and students: Math TREE Online Education Resources A curated set of aligned internet resources for WV elementary teachers	fractional parts must be equal sized. Students express fractions as fair sharing, parts of a whole and parts of a set. Students transfer their understanding of
standard are limited to denominators of 2, 3, 4, 6, and 8. M.3.14 Understand a fraction as a number on	Students use the Mathematical Habits of Mind to interact with the grade level content standards. The teacher	Quantile Teacher Assistant This tool is aligned to WV standards and is	fractions to the number line. • Students reason about size to compare fractions.
the number line and represent fractions on a number line diagram. a. Represent a fraction 1/b on a number line diagram by defining the interval from 0 to 1 as the whole and partitioning it into b equal parts. Recognize that each part has size 1/b	needs to craft instructional tasks that connect the Mathematical Habits of Mind to the content standards. The standards in the	designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	 Students may identify a shape as being divided into three unequal pieces as being divided into thirds. They do not
and that the endpoint of the part based at 0 locates the number 1/b on the	Number and Operations Fraction Domain develop an	Illustrative Mathematics	consider that each piece must be the same size.



number line. (e.g., Given that b parts is 4 parts, then 1/b represents 1/4. Students partition the number line into fourths and locate 1/4 on the number line.)

b. Represent a fraction a/b on a number line diagram by marking off a lengths 1/b from 0. Recognize that the resulting interval has size a/b and that its endpoint locates the number a/b on the number line. (e.g., Given that a/b represents 3/4 or 6/4, students partition the number line into fourths and represent these fractions accurately on the same number line; students extend the number line to include the number of wholes required for the given fractions.) Instructional Note: Fractions in this standard are limited to denominators of 2, 3, 4, 6, and 8.

M.3.15

Explain equivalence of fractions in special cases and compare fractions by reasoning about their size.

- a. Understand two fractions as equivalent (equal) if they are the same size or the same point on a number line.
- b. Recognize and generate simple equivalent fractions (e.g., 1/2 = 2/4, 4/6 = 2/3). Explain why the fractions are equivalent (e.g., by using a visual fraction model).

understanding of fraction as numbers. Students will use various visual representations including number line, area model, bar representations, and tape diagrams.

The standards in the Number and Operations Fraction domain focus on building an understanding of fractions and NOT on operations related to fractions. Operations related to fractions begin in grade four and extend to grade six.

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Inside Mathematics

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- Students may not identify a shape as being divided into thirds if the parts do not have the same shape. They focus on the shape and not on the size.
- believe that a fraction can be placed on the number line. They do not see a fraction as a number.
- Students do not understand that there are an infinite number of numbers between each whole number.
- Students do not understand the number line as a distance model.



- c. Express whole numbers as fractions, and recognize fractions that are equivalent to whole numbers. (e.g., Express 3 in the form 3 = 3/1; recognize that 6/1 = 6; locate 4/4 and 1 at the same point of a number line diagram.)
- d. Compare two fractions with the same numerator or the same denominator by reasoning about their size. Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with the symbols >, = or < and justify the conclusions (e.g., by using a visual fraction model). Instructional Note: Fractions in this standard are limited to denominators of 2, 3, 4, 6, and 8.

world. This site includes videos, learning tasks, and performance assessment tasks.

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Measurement and Data

Standards	Teacher Understandings	Resources	Student Understandings
	In the transfer of Court of the	The College Control of the Co	
Solve problems involving measurement	It is important for teachers	The following is a list of	 Students tell time to
and estimation of intervals of time,	to understand that	resources for teachers and	the nearest minute
liquid volumes, and masses of objects.	neglecting any grade-level	students:	and solve problems
	standards will leave gaps in		related to intervals
M.3.16	students' skills and	Math TREE Online	of time.
Tell and write time to the nearest	understandings. This will	Education Resources	 Students extend
minute, measure time intervals in	leave students unprepared	A curated set of aligned	their knowledge of
minutes. Solve word problems	for the challenges they face	internet resources for WV	measurement to
involving addition and subtraction of	in later grades.	elementary teachers	volume and mass.
time intervals in minutes (e.g., by representing the problem on a number	Students use the		 Students create
line diagram).	Mathematical Habits of	Quantile Teacher Assistant	scaled picture and
tille diagrami.	Mind to interact with the	This tool is aligned to WV	bar graphs. (1 picture
M.3.17	grade level content	standards and is designed	represents a count of
Measure and estimate liquid volumes	standards. The teacher	to help educators locate	5)
and masses of objects using standard	needs to craft instructional	resources that can support	 Students measure to
units of grams (g), kilograms (kg) and	tasks that connect the	instruction and identify	the nearest ½ and ¼
liters (l). Add, subtract, multiply or	Mathematical Habits of	skills most relevant to	of the unit of
divide to solve one-step word problems	Mind to the content	standards.	
involving masses or volumes that are	standards.		measure.
given in the same units (e.g., by using		Illustrative Mathematics	Students understand
drawings, such as a beaker with a	The standards in the	http://www.illustrativemat	area as an attribute
measurement scale) to represent the	Measurement and Data	hmatics.org	of a plane figure.
problem. Instructional Note: Exclude	domain directly relate to	This website provides	Area is the number
compound units such as cm ³ and	the standards in the	teachers with learning	of unit squares
finding the geometric	Operation and Algebraic	tasks that develop the WV	needed to cover a
volume of a container.	Thinking domain and the	College- and Career-	plane figure without
	Number and Operations	Readiness Standards for	gaps or overlaps.
Represent and interpret data.	Base Ten domain.	Mathematics and support	 Students measure
		that extends the teacher's	the area of a plane
M.3.18			shape by counting



Draw a scaled picture graph and a scaled bar graph to represent a data set with several categories. Solve one-and two-step "how many more" and "how many less" problems using information presented in scaled bar graphs (e.g., draw a bar graph in which each square in the bar graph might represent 5 pets).

M.3.19

Generate measurement data by measuring lengths using rulers marked with halves and fourths of an inch. Show the data by making a line plot, where the horizontal scale is marked off in appropriate units—whole numbers, halves or quarters.

Geometric measurement: understand concepts of area and relate area to multiplication and to addition.

M.3.20

Recognize area as an attribute of plane figures and understand concepts of area measurement.

- a. A square with side length 1 unit, called "a unit square," is said to have "one square unit" of area and can be used to measure area.
- b. A plane figure which can be covered without gaps or overlaps by b unit squares is said to have an area of b square units.

Grade three students need to develop an understanding of area and perimeter and how to calculate area. It is not a grade three requirement for students to use the formulas to calculate the measurements.

A rectilinear shape is a polygon that can be divided into smaller rectangles. This enables the student to easily calculate the area of the shape.

content knowledge of mathematics.

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learning progression
videos related to counting
and 3-Act tasks that may
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- the squares needed to cover the shape and relating area to multiplication.
- Students develop an understanding of perimeter as the distance around a shape.

Common Misconceptions

 Students use the terms area and perimeter interchangeably.



M.3.21

Measure areas by counting unit squares (square cm, square m, square in, square ft. and improvised units).

- M.3.22 Relate area to the operations of multiplication and addition.
- a. Find the area of a rectangle with whole-number side lengths by tiling it, and show that the area is the same as would be found by multiplying the side lengths.
- b. Multiply side lengths to find areas of rectangles with whole number side lengths in the context of solving real world and mathematical problems, and represent whole-number products as rectangular areas in mathematical reasoning.
- c. Use tiling to show in a concrete case that the area of a rectangle with whole-number side lengths a and b + c is the sum of a × b and a × c. Use area models to represent the distributive property in mathematical reasoning.
- d. Recognize area as additive and find areas of rectilinear figures by decomposing them into non-overlapping rectangles and adding the areas of the non-overlapping parts, applying this technique to solve real world problems.

Geometric measurement: recognize

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perimeter as an attribute of plane figures and distinguish between linear and area measures.	
M.3.23 Solve real world and mathematical problems involving perimeters of polygons, including finding the perimeter given the side lengths, finding an unknown side length and exhibiting rectangles with the same perimeter and different areas or with the same area and different perimeters.	

Geometry

Standards	Teacher Understandings	Resources	Student Understandings
Reason with shapes and their attributes. M.3.24 Understand that shapes in different categories (e.g., rhombuses, rectangles, and others) may share attributes (e.g., having four sides), that the shared attributes can define a larger category (e.g. quadrilaterals). Recognize rhombuses, rectangles, and squares as examples of quadrilaterals, and draw examples of quadrilaterals that do not belong to any of these subcategories.	It is important for teachers to understand that neglecting any grade-level standards will leave gaps in students' skills and understandings. This will leave students unprepared for the challenges they face in later grades. Students use the Mathematical Habits of Mind to interact with the grade level content	The following is a list of resources for teachers and students: Math TREE Online Education Resources A curated set of aligned internet resources for WV elementary teachers Quantile Teacher Assistant This tool is aligned to WV standards and is designed	 Students understand that shapes are categorized based upon attributes and each category may have subsets. Students partition shapes into equal parts and identify the fractional representation of each part.
M.3.25	standards. The teacher	to help educators locate	



Partition shapes into parts with equal areas. Express the area of each part as a unit fraction of the whole. For example, partition a shape into 4 parts with equal area, and describe the area of each part as 1/4 or the area of the shape.

needs to craft instructional tasks that connect the Mathematical Habits of Mind to the content standards.

The partitioning of shapes in to equal parts with equal areas relate the Number and Operations Fraction domain.

Shapes can be classified into categories with each category having subsets of shapes with more specific characteristics.

Quadrilaterals-rectanglessquares resources that can support instruction and identify skills most relevant to standards.

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