Frameworks for Mathematics Grade 4

West Virginia Board of Education 2018-2019

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## Grade 4

Instructional time should focus on three critical areas: (1) developing understanding and fluency with multi-digit multiplication and developing understanding of dividing to find quotients involving multi-digit dividends; (2) developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of fractions by whole numbers; and (3) understanding that geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and Students also work toward fluency in addition and subtraction within 1,000,000 using the standard algorithm.

## Operations and Algebraic Thinking

| Standards | Teacher Understandings | Resources | Student Understandings |
| :---: | :---: | :---: | :---: |
| Use the four operations with whole numbers to solve problems. <br> M.4.1 <br> Interpret a multiplication equation as a comparison (e.g., interpret $35=5 \times 7$ as a statement that 35 is 5 times as many as 7 and 7 times as many as 5). Represent verbal statements of multiplicative comparisons as multiplication equations. <br> M.4.2 <br> Multiply or divide to solve word problems involving multiplicative comparison (e.g., by using drawings and equations with a symbol for | It is important for teachers to understand that neglecting any gradelevel standards will leave gaps in students' skills and understandings. This will leave students unprepared for the challenges they face in later grades. <br> Students use the Mathematical Habits of Mind to interact with the grade level content standards. The teacher needs to craft instructional tasks that connect the Mathematical Habits of Mind to the content standards. <br> Students need to understand if the value in an equation or an expression can change (or vary), we replace it with a symbol called a variable. Remember, an equation contains an equal sign and an expression does not contain an equal sign. | The following provides resources for teachers and students: <br> Math TREE Online <br> Education Resources <br> A curated set of aligned internet resources for WV elementary teachers <br> Quantile Teacher Assistant <br> This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills | - Develop an understanding of the order and meaning of numbers in multiplicative comparison problems. <br> - Know multiplication terminology: The numbers multiplied are called factors. The answer is called a product. <br> - Know division terminology: The number divided is called the dividend; what it is divided by is the divisor; the |



## Determine whether a given

 whole number in the range 1 100 is prime or composite.
## Generate and analyze patterns.

## M.4.5

Generate a number or shape pattern that follows a given rule. Identify apparent features of the pattern that were not explicit in the rule itself. (e.g., Given the rule "Add 3 " and the starting number 1 , generate terms in the resulting sequence and observe that the terms appear to alternate between odd and even numbers. Explain informally why the numbers will continue to alternate in this way.)

Using tape diagrams or bar models can help students visualize and solve multiplication and division word problems.


Encourage representation of the remainder in a division problem in a manner consistent with the content of the problem. For example, in a simple division problem, $100 \div 33=3 \mathrm{R} 1$ is not as accurate as $100 \div 33=3 \frac{1}{33}$ or by saying "When we divide 100 by 33 , the quotient is 3 with 1 left over." Writing the equation as $100=33 \times 3+1$ is also accurate. Students need to be able to interpret the remainder: If 250 students travel on buses that hold 45 students

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 a nationally recognized multimedia website for educators around the world. This site includes videos, learning tasks, and performance assessment tasks.
## NCTM Illuminations

https://illuminations.n ctm.org/
Illuminations is a project designed by NCTM. The site includes lessons, activities and computer applets.

## Math Coach's Corner

## Donna Boucher

 http://www.mathcoac hscorner.comThis site is a blog by an elementary mathematics coach. Her blog includes mathematical background on concepts, as well as, mathematical tasks.
larger number" concept is no longer true.

- "More than" can be both additive ( 5 more than a number) and multiplicative (3 times a number)
- Find all factor pairs for whole numbers from 1 to 100.
- Understand the difference between Prime (exactly 2 unique whole number factors) and Composite (more than 2 factors).
- Know that " 1 " is neither Prime nor composite.
- Be able to search systematically to find all factor pairs by checking if 2 is a factor, then 3 , then 4 , and so on until they find a "reversal" in the pairs.
- Know that not all prime numbers are






## Number and Operations in Base Ten

## Standards

## Generalize place value understanding for multi-digit whole numbers.

## M.4.6

Recognize that in a multi-digit whole number, a digit in one place represents

It is important for teachers to understand that neglecting any grade-level standards will leave gaps in students' skills and understandings. Therefore,

The following resources provide teachers with resources for teachers and students:

- Know place value positions
- Be able to expand numerals
ten times what it represents in the place to its right (e.g., recognize that $700 \div 70=10$ by applying concepts of place value and division).


## M.4.7

Read and write multi-digit whole numbers using base-ten numerals, number names, and expanded form. Compare two multi-digit numbers based on meanings of the digits in each place, using >, = and < symbols to record the results of comparisons.

## M.4.8

Use place value understanding to round multi-digit whole numbers to any place.

## Use place value understanding and properties of operations to perform multi-digit arithmetic.

## M.4.9

Fluently add and subtract multi-digit whole numbers using the standard algorithm.
M.4.10

Multiply a whole number of up to four digits by a one-digit whole number, multiply two two-digit numbers, using strategies based on place value and the properties of operations and illustrate and explain the calculation by using
students will be unprepared for the challenges they face in later grades.

## Students use the

## Mathematical Habits of

Mind to interact with the grade level content standards. The teacher needs to craft instructional tasks that connect the Mathematical Habits of Mind to the content standards.

To represent multiplication instruct students to use equations, rectangular arrays and/or area models.

Instruct students to use commas to assist in reading and writing numerals.

Math TREE Online
Education Resources A curated set of aligned internet resources for WV elementary teachers

## Quantile Teacher

## Assistant

This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.

## Illustrative Mathematics

http://www.illustrative mathmatics.org
This website provides teachers with learning tasks that develop the WV College- and CareerReadiness Standards for Mathematics and that support the extension of the teacher's content knowledge of mathematics.

## Graham Fletcher Site G

 Fletchyhttp://www.gfletchy.co

- Round multi-digit whole numbers
- Stack numbers vertically to see relationships when rounding
- Work from inner to outer grouping symbols.
- When working with expressions do not insert equal signs changing the expression into an equation.
- The word "of" implies multiplication.

| equations, rectangular arrays and/or area models. <br> M.4.11 <br> Find whole-number quotients and remainders with up to four-digit dividends and one-digit divisors, using strategies based on place value, the properties of operations and/or the relationship between multiplication and division. Illustrate and explain the calculation by using equations, rectangular arrays and/or area models. |  | This website includes learning progression videos related to counting and 3-Act tasks that may be connected to the WV College- and Career- Readiness Standards for Mathematics. <br> Inside Mathematics http://insidemathemati cs.org <br> Inside Mathematics is a nationally recognized multimedia website for educators around the world. This site includes videos, learning tasks, and performance assessment tasks. <br> NCTM Illuminations <br> https://illuminations.nc tm.org/ <br> Illuminations is a project designed by NCTM. The side includes lessons, activities and computer applets. <br> Math Coach's Corner Donna Boucher http://www.mathcoachs corner.com |
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|  |  | This site is a blog by an <br> elementary <br> mathematics coach. Her <br> blog includes <br> mathematical <br> background on <br> concepts, as well as, <br> mathematical tasks. |  |
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## Number and Operations Fractions

## Standards

## Extend understanding

 of fraction equivalence and ordering.M.4.12

Explain why a fraction $a / b$ is equivalent to a fraction $(n \times a) /(n \times b)$ by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

Teacher Understandings
It is important for teachers to understand that neglecting any grade-level standards will leave gaps in students' skills and understandings. This will leave students unprepared for the challenges they face in later grades.

Students use the Mathematical Habits of
Mind to interact with the grade level content standards. The teacher needs to craft instructional tasks that connect the Mathematical Habits of Mind to the content standards.

Help student make sense of writing mixed numbers as unit fractions. Visual models such as the one below support student understanding.

Resources

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Student Understandings

- Understand a fraction, $\frac{a}{b}$ as a sum of the unit fractions, $\frac{1}{b}$.
- Write and use unit fractions.
- Understand that every fraction is composed of unit fractions.
- A mixed number is the sum of a whole number and a fraction.
- Know how to convert mixed numbers to fractions.


Compare two fractions with different numerators and different denominators (e.g., by creating common denominators numerators, or by comparing to a benchmark fraction such as $1 / 2$ ). Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, = or <, and justify the conclusions by using a visual fraction model.

Build fractions from unit fractions by applying and extending previous understandings of operations on whole numbers.
M.4. 14

Understand the fraction a/b, with $a>1$, as the sum of a of the fractions
a. Understand
addition and

| subtraction of fractions as joining and separating parts referring to the same whole. <br> b. Decompose a fraction into a sum of fractions with the same denominator in more than one way, recording each decomposition by an equation and justify decompositions by using a visual fraction model (e.g., $3 / 8=1 / 8+$ $1 / 8+1 / 8 ; 3 / 8=1 / 8+$ $2 / 8 ; 21 / 8=1+1+1 / 8=$ $8 / 8+8 / 8+1 / 8)$. <br> c. Add and subtract mixed numbers with like denominators by replacing each mixed number with an equivalent fraction and/or by using properties of operations and the relationship between addition and subtraction. <br> d. Solve word problems involving addition and subtraction of fractions referring to the same whole and having like |  | Inside Mathematics is a nationally recognized multimedia website for educators around the world. This site includes videos, learning tasks, and performance assessment tasks. <br> NCTM Illuminations <br> https://illuminations.nc tm.org/ <br> Illuminations is a project designed by NCTM. The side includes lessons, activities and computer applets. <br> Math Coach's Corner Donna Boucher <br> http://www.mathcoachs corner.com <br> This site is a blog by an elementary mathematics coach. Her blog includes mathematical background on concepts; as well as, mathematical tasks. |
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| denominators by using visual fraction models and equations to represent the problem. <br> M.4.15 <br> Apply and extend previous understandings of multiplication to multiply a fraction by a whole number. <br> a. Understand a fraction $a / b$ as a multiple of $1 / \mathrm{b}$, (e.g., use a visual fraction model to represent 5/4 as the product $5 \times(1 / 4)$, recording the conclusion by the equation $5 / 4=5 \times(1 / 4)$ ). <br> b. Understand a multiple of $a / b$ as a multiple of $1 / b$, and use this understanding to multiply a fraction by a whole number (e.g., use a visual fraction model to express $3 \times(2 / 5)$ as 6 $\times(1 / 5)$, recognizing this product as $6 / 5$. In general, $\mathrm{n} \times(\mathrm{a} / \mathrm{b})=(\mathrm{n} \times$ a)/b). <br> c. Solve word problems involving |  |  |  |
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multiplication of a
fraction by a whole
number by using visual
fraction models and
equations to represent
the problem (e.g., If
each person at a party
will eat 3/8 of a pound
of roast beef, and there
will be 5 people at the
party, how many pounds
of roast beef will be
needed? Between what
two whole numbers
does your answer lie?).
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Understand decimal
notation for fractions,
and compare decimal
fractions.
M.4.16
Express a fraction with
denominator 10 as an
equivalent fraction with
denominator 100, and
use this technique to
add two fractions with
respective
denominators }10\mathrm{ and
100 (e.g., express 3/10 as
30/100, and add 3/10 +
4/100 = 34/100).
Instructional Note:
Students who can
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| generate equivalent |  |  |
| :--- | :--- | :--- | :--- |
| fractions can develop |  |  |
| strategies for adding |  |  |
| fractions with unlike |  |  |
| denominators in |  |  |
| general. But addition |  |  |
| and subtraction with |  |  |
| unlike denominators in |  |  |
| general is not a |  |  |
| requirement at this |  |  |
| grade. |  |  |
| M.4.17 |  |  |
| Use decimal notation for |  |  |
| fractions with |  |  |
| denominators 10 or 100 |  |  |
| (e.g., rewrite 0.62 as |  |  |
| 62/100; describe a |  |  |
| length as 0.62 meters; |  |  |
| locate 0.62 on a number |  |  |
| line diagram). |  |  |
| M.4.18 |  |  |
| Compare two decimals |  |  |
| to hundredths by |  |  |
| reasoning about their |  |  |
| size. Recognize that |  |  |
| comparisons are valid |  |  |
| only when the two |  |  |
| decimals refer to the |  |  |
| same whole. Record the |  |  |
| results of comparisons |  |  |
| with the symbols >, = or |  |  |
| <, and justify the |  |  |
| conclusions by using a |  |  |

## visual model.

## Measurement and Data

## Solve problems involving measurement

 and conversion of measurements from a larger unit to a smaller unit.M.4.19 Know relative sizes of measurement units within a system of units, including the metric system (km, $\mathrm{m}, \mathrm{cm} ; \mathrm{kg}, \mathrm{g} ; \mathrm{l}, \mathrm{ml})$, the standard system (lb, oz), and time (hr, min, sec.). Within a single system of measurement, express measurements in a larger unit in terms of a smaller unit. Record measurement equivalents in a twocolumn table. (e.g., Know that 1 ft is 12 times as long as 1 in . Express the length of a 4 ft snake as 48 in . Generate a conversion table for feet and inches listing the number pairs $(1,12),(2,24)$, $(3,36), \ldots$ )

## M.4.20

Use the four operations to solve word problems involving distances, intervals of time, liquid volumes, masses of objects, and money, including problems involving simple fractions or decimals and problems that require expressing measurements given in a larger unit in

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## Students use the

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Work with angle measures connects to and supports addition of fractions.

Students need to gain experience measuring angles prior to working with

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- Understand how to record information on a line plot.
- Have experience with benchmark angles.
- Transfer understanding that a $360^{\circ}$ rotation about a point makes a complete circle to recognize and sketch angles that measure approximately $90^{\circ}$ and $180^{\circ}$. This understanding is extended to recognize and sketch angles that measure approximately $45^{\circ}$ and $30^{\circ}$.
- Use appropriate terminology (acute, right, and obtuse) to describe angles and rays (perpendicular).

| terms of a smaller unit. Represent measurement quantities using diagrams such as number line diagrams that feature a measurement scale. <br> M.4.21 A <br> pply the area and perimeter formulas for rectangles in real world and mathematical problems by viewing the area formula as a multiplication equation with an unknown factor. (e.g., find the width of a rectangular room given the area of the flooring and the length.) <br> Represent and interpret data. <br> M.4.22 <br> Make a line plot to display a data set of measurements in fractions of a unit ( $1 / 2,1 / 4,1 / 8$ ). Solve problems involving addition and subtraction of fractions by using information presented in line plots (e.g., from a line plot find and interpret the difference in length between the longest and shortest specimens in an insect collection). <br> Geometric measurement: understand concepts of angle and measure angles. <br> M.4.23 <br> Recognize angles as geometric shapes that are formed wherever two rays | word problems involving angle measurement. <br> Students need to understand geometric terms prior to solving word problems. <br> When a new concept is introduced, concrete objects help students visualize the meaning. <br> Base-ten blocks are a useful tool when developing an understanding of equivalent representations. <br> Previously, students built a conceptual understanding of multiplication with whole numbers as they applied multiple strategies to compute and solve problems. Students can continue to use different strategies and methods learned previously-as long as the methods are efficient. | Mathematics and support that extends the teacher's content knowledge of mathematics. <br> Graham Fletcher Site G Fletchy <br> http://www.gfletchy.co m <br> This website includes learning progression videos related to counting and 3-Act tasks that may be connected to the WV College- and Career- Readiness Standards for Mathematics. <br> Inside Mathematics http://insidemathemati cs.org <br> Inside Mathematics is a nationally recognized multimedia website for educators around the world. This site includes videos, learning tasks, and performance assessment tasks. <br> NCTM Illuminations https://illuminations.nc tm.org/ | - Recognize angle measure as additive and use this to solve addition and subtraction problems to find unknown angles on a diagram. |
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share a common endpoint, and
understand concepts of angle
measurement:
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a. An angle is measured with reference to a circle with its center at the common endpoint of the rays, by considering the fraction of the circular arc between the points where the two rays intersect the circle. An angle that turns through $1 / 360$ of a circle is called a "one-degree angle," and can be used to measure angles.
b. An angle that turns through b one-degree angles is said to have an angle measure of $b$ degrees.

## M.4.24

Measure angles in whole-number degrees using a protractor and sketch angles of specified measure.

## M.4.25

Recognize angle measure as additive. When an angle is decomposed into non-overlapping parts, the angle measure of the whole is the sum of the angle measures of the parts. Solve addition and subtraction problems to find unknown angles on a diagram in real world and mathematical problems (e.g., by using an equation with a symbol for the unknown angle measure).

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## Geometry

| Standards | Teacher Understandings | Resources | Student Understandings |
| :---: | :---: | :---: | :---: |
| Draw and identify lines and angles and classify shapes by properties of their lines and angles. <br> M.4.26 <br> Draw points, lines, line segments, rays, angles (right, acute, obtuse) and perpendicular and parallel lines. Identify these in two-dimensional figures. <br> M.4.27 <br> Classify two-dimensional figures based on the presence or absence of parallel or perpendicular lines or the presence or absence of angles of a specified size. Recognize right triangles as a category, and identify right triangles. <br> M.4.28 <br> Recognize a line of symmetry for a twodimensional figure as a line across the figure such that the figure can be folded along the line into matching parts. Identify line-symmetric figures and draw lines of symmetry. | It is important for teachers to understand that neglecting any grade-level standards will leave gaps in students' skills and understandings. This will leave students unprepared for the challenges they face in later grades. <br> Students use the Mathematical Habits of Mind to interact with the grade level content standards. The teacher needs to craft instructional tasks that connect the Mathematical Habits of Mind to the content standards. <br> Geometric figures can be analyzed and classified based on their properties, such as having parallel sides, perpendicular sides, particular angle measures, and symmetry. <br> For the first time, students in grade four are exposed to | The following is a list of resources for teachers and students: <br> Math TREE Online Education Resources A curated set of aligned internet resources for WV elementary teachers <br> Quantile Teacher Assistant <br> This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards. <br> Illustrative Mathematics http://www.illustrative mathmatics.org <br> This website provides teachers with learning tasks that develop the WV College- and CareerReadiness Standards for Mathematics and that support the extension | - Draw and identify right, acute, and obtuse angles. <br> - Classify twodimensional figures based on specific angle measurements. <br> - Use the benchmark angles of $90^{\circ}, 180^{\circ}$, and $360^{\circ}$ to approximate the measurement of angles. <br> - Know that right triangles (triangles with one right angle) can be a category for classification, with subcategories-for example, an isosceles right triangle has two or more congruent sides and a scalene right triangle has no congruent sides. |

$\left.\begin{array}{|l|l|l|l|}\hline & \begin{array}{l}\text { the concepts of rays, angles, } \\ \text { and perpendicular and } \\ \text { parallel lines. }\end{array} & \begin{array}{l}\text { ofs the teacher's } \\ \text { content knowledge of } \\ \text { mathematics. }\end{array} \\ \text { It is helpful to provide }\end{array} \quad \begin{array}{l}\text { Graham Fletcher Site G } \\ \text { Fletchy }\end{array}\right]$

|  | to characteristics, such as <br> the presence of parallel or <br> perpendicular lines or by <br> angle measurements. <br> Students may use <br> transparencies with lines <br> drawn on them to arrange <br> two lines in different ways <br> to determine that the two includes <br> lines might intersect at one <br> point or might never <br> intersect, thereby <br> understanding the notion of <br> parallel lines. Further <br> investigations may be <br> initiated with geometry <br> lessons, activities and <br> software. These types of <br> explorations may lead to a <br> discussion on angles. | Math Coach's Corner <br> Donna Boucher <br> http://www.mathcoachs <br> This site is a blog by an <br> elementary <br> mathematics coach. Her <br> blog includes <br> mathematical <br> background on <br> concepts; as well as, <br> mathematical tasks. |
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Steven L. Paine, Ed.D.
West Virginia Superintendent of Schools

