Frameworks for Mathematics Grade 7

West Virginia Board of Education 2018-2019

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## Grade Seven

As students enter grade seven, they have an understanding of variables and how to apply properties of operations to write and solve simple one-step equations. They are fluent in all positive rational number operations. Students who are entering grade seven have been introduced to ratio concepts and applications, concepts of negative rational numbers, absolute value, and all four quadrants of the coordinate plane. They have a solid foundation for understanding area, surface area, and volume of geometric figures and have been introduced to statistical variability and distributions.

## Ratios and Proportional Relationships

Standards
Teacher Understandings
A critical area of instruction in grade seven is developing an understanding and application of proportional relationships, including percentages. In grade seven, students extend their reasoning about ratios and proportional relationships in several ways. Students use ratios in cases that involve pairs of rational number entries and compute associated rates. They identify unit rates in representations of proportional relationships and work with equations in two variables to represent and analyze proportional relationships. They also solve multi-step ratio and percent

Resources

## Educators' Guide

Organized by Grade Seven domains, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application.

Math TREE Online Education Resources A curated set of aligned, internet resources for WV

Student Understandings

- Students will find unit rates in ratios involving fractional quantities.
- Proportional quantities are represented in a table, pairs of entries represent equivalent ratios.
- The graph of a proportional relationship lies on a straight line that passes through the origin.
- Equations of proportional relationships in a ratio of $a$ : $b$ always take the form, $y=k \cdot x$ where $k$ is the constant $\frac{a}{b}$ if the variables $x$ and $y$ are defined so that the ratio $x: y$ is equivalent to $a$ : $b$. (The number $k$ is also known as the constant of
b. Identify the constant of
proportionality (unit rate) in tables, graphs, equations, diagrams and verbal descriptions of proportional relationships.
c. Represent proportional relationships by equations. (e.g., If total cost $t$ is proportional to the number $n$ of items purchased at a constant price $p$, the relationship between the total cost and the number of items can be expressed as $t=$ pn.)
d. Explain what a point $(x, y)$ on the graph of a proportional relationship means in terms of the situation. Focus special attention on the points $(0,0)$ and $(1, r)$ where $r$ is the unit rate.


## M.7.3

Use proportional relationships to solve multistep ratio and percent problems (e.g., simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, and/or percent error).
problems, such as problems involving percent increase and decrease.

## Content by Cluster

Teachers must provide students opportunity to master each grade-level content standard. This document provides a sample content plan for Grade Seven. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.
middle and high school math teachers.

## Quantile Teacher

## Assistant

This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.

Open-Up Resources This resource provides opportunities for students to learn math by doing math, solving problems in mathematical and realworld contexts, and constructing arguments using precise language. Tasks spark discussion, perseverance, and enjoyment of mathematics. Highleverage routines guide students in understanding and making connections between concepts and procedures.
proportionality)

- Students extend their work of grade six percent problems to solve multi-step ratio and percent problems.


## Common Misconceptions

- For a ratio $a: b$ with $a$ and $b \neq 0$, the unit rate is the number $\frac{a}{b}$.
- A typical strategy for solving proportional relationship problems has been to "set up a proportion and crossmultiply." Instead, students should be prompted to reason about solution strategies and why they work. Setting up an equation to solve a proportional relationship problem makes perfect sense if students understand that they are setting unit rates equal to each other. However, introducing the term proportion (or proportion equation) may needlessly clutter up the curriculum; rather, students should see

|  |  |  | this as setting up an <br> equation in a single variable. |
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## The Number System

## Standards

## Apply and extend previous understandings

 of operations with fractions to add, subtract, multiply, and divide rational numbers.
## M.7.4

Apply and extend previous understandings of addition and subtraction to add and subtract rational numbers; represent addition and subtraction on a horizontal or vertical number line diagram.
a. Describe situations in which opposite quantities combine to make 0 . (e.g., A hydrogen atom has 0 charge because its two constituents are oppositely charged.)
b. Understand $\mathrm{p}+\mathrm{q}$ as the number located a distance |q| from $p$, in the positive or negative direction, depending on whether $q$ is positive or negative. (i.e., To add " $p+q$ " on the number line, start at " 0 " and move to " p " then move $|\mathrm{q}|$ in the positive or negative direction depending on whether " q " is positive or negative.) Show that a number and its opposite have a sum

Teacher Understandings
In grade six, students complete their understanding of division of fractions and achieve fluency with multi-digit division and multi-digit decimal operations. They also work with concepts of positive and negative rational numbers. They learned about signed numbers and the types of quantities that can be represented with these numbers. Students located signed numbers on a number line and, as a result of this study, should have concluded that the negative side of the number line is a mirror reflection of the positive side. Grade-six students also learned about absolute value and ordering of rational numbers, including in realworld contexts. In grade seven, a critical area of instruction is developing an understanding

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Quantile Teacher
Assistant
This tool is aligned to WV

Student Understandings

- Students represent addition and subtraction with positive and negative rational numbers on a horizontal or vertical number line diagram.
- Students add and subtract, understanding $p+q$ as the number located a distance $|q|$ from $p$ on a number line, in the positive or negative direction depending on whether $q$ is positive or negative.
- Students demonstrate that a number and its opposite have a sum of 0 (meaning they are additive inverses) and understand subtraction of rational numbers as adding the additive inverse.
- Students understand that when adding two numbers with different signs, the sum

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of 0 (are additive inverses). Interpret sums
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of rational numbers by describing real-
world contexts.
c. Understand subtraction of rational numbers as adding the additive inverse, $\mathrm{p}-$ $q=p+(-q)$. Show that the distance between two rational numbers on the number line is the absolute value of their difference and apply this principle in realworld contexts.
d. Apply properties of operations as strategies to add and subtract rational numbers.

## M.7.5

Apply and extend previous understandings of multiplication and division and of fractions to multiply and divide rational numbers.
a. Understand that
multiplication is extended from fractions to
rational numbers by requiring that
operations continue to satisfy the
properties of operations, particularly the
distributive property, leading to products
such as $(-1)(-1)=1$ and the rules for
multiplying signed numbers. Interpret
products of rational numbers by describing
real-world contexts.
b. $\quad$ Understand that integers can
be divided, provided that the divisor is not
zero, and every quotient of integers (with
non-zero divisor) is a rational number. If $p$ multiplication is extended from fractions to rational numbers by requiring that operations continue to satisfy the properties of operations, particularly the distributive property, leading to products such as $(-1)(-1)=1$ and the rules for multiplying signed numbers. Interpret products of rational numbers by describing real-world contexts.
b. Understand that integers can be divided, provided that the divisor is not zero, and every quotient of integers (with non-zero divisor) is a rational number. If $p$
of operations with rationa numbers. Grade-seven students extend addition, subtraction, multiplication, and division to all rational numbers by applying these operations to both positive and negative numbers.

Adding, subtracting, multiplying, and dividing rational numbers is the culmination of numerical work with the four basic operations. The number system continues to develop in grade eight, expanding to become the real numbers by the introduction of irrational numbers. Because there are no specific standards for arithmetic with rational numbers in later grades-and because so much other work in grade seven depends on that arithmetic-fluency in arithmetic with rational numbers should be a primary goal of grade-seven instruction.

## Content by Cluster

Teachers must provide students opportunity to
standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.

## Open-Up Resources

This resource provides opportunities for students to learn math by doing math, solving problems in mathematical and realworld contexts, and constructing arguments using precise language. Tasks spark discussion, perseverance, and enjoyment of mathematics. Highleverage routines guide students in understanding and making connections between concepts and procedures.
is equal to the difference of the absolute values of the two numbers and has the same sign as the number with the larger absolute value.

- By the end of seventh grade, students' arithmetic repertoire includes adding, subtracting, multiplying, and dividing with rational numbers including whole numbers, fractions, decimals, and signed numbers.

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and q are integers, then -(p/q) = (-p)/q =
p/(-q). Interpret quotients of rational
numbers by describing real world contexts.
c. Apply properties of
operations as strategies to multiply and
divide rational numbers.
d. Convert a rational number to
a decimal using long division; know that the
decimal form of a rational number
terminates in Os or eventually repeats.
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## M.7.6

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Solve real-world and mathematical problems involving the four operations with rational numbers. Instructional Note: Computations with rational numbers extend the rules for manipulating fractions to complex fractions.
master each grade-level content standard. This document provides a sample content plan for Grade Seven. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.
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## Expressions and Equations

## Standards

Use properties of operations to generate equivalent expressions.

## M.7.7

Apply properties of operations as strategies to add, subtract, factor and expand linear expressions with rational coefficients. M.7.8

Teacher Understandings
Starting in grade six, students began the study of equations and inequalities and methods for solving them. In grade seven, students build on this understanding and use the arithmetic of rational numbers

Resources
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Student Understandings

- Students work with the linear expressions where the distributive property plays a prominent role.
- Students become familiar with multiple ways of writing an expression, they

Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related. (e.g., $a+0.05 a=$ 1.05a means that "increase by $5 \%$ " is the same as "multiply by 1.05.")
Solve real-life and mathematical problems using numerical and algebraic expressions and equations.

## M.7.9

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies. (e.g., If a woman making $\$ 25$ an hour gets a $10 \%$ raise, she will make an additional $1 / 10$ of her salary an hour, or $\$ 2.50$, for a new salary of $\$ 27.50$. If you want to place a towel bar 9 $3 / 4$ inches long in the center of a door that is $271 / 2$ inches wide, you will need to place the bar about 9 inches from each edge; this estimate can be used as a check on the exact computation.)

## M.7.10

Use variables to represent quantities in a real-world or mathematical problem and
as they formulate expressions and equations in one variable and use these equations to solve problems. Students also work toward fluently solving equations of the form $p x+$ $q=r$ and $p(x+q)=r$.

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Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. It highlights some necessary foundational skills from previous grade levels.

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Quantile Teacher Assistant
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Open-Up Resources
This resource provides opportunities for students to learn math by doing
also learn that different ways of writing expressions can serve varied purposes and provide different ways of seeing a problem.

- Students solve multi-step problems involving rational numbers presented in various forms (whole numbers, fractions, and decimals).


## Common Misconceptions

- Students see expressions like $7-2(8-1.5 x)$ and realize that the expression $(8-1.5 x)$ is treated as a separate quantity in its own right, being multiplied by 2 and the result being subtracted from 7. Students may mistakenly come up with the expressions below, and each case offers a chance for class discussion about why it is not equivalent to the original:

math, solving problems in mathematical and realworld contexts, and constructing arguments using precise language. Tasks spark discussion, perseverance, and enjoyment of mathematics. Highleverage routines guide students in understanding and making connections between concepts and procedures.
- $5(8-1.5 x)$, subtracting $7-2$ without realizing the multiplication must be done first
- $7-2(6.5 x)$, erroneously combining 8 and $-1.5 x$ by neglecting to realize that these are not like terms
- $7-16-3 x$, by misapplying the distributive property or not being attentive to the rules for multiplying negative numbers
- Students should have the opportunity to see this expression as equivalent to both $7+$ $(-2)(8-1.5 x)$ and $7-$ $(2(8-1.5 x))$, which can aid in seeing the correct way to handle the -2 part of the expression.


## Geometry

## Standards

## Draw, construct and describe geometrical

 figures and describe the relationships between them.
## M.7.11

Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

## M.7.12

Draw (freehand, with ruler and protractor, and with technology) geometric shapes
with given conditions. Focus on constructing triangles from three measures of angles or sides, noticing when the conditions determine a unique triangle, more than one triangle, or no triangle.

## M.7.13

Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.
Solve real-life and mathematical problems involving angle measure, area, surface area, and volume.

## M.7.14

Know the formulas for the area and

Teacher Understandings
In grade seven, a critical area of instruction is for students to extend their study of geometry as they solve problems involving scale drawings and informal geometric constructions. Students also work with two- and threedimensional shapes to solve problems involving area, surface area, and volume.

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Student Understandings

- Students will understand the concept of similar shapes in eighth grade: shapes that can be obtained from one another through dilatation.
- Students will need the concept of dilation to understand the derivation of the equations $y=m x$ and $y=m x+b$ by using similar triangles and the relationship between them.
- Students draw (freehand, with a ruler and a protractor, and with technology) geometric shapes, focusing on triangles.
- Students work with threedimensional figures and relate them to twodimensional figures by examining cross-sections that result when threedimensional figures are split.
- Students describe how two or more objects are related in space (like skewed likes)

| circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle. <br> M.7.15 <br> Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure. <br> M.7.16 <br> Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. | students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year. | standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards. <br> Open-Up Resources <br> This resource provides opportunities for students to learn math by doing math, solving problems in mathematical and realworld contexts, and constructing arguments using precise language. Tasks spark discussion, perseverance, and enjoyment of mathematics. Highleverage routines guide students in understanding and making connections between concepts and procedures. | and the possible ways in which three planes might intersect. <br> - Students know the formulas for the area and circumference of a circle and use them to solve problems. Students need to know why the formula works and how the formula relates to the measure (area and circumference) and the figure. <br> - Students work to solve problems involving area, surface area, and volume of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms. |
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## Statistics and Probability

## Standards

## Use random sampling to draw inferences

 about a population.
## M.7.17

Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population.
Understand that random sampling tends to produce representative samples and support valid inferences.

## M.7.18

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions. (e.g., Estimate the mean word length in a book by randomly sampling words from the book; predict the winner of a school election based on randomly sampled survey data. Gauge how far off the estimate or prediction might be.)
Draw informal comparative inferences about two populations.
M.7.19

Teacher Understandings
Students were introduced to statistics in grade six. In grade seven, they extend their work with single-data distributions to compare two different data distributions and address questions about differences between populations. They also begin informal work with random sampling.

Grade seven continues the characterization of data distributions by measures of center and spread. To be useful, center and spread must have well-defined numerical descriptions that are commonly understood by those using the results of a statistical investigation. Students analyze and/or compare data sets, they consider the context in which the data are collected and identify clusters, peaks, gaps, and symmetry in the data. Students understand that data sets contain many numerical values that can be summarized

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Quantile Teacher Assistant

Student Understandings

- Students use data from a random sample to draw inferences about a population with an unknown characteristic.
- Students learn that a random sample can be representative of the total population and will generate valid predictions. Students use this information to draw inferences from data.
- Students understand that data requires consideration of the measures of variability as well as the mean or median.
- Students know that variability is responsible for the overlap of two data sets, and an increase in the variability can increase the overlap.

Recognize that a measure of center for a numerical data set summarizes all of its values with a single number, while a measure of variation describes how its values vary with a single number.

## M.7.20

Summarize numerical data sets in relation to their context, such as by:
a. Reporting the number of observations.
b. Describing the nature of the attribute under investigation, including how it was measured and its units of measurement.
c. Giving quantitative measures of center (median and/or mean) and variability (interquartile range and/or mean absolute deviation), as well as describing any overall pattern and any striking deviations from the overall pattern with reference to the context in which the data were gathered.
Relating the choice of measures of center and variability to the shape of the data distribution and the context in which the data were gathered.

## M.7.21

Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of
by one number, such as a measure of center (mean and median) and range.

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## Open-Up Resources

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- Students understand that the median is paired with the interquartile range and the mean is paired with the mean absolute deviation.
- Students interpret probability as indicating the long-run relative frequency of the occurrence of an event.
- Student use and develop probability models to find the probabilities of events and investigate both empirical probabilities and theoretical probabilities.
- Students will examine compound events and use basic counting ideas for finding the total number of equally likely outcomes for such an event.
- Students use simulations to determine probabilities for compound events.

| variability. (e.g., The mean height of players |  | Students learn to <br> on the basketball team is 10 cm greater <br> than the mean height of players on the <br> soccer team, about twice the variability <br> (mean absolute deviation) on either team; |
| :--- | :--- | :--- |
| determine the |  |  |
| on a dot plot, the separation between the |  | appropriate graph for <br> displaying data and how <br> two distributions of heights is noticeable.) |
| M.7.22 read data from |  |  |
| Use measures of center and measures of |  |  |
| variability for numerical data from random |  | graphs generated by |
| others. |  |  |
| samples to draw informal comparative |  |  |
| inferences about two populations. (e.g., |  |  |
| Decide whether the words in a chapter of a |  |  |
| seventh-grade science book are generally |  |  |
| longer than the words in a chapter of a |  |  |
| fourth-grade science book.) |  |  |
| Investigate chance processes and develop, |  |  |
| use, and evaluate probability models. |  |  |
| M.7.23 |  |  |
| Understand that the probability of a chance |  |  |
| event is a number between 0 and 1 that |  |  |
| expresses the likelihood of the event |  |  |
| occurring. Larger numbers indicate greater |  |  |
| likelihood. A probability near 0 indicates an |  |  |
| unlikely event, a probability around $1 / 2$ |  |  |
| indicates an event that is neither unlikely |  |  |
| nor likely and a probability near 1 indicates |  |  |
| a likely event. |  |  |
| M.7.24 |  |  |
| Approximate the probability of a chance |  |  |
| event by collecting data on the chance |  |  |
| process that produces it and observing its |  |  |

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long-run relative frequency, and predict the
approximate relative frequency given the
probability. (e.g., When rolling a number
cube }600\mathrm{ times, predict that a }3\mathrm{ or }6\mathrm{ would
be rolled roughly 200 times, but probably
not exactly 200 times.)
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## M.7.25

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Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy. a. Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events. (e.g., If a student is selected at random from a class, find the probability that Jane will be selected and the probability that a girl will be selected.)
b. Develop a probability model (which may not be uniform) by observing
frequencies in data generated from a chance process. (e.g., Find the approximate probability that a spinning penny will land heads up or that a tossed paper cup will land open-end down. Do the outcomes for the spinning penny appear to be equally likely based on the observed frequencies?)
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## M.7.26

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Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.
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a. Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.
b. Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams.
For an event described in everyday
language (e.g., "rolling double sixes"),
identify the outcomes in the sample space which compose the event.
c. Design and use a simulation to generate frequencies for compound events. (e.g., Use random digits as a simulation tool to approximate the answer to the question: If $40 \%$ of donors have type A blood, what is the probability that it will take at least 4 donors to find one with type A blood?)


Steven L. Paine, Ed.D.
West Virginia Superintendent of Schools

