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# Frameworks for Mathematics

## *Grade 8*



West Virginia DEPARTMENT OF  
EDUCATION



**West Virginia Board of Education  
2018-2019**

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## Grade EIGHT

Prior to entering grade eight, students write and interpret expressions, solve equations and inequalities, explore quantitative relationships between dependent and independent variables, and solve problems involving area, surface area, and volume. Students who are entering grade eight have also begun to develop an understanding of statistical thinking.

### The Number System

Standards	Teacher Understandings	Resources	Student Understandings
<p><b>Know that there are numbers that are not rational, and approximate them by rational numbers.</b></p> <p><b>M.8.1</b> Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually and convert a decimal expansion which repeats eventually into a rational number. Instructional Note: A decimal expansion that repeats the digit 0 is often referred to as a “terminating decimal.”</p> <p><b>M.8.2</b> Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram and estimate the value of</p>	<p>In grade seven, adding, subtracting, multiplying, and dividing rational numbers was the culmination of numerical work with the four basic operations. The number system continues to develop in grade eight, expanding to the real numbers with the introduction of irrational numbers, and develops further in higher mathematics, expanding to become the complex numbers with the introduction of imaginary numbers.</p>	<p><b>Educators’ Guide</b> Organized by Grade 8 domains, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. It highlights some necessary foundational skills from previous grade levels.</p>	<ul style="list-style-type: none"> <li>• Students learn irrational numbers, and students explore cases of both rational and irrational numbers and their decimal expansions to begin to understand distinction.</li> <li>• Student approximate irrational numbers by rational numbers in simple cases. (e.g., <math>\pi</math> is approximated to 3.14)</li> <li>• A set of real numbers consists of rational numbers and irrational numbers.</li> </ul>



<p>expressions such as <math>\pi^2</math>. (e.g., By truncating the decimal expansion of <math>\sqrt{2}</math>, show that <math>\sqrt{2}</math> is between 1 and 2, then between 1.4 and 1.5, and explain how to continue on to get better approximations.)</p>	<p><b>Content by Cluster</b></p> <p>Teachers must provide students opportunity to master each grade-level content standard. It is important to understand that neglecting grade-level content standards, whether standards found in the major or additional/supporting clusters, will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.</p>	<p><a href="#">Math TREE Online Education Resources</a> A curated set of aligned, internet resources for WV middle and high school math teachers.</p> <p><a href="#">Quantile Teacher Assistant</a> This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.</p> <p><a href="#">Open-Up Resources</a> This resource provides opportunities for students to learn math by doing math, solving problems in mathematical and real-world contexts, and constructing arguments using precise language. Tasks spark discussion, perseverance, and enjoyment of</p>	
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		<p>mathematics. High-leverage routines guide students in understanding and making connections between concepts and procedures.</p>	
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## Expressions and Equations

Standards	Teacher Understandings	Resources	Student Understandings
<p><b>Work with radicals and integer exponents.</b></p> <p><b>M.8.3</b> Know and apply the properties of integer exponents to generate equivalent numerical expressions. (e.g., <math>3^2 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27</math>.)</p> <p><b>M.8.4</b> Use square root and cube root symbols to represent solutions to equations of the form <math>x^2 = p</math> and <math>x^3 = p</math>, where <math>p</math> is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that <math>\sqrt{2}</math> is irrational.</p> <p><b>M.8.5</b> Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small</p>	<p>In grade seven, students formulate expressions and equations in one variable, using these equations to solve problems and fluently solving equations of the form <math>px + q = r</math> and <math>p(x + q) = r</math>.</p> <p>In grade eight, students apply their previous understandings of ratio and proportional reasoning to the study of linear equations and pairs of simultaneous linear equations, which is a critical area of instruction for this grade level.</p>	<p><b>Educators' Guide</b> Organized by Grade 8 domains, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. It highlights some necessary foundational</p>	<ul style="list-style-type: none"> <li>• Students learn the properties of exponents and then use the properties of exponents to generate equivalent expressions.</li> <li>• Students start to work systemically with the symbols for square root and cube root.</li> <li>• Students recognize perfect squares and cubes, understanding that square roots of non-perfect squares and cube roots of non-perfect cubes are irrational.</li> <li>• Squaring/Cubing a number is the inverse</li> </ul>



<p>quantities, and to express how many times as much one is than the other. (e.g., Estimate the population of the United States as <math>3 \times 10^8</math> and the population of the world as <math>7 \times 10^9</math>, and determine that the world population is more than 20 times larger.)</p> <p><b>M.8.6</b> Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. (e.g., Use millimeters per year for seafloor spreading.) Interpret scientific notation that has been generated by technology.</p> <p><b>Understand the connections between proportional relationships, lines, and linear equations.</b></p> <p><b>M.8.7</b> Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. (e.g., Compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.)</p> <p><b>M.8.8</b></p>	<p><b>Content by Cluster</b></p> <p>Teachers must provide students opportunity to master each grade-level content standard. This is important to understand that neglecting grade-level content standards, whether standards found in the major or additional/supporting clusters, will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.</p>	<p>skills from previous grade levels.</p> <p><a href="#">Math TREE Online Education Resources</a> A curated set of aligned, internet resources for WV middle and high school math teachers.</p> <p><a href="#">Quantile Teacher Assistant</a> This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.</p> <p><a href="#">Open-Up Resources</a> This resource provides opportunities for students to learn math by doing math, solving problems in mathematical and real-world contexts, and constructing arguments using precise language. Tasks spark discussion,</p>	<p>operation of taking a square root/cube root of a number.</p> <ul style="list-style-type: none"> <li>• Students perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used.</li> <li>• Students compare and interpret scientific notation quantities in the context of the situation, recognizing that the powers of 10 indicated in quantities expressed in scientific notation follow the rules of exponents.</li> <li>• Students connect their work on ratios and proportional reasoning to concepts of the slope of a line.</li> <li>• Students identify that the unit rate is the slope and use the slope to compare two proportional relationships represented in different ways.</li> </ul>
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<p>Use similar triangles to explain why the slope <math>m</math> is the same between any two distinct points on a non-vertical line in the coordinate plane; derive the equation <math>y = mx</math> for a line through the origin and the equation <math>y = mx + b</math> for a line intercepting the vertical axis at <math>b</math>.</p> <p><b>Analyze and solve linear equations and pairs of simultaneous linear equations.</b></p> <p><b>M.8.9</b> Solve linear equations in one variable.</p> <p>a. Give examples of linear equations in one variable with one solution, infinitely many solutions or no solutions. Show which of these possibilities is the case by successively transforming the given equation into simpler forms, until an equivalent equation of the form <math>x = a</math>, <math>a = a</math>, or <math>a = b</math> results (where <math>a</math> and <math>b</math> are different numbers).</p> <p>b. Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.</p> <p><b>M.8.10</b> Analyze and solve pairs of simultaneous linear equations.</p> <p>a. Understand that solutions to a system of two linear equations in two</p>		<p>perseverance, and enjoyment of mathematics. High-leverage routines guide students in understanding and making connections between concepts and procedures.</p>	<ul style="list-style-type: none"> <li>• Student identify slope as a constant rate of change and understand why slope is the same through any two points on a given line.</li> <li>• Students analyze and solve linear equations and pairs of simultaneous linear equations.</li> <li>• Students solve linear equations in one variable, including cases with one solution, an infinite number of solutions, and no solutions.</li> <li>• Students analyze and solve pairs of simultaneous equations.</li> <li>• Students solve real-world and mathematical problems leading to two linear equations in two variables.</li> </ul>
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<p>variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.</p> <p>b. Solve systems of two linear equations in two variables algebraically and estimate solutions by graphing the equations. Solve simple cases by inspection. (e.g., <math>3x + 2y = 5</math> and <math>3x + 2y = 6</math> have no solution because <math>3x + 2y</math> cannot simultaneously be 5 and 6.)</p> <p>c. Solve real-world and mathematical problems leading to two linear equations in two variables. (e.g., Given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.)</p>			
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## Functions

Standards	Teacher Understandings	Resources	Student Understandings
<p><b>Define, evaluate, and compare functions.</b>  <b>M.8.11</b>            Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an</p>	<p>In grade seven, students learn to determine if two quantities represent a proportional relationship. Proportional reasoning is a transitional topic that falls between arithmetic and</p>	<p><b>Educators' Guide</b>            Organized by Grade 8 domains, this document provides exemplars to explain the content standards, highlight connections</p>	<ul style="list-style-type: none"> <li>• Students understand that a function is a rule that assigns to each input exactly one output.</li> <li>• Students know that a graph of a function is a set of ordered pairs</li> </ul>



<p>input and the corresponding output. Instructional Note: Function notation is not required in grade 8.</p> <p><b>M.8.12</b> Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.)</p> <p><b>M.8.13</b> Interpret the equation <math>y = mx + b</math> as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. (e.g., The function <math>A = s^2</math> giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.)</p> <p><b>Use functions to model relationships between quantities</b></p> <p><b>M.8.14</b> Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from</p>	<p>algebra. Underlying the progression from proportional reasoning through algebra and beyond is the idea of a <i>function</i>—a rule that assigns to each input exactly one output. The concept of a function is a critical area of instruction in grade eight. Students are introduced to functions and learn that proportional relationships are part of a broader group of linear functions.</p> <p><b>Content by Cluster</b></p> <p>Teachers must provide students opportunity to master each grade-level content standard. It is important to understand that neglecting grade-level content standards, whether standards found in the major or additional/supporting clusters, will leave gaps in students' skills and understandings and will leave students unprepared</p>	<p>to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. It highlights some necessary foundational skills from previous grade levels.</p> <p><a href="#">Math TREE Online Education Resources</a> A curated set of aligned, internet resources for WV middle and high school math teachers.</p> <p><a href="#">Quantile Teacher Assistant</a> This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.</p> <p><a href="#">Open-Up Resources</a></p>	<p>consisting of an input and the corresponding output.</p> <ul style="list-style-type: none"> <li>• Student expand their knowledge of linear functions to informally compare them to various types of functions recognizing them as linear or non-linear functions.</li> <li>• Students learn to use functions to represent relationships between quantities.</li> <li>• Students find the initial value of a linear function.</li> <li>• Students analyze graphs and then describe qualitatively the functional relationship between two quantities.</li> </ul>
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<p>two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.</p> <p><b>M.8.15</b> Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.</p>	<p>for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.</p>	<p>This resource provides opportunities for students to learn math by doing math, solving problems in mathematical and real-world contexts, and constructing arguments using precise language. Tasks spark discussion, perseverance, and enjoyment of mathematics. High-leverage routines guide students in understanding and making connections between concepts and procedures.</p>	
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## Geometry

Standards	Teacher Understandings	Resources	Student Understandings
<p><b>Understand congruence and similarity using physical models, transparencies, or geometry software.</b></p> <p><b>M.8.16</b> Verify experimentally the properties of rotations, reflections and translations: a. Lines are taken to lines, and line segments to line segments of the</p>	<p>In grade seven, students solve problems involving scale drawings and informal geometric constructions, and they work with two- and three-dimensional shapes to solve problems involving</p>	<p><b>Educators' Guide</b> Organized by Grade 8 domains, this document provides exemplars to explain the content standards, highlight connections to the Mathematical</p>	<ul style="list-style-type: none"> <li>• Students gain an understanding of transformations and their relationship to congruent shapes.</li> <li>• Students study dilations.</li> </ul>



<p>same length.</p> <p>b. Angles are taken to angles of the same measure.</p> <p>c. Parallel lines are taken to parallel lines.</p> <p><b>M.8.17</b> Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.</p> <p><b>M.8.18</b> Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates.</p> <p><b>M.8.19</b> Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them.</p> <p><b>M.8.20</b> Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a</p>	<p>area, surface area, and volume. Students in grade eight complete their work on volume by solving problems involving cones, cylinders, and spheres. They also analyze two- and three-dimensional space and figures using distance, angle, similarity, and congruence and by understanding and applying the Pythagorean Theorem, which is a critical area of instruction at this grade level.</p> <p><b>Content by Cluster</b> Teachers must provide students opportunity to master each grade-level content standard. It is important to understand that neglecting grade-level content standards, whether standards found in the major or additional/supporting clusters, will leave gaps in students' skills and understandings and will leave students unprepared</p>	<p>Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. It highlights some necessary foundational skills from previous grade levels.</p> <p><a href="#">Math TREE Online Education Resources</a> A curated set of aligned, internet resources for WV middle and high school math teachers.</p> <p><a href="#">Quantile Teacher Assistant</a> This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.</p> <p><a href="#">Open-Up Resources</a> This resource provides</p>	<ul style="list-style-type: none"> <li>• Students know the definition of similar shapes.</li> <li>• Students explain a proof of the Pythagorean Theorem and apply the knowledge to solve problems involving unknown sides.</li> <li>• Students use the knowledge of the Pythagorean Theorem between two points in a coordinate system.</li> <li>• Students learn the formulas for calculating the volumes of cones, cylinders, and spheres and use the formulas to solve real-world and mathematical problems.</li> </ul>
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<p>transversal, and the angle-angle criterion for similarity of triangles. (e.g., Arrange three copies of the same triangle so that the sum of the three angles appears to form a line, and give an argument in terms of transversals why this is so.)</p> <p><b>Understand and apply the Pythagorean Theorem.</b></p> <p><b>M.8.21</b> Explain a proof of the Pythagorean Theorem and its converse.</p> <p><b>M.8.22</b> Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions.</p> <p><b>M.8.23</b> Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.</p> <p><b>Solve real-world and mathematical problems involving volume of cylinders, cones, and spheres.</b></p> <p><b>M.8.24</b> Know the formulas for the volumes of cones, cylinders and spheres and use them to solve real-world and mathematical problems.</p>	<p>for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.</p>	<p>opportunities for students to learn math by doing math, solving problems in mathematical and real-world contexts, and constructing arguments using precise language. Tasks spark discussion, perseverance, and enjoyment of mathematics. High-leverage routines guide students in understanding and making connections between concepts and procedures.</p>	
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## Statistics and Probability

Standards	Teacher Understandings	Resources	Student Understandings
<p><b>Investigate patterns of association in bivariate data.</b></p> <p><b>M.8.25</b> Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association and nonlinear association.</p> <p><b>M.8.26</b> Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line and informally assess the model fit by judging the closeness of the data points to the line.</p> <p><b>M.8.27</b> Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. (e.g., In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated</p>	<p>Building on work in earlier grades with univariate measurement data and analyzing data on line plots and histograms, grade-eight students begin to work with bivariate measurement data and use scatter plots to represent and analyze the data.</p> <p>Bivariate measurement data represent two separate (but usually related) measurements. Scatter plots can show the relationship between the two measured variables. Collecting and analyzing bivariate measurement data help students to answer questions such as: “How does more time spent on homework affect test grades?” and “What is the relationship between annual income and the number of years of formal education a person has?”</p>	<p><b>Educators’ Guide</b> Organized by Grade 8 domains, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. It highlights some necessary foundational skills from previous grade levels.</p> <p><a href="#">Math TREE Online Education Resources</a> A curated set of aligned, internet resources for WV</p>	<ul style="list-style-type: none"> <li>• Students analyze scatter plots to examine the relationships between variables to determine positive and negative associations, the degree of association, and the type of association.</li> <li>• Students analyze outliers to determine if data points are valid or represent a recording or measure of error.</li> <li>• Students use knowledge or linear association, students informally fit a line and informally assess the fit by judging the closeness of the data points to the straight line.</li> <li>• Students solve problems in the context of bivariate measurement data by using the equation of a linear model. They interpret the slope and the y-intercept</li> </ul>



<p>with an additional 1.5 cm in mature plant height.)</p> <p><b>M.8.28</b></p> <p>Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. (e.g., Collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?)</p>	<p><b>Content by Cluster</b></p> <p>Teachers must provide students opportunity to master each grade-level content standard. It is important to understand that neglecting grade-level content standards, whether standards found in the major or additional/supporting clusters, will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.</p>	<p>middle and high school math teachers.</p> <p><a href="#">Quantile Teacher Assistant</a></p> <p>This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.</p> <p><a href="#">Open-Up Resources</a></p> <p>This resource provides opportunities for students to learn math by doing math, solving problems in mathematical and real-world contexts, and constructing arguments using precise language. Tasks spark discussion, perseverance, and enjoyment of mathematics. High-leverage routines</p>	<p>in the context of the problem.</p> <ul style="list-style-type: none"> <li>• Students learn to see patterns of association in bivariate categorical data in a two-way table.</li> <li>• Students use relative frequencies calculated from rows or columns to describe a possible association between the two variables.</li> </ul>
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