



Frameworks for Mathematics

Mathematics I



West Virginia DEPARTMENT OF
EDUCATION



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High School Mathematics I

Students in Mathematics I continue their work with expressions and modeling and analysis of situations. In previous grade levels, students informally defined, evaluated, and compared functions, using them to model relationships between quantities. In Mathematics I, students learn function notation and develop the concepts of *domain* and *range*. Students move beyond viewing functions as processes that take inputs and yield outputs and begin to view functions as objects that can be combined with operations (e.g., finding $(f + g)(x) = f(x) + g(x)$). They explore many examples of functions, including sequences. They interpret functions that are represented graphically, numerically, symbolically, and verbally, translating between representations and understanding the limitations of various representations. They work with functions given by graphs and tables, keeping in mind that these representations are likely to be approximate and incomplete, depending upon the context. Students' work includes functions that can be described or approximated by formulas, as well as those that cannot. When functions describe relationships between quantities arising from a context, students reason with the units in which those quantities are measured. Students build on and informally extend their understanding of integer exponents to consider exponential functions. They compare and contrast linear and exponential functions, distinguishing between additive and multiplicative change. They also interpret arithmetic sequences as linear functions and geometric sequences as exponential functions.

Students who are prepared for Mathematics I have learned to solve linear equations in one variable and have applied graphical and algebraic methods to analyze and solve systems of linear equations in two variables. Mathematics I builds on these earlier experiences by asking students to analyze and explain the process of solving an equation and to justify the process used in solving a system of equations. Students develop fluency in writing, interpreting, and translating between various forms of linear equations and inequalities and using them to solve problems. They master solving linear equations and apply related solution techniques and the laws of exponents to the creation and solving of simple exponential equations. Students explore systems of equations and inequalities, finding and interpreting solutions. All of this work is based on understanding quantities and the relationships between them.

In Mathematics I, students build on their prior experiences with data, developing more formal means of assessing how a model fits data. Students use regression techniques to describe approximately linear relationships between quantities. They use graphical representations and knowledge of the context to make judgments about the appropriateness of linear models. With linear models, they look at residuals to analyze the goodness of fit.

In previous grade levels, students were asked to draw triangles based on given measurements. They also gained experience with rigid motions (translations, reflections, and rotations) and developed notions about what it means for two objects to be congruent. In Mathematics I, students establish triangle congruence criteria based on analyses of rigid motions and formal constructions. They solve problems about triangles, quadrilaterals, and other polygons. They apply reasoning to complete



geometric constructions and explain why the constructions work. Finally, building on their work with the Pythagorean Theorem in the grade-eight standards to find distances, students use a rectangular coordinate system to verify geometric relationships, including properties of special triangles and quadrilaterals and slopes of parallel and perpendicular lines.

Modeling

Standards	Teacher Understandings	Resources	Student Understandings
<p>Although the Modeling category does not include specific standards, the idea of using mathematics to model the world pervades all higher mathematics courses and should hold a significant place in instruction. Modeling is addressed first here to emphasize its importance in the higher mathematics curriculum.</p>	<p>Modeling at the higher mathematics level goes beyond the simple application of previously constructed mathematics and includes real-world problems. True modeling begins with students asking a question about the world around them and then constructing the mathematics in the process of attempting to answer the question.</p> <p>Modeling problems have an element of being genuine problems, in the sense that students care about answering the question under consideration. In modeling, mathematics is used as a tool to answer questions that students really want answered.</p>	<p>Educators' Guide for Mathematics Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. It highlights some necessary foundational skills</p>	<ul style="list-style-type: none"> • When presented with a real-world situation and challenged to ask a question, students encounter all sorts of new issues: Which of the quantities present in this situation are known and unknown? Can a table of data be made? Is there a functional relationship in this situation? • Students decide on a solution path that may need to be revised. They make use of tools such as calculators, dynamic geometry software, or spreadsheets. They use previously derived models (e.g., linear functions) as well as new formulas or functions that apply.



	<p>Students examine a problem and formulate a <i>mathematical model</i> (an equation, table, graph, and the like), compute an answer or rewrite their expression to reveal new information, interpret and validate the results, and report out. This is a new approach for many teachers and may be challenging to implement, but the effort should show students that mathematics is relevant to their lives. From a pedagogical perspective, modeling gives a concrete basis from which to abstract the mathematics and often serves to motivate students to become independent learners.</p> <p>Content by Cluster Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and</p>	<p>from previous grade levels.</p> <p>Math TREE Online Education Resources A curated set of aligned internet resources for WV middle and high school math teachers.</p> <p>Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.</p>	<ul style="list-style-type: none"> • Students may discover that answering their question requires solving an equation and knowing the output value of a function at an unknown input value.
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	<p>understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.</p>		
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Functions

Standards	Teacher Understandings	Resources	Student Understandings
<p>LINEAR AND EXPONENTIAL RELATIONSHIPS Understand the concept of a function and use function notation. M.1HS.12 Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then $f(x)$ denotes the output of f corresponding to the input x. The graph of f is the graph of the equation</p>	<p>The standards in the Functions conceptual category can serve as motivation for the study of standards in the other Mathematics I conceptual categories. For instance, an equation wherein one is asked to “solve for x” can be seen as a search for the input of a function f that gives a specified output, and solving the equation amounts to undoing the work</p>	<p>Educators’ Guide Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing</p>	<ul style="list-style-type: none"> • While the grade-eight standards called for students to work informally with functions, students in Mathematics I refine their understanding and use the formal mathematical language of functions. • Students learn the language of functions and that a function has a domain that must be



<p>$y = f(x)$. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear and exponential functions.</p> <p>M.1HS.13 Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear and exponential functions.</p> <p>M.1HS.14 Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n) + f(n-1)$ for $n \geq 1$. Instructional Note:</p>	<p>of the function. Or, the graph of an equation such as $y = \frac{1}{3}x + 5$ can be seen as a representation of a function f where $f(x) = \frac{1}{3}x + 5$. Solving a more complicated equation can be seen as asking, "For which values of do two functions and agree? (i.e., when does $f(s) = g(x)$)," and the intersection of the two graphs $y = f(x)$ and $y = g(x)$ is then connected to the solution of this equation. In general, functions describe in a precise way how two quantities are related and can be used to make predictions and generalizations, keeping true to the emphasis on modeling in higher mathematics.</p> <p>Content by Cluster Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in</p>	<p>conceptual understanding, procedural skill and fluency, and application.</p> <p>Math TREE Online Education Resources A curated set of aligned internet resources for WV middle and high school math teachers.</p> <p>Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.</p>	<p>specified as well as a corresponding range.</p> <ul style="list-style-type: none"> • Students develop the connection between the graph of an equation and the function itself, understanding that the graph is a <i>representation</i> of a function. • Students recognize sequences as functions. They are introduced to arithmetic and geometric sequences, written both explicitly and recursively. • Students represent linear, exponential, and absolute value functions with graphs and identify key features in the graph. They represent the same function algebraically in different forms and interpret these differences in terms of the graph or context. • Students recognize and understand the defining characteristics of linear and exponential functions.
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<p>Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear and exponential functions. Draw connection to M.1HS.21, which requires students to write arithmetic and geometric sequences. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions.</p> <p>Interpret functions that arise in applications in terms of a context. M.1HS.15 For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on linear and exponential functions.</p>	<p>students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.</p>		
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<p>M.1HS.16 Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (e.g., If the function $h(n)$ gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Instructional Note: Focus on linear and exponential functions.</p> <p>M.1HS.17 Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers. Mathematics II and III will address other function types.</p> <p>Analyze functions using different representations.</p> <p>M.1HS.18 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.</p> <p>a. Graph linear and quadratic</p>			
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<p>functions and show intercepts, maxima, and minima.</p> <p>b. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.</p> <p>M.1HS.19 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \cdot 2^n$.</p> <p>Build a function that models a relationship between two quantities.</p> <p>M.1HS.20 Write a function that describes a relationship between two quantities.</p> <p>a. Determine an explicit expression, a recursive process or steps for calculation from a context.</p> <p>b. Combine standard function types using arithmetic operations. (e.g.,</p>			
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<p>Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.) Instructional Note: Limit to linear and exponential functions.</p> <p>M.1HS.21 Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Instructional Note: Limit to linear and exponential functions. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.</p> <p>Build new functions from existing functions.</p> <p>M.1HS.22 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on vertical translations of graphs of linear</p>			
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and exponential functions. Relate the vertical translation of a linear function to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.

Construct and compare linear, quadratic, and exponential models and solve problems.

M.1HS.23

Distinguish between situations that can be modeled with linear functions and with exponential functions.

- a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals.
- b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another.
- c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.

M.1HS.24

Construct linear and exponential functions, including arithmetic and



<p>geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</p> <p>M.1HS.25 Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Limit to comparisons between exponential and linear models.</p> <p>Interpret expressions for functions in terms of the situation they model.</p> <p>M.1HS.26 Interpret the parameters in a linear or exponential function in terms of a context. Instructional Note: Limit exponential functions to those of the form $f(x) = b^x + k$.</p>			
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Number and Quantity

Standards	Teacher Understandings	Resources	Student Understandings
<p>RELATIONSHIPS BETWEEN QUANTITIES Reason quantitatively and use units to solve problems. M.1HS.1</p>	<p>In real-world problems, the answers are usually not numbers, but quantities: numbers with units, which</p>	<p>Educators' Guide Organized by conceptual categories, this</p>	<ul style="list-style-type: none"> Students create expressions that describes a computation involving a general



<p>Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.</p> <p>M.1HS.2 Define appropriate quantities for the purpose of descriptive modeling. Instructional Note: Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.</p> <p>M.1HS.3 Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.</p>	<p>involve measurement. In their work in measurement up through grade eight, students primarily measure commonly used attributes such as length, area, and volume. In higher mathematics, students encounter a wider variety of units in modeling—for example, when considering acceleration, currency conversions, derived quantities such as person-hours and heating degree-days, social science rates such as per-capita income, and rates in everyday life such as points scored per game or batting averages.</p> <p>Content by Cluster Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students’ skills and understandings and will leave students unprepared</p>	<p>document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application.</p> <p>Math TREE Online Education Resources A curated set of aligned internet resources for WV middle and high school math teachers.</p>	<p>quantity. This skill requires the ability to express computation in general terms, abstracting from specific instances.</p>
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	for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.		
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Algebra

Standards	Teacher Understandings	Resources	Student Understandings
<p>Interpret the structure of expressions. M.1HS.4 Interpret expressions that represent a quantity in terms of its context.</p> <p>a. Interpret parts of an expression, such as terms, factors, and coefficients.</p> <p>b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.</p> <p>Instructional Note: Limit to linear expressions and to exponential expressions with integer exponents.</p> <p>Create equations that describe numbers or relationships.</p>	<p>In the Algebra conceptual category, students extend the work with expressions that they started in grades six through eight. They create and solve equations in context, utilizing the power of variable expressions to model real-world problems and solve them with attention to units and the meaning of the answers they obtain. They continue to graph equations, understanding the resulting picture as a representation of the points satisfying the</p>	<p>Educators' Guide Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application.</p>	<ul style="list-style-type: none"> • Students read expressions with comprehension. This skill requires an analysis of the underlying structure of the expression. • Students solve linear equations and inequalities in one variable, including equations and inequalities with coefficients represented by letters. • Students solve equations <i>and use properties to</i>



<p>M.1HS.5 Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions. Instructional Note: Limit to linear and exponential equations and in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.</p> <p>M.1HS.6 Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Limit to linear and exponential equations and in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.</p> <p>M.1HS.7 Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on</p>	<p>equation. This conceptual category accounts for a large portion of the Mathematics I course and, along with the Functions category, represents the main body of content.</p> <p>The Algebra conceptual category in higher mathematics is very closely related to the Functions conceptual category:</p> <ul style="list-style-type: none"> • An expression in one variable can be viewed as defining a function: the act of evaluating the expression is an act of producing the function's output given the input. • An equation in two variables can sometimes be viewed as defining a function, if one of the variables is designated as the input variable and the other as the output variable, and if there is just one output for each input. This is the case if the expression is of the form $y =$ (expression in x) or if it 	<p>Math TREE Online Education Resources A curated set of aligned internet resources for WV middle and high school math teachers.</p> <p>Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.</p>	<p><i>explain</i> the steps as resulting from previous true equations. In this way, the idea of <i>proof</i>, while not explicitly named, is given a prominent role in the solving of equations, and the reasoning and justification process is not simply relegated to a future mathematics course.</p> <ul style="list-style-type: none"> • Students understand that if the two sides of one equation are equal, and the two sides of another equation are equal, then the sum (or difference) of the left sides of the two equations is equal to the sum (or difference) of the right sides. The reversibility of these steps justifies that an equivalent system of equations has been achieved. This is a crucial understanding as students' reason about solving systems of equations.
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<p>combinations of different foods.) Instructional Note: Limit to linear equations and inequalities.</p> <p>M.1HS.8 Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm’s law $V = IR$ to highlight resistance R. Instructional Note: Limit to formulas with a linear focus.</p> <p>Represent and solve equations and inequalities graphically.</p> <p>M.1HS.9 Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Instructional Note: Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.</p> <p>M.1HS.10 Explain why the x-coordinates of the points where the graphs of the equations $y = f(x)$ and $y = g(x)$ intersect are the solutions of the equation $f(x) = g(x)$; find the solutions approximately, (e.g., using technology to graph the</p>	<p>can be put into that form by solving for y.</p> <ul style="list-style-type: none"> • The notion of equivalent expressions can be understood in terms of functions: if two expressions are equivalent, they define the same function. • The solutions to an equation in one variable can be understood as the input values that yield the same output in the two functions defined by the expressions on each side of the equation. This insight allows for the method of finding approximate solutions by graphing functions defined by each side and finding the points where the graphs intersect. <p>Thus, in light of understanding functions, the main content of the Algebra category (solving equations, working with expressions, and so forth)</p>		<ul style="list-style-type: none"> • Students recognize the relationship between the algebraic representation of an equation and its graph plotted in the coordinate plane and understand geometric interpretations of solutions to equations and inequalities.
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<p>functions, make tables of values, or find successive approximations). Include cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value exponential, and logarithmic functions. Instructional Note: Focus on cases where $f(x)$ and $g(x)$ are linear or exponential.</p> <p>M.1HS.11 Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.</p> <p>Understand solving equations as a process of reasoning and explain the reasoning.</p> <p>M.1HS.27 Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Instructional Note: Students should focus on linear equations and be able to extend and apply their reasoning to other types of</p>	<p>serves a very important purpose.</p> <p>When solving equations, students make use of the symmetric and transitive properties and particular properties of equality regarding operations (e.g., “Equals added to equals are equal”). Students solve an equation <i>and explain</i> the steps as resulting from previous true equations and using the symmetric and transitive properties and particular properties of equality regarding operations. In this way, the idea of <i>proof</i>, while not explicitly named, is given a prominent role in the solving of equations, and the reasoning and justification process is not simply relegated to a future mathematics course. The following example illustrates the justification process that may be expected in Mathematics I.</p>		
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<p>equations in future courses. Students will solve exponential equations with logarithms in Mathematics III.</p> <p>Solve equations and inequalities in one variable.</p> <p>M.1HS.28 Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Instructional Note: Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.</p> <p>Solve systems of equations.</p> <p>M.1HS.29 Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. Instructional Note: Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations</p>	<p>Content by Cluster Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.</p>		
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<p>describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to M.1HS.50, which requires students to prove the slope criteria for parallel lines.</p> <p>M.1HS.30 Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Instructional Note: Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to M.1HS.50, which requires students to prove the slope criteria for parallel lines.</p>			
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Geometry

Standards	Teacher Understandings	Resources	Student Understandings
<p>Experiment with transformations in the plane.</p> <p>M.1HS.39 Know precise definitions of angle, circle, perpendicular line, parallel line and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.</p> <p>M.1HS.40 Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a</p>	<p>The standards for grades seven and eight introduce students to seeing two-dimensional shapes as part of a generic plane (the <i>Euclidean plane</i>) and exploring transformations of this plane as a way to determine whether two shapes are congruent or similar. These concepts are formalized In Mathematics I, and students use transformations to prove geometric theorems about triangles. Students then apply these triangle congruence theorems to prove other geometric results.</p> <p>Content by Cluster Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting</p>	<p>Educators' Guide Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application.</p> <p>Math TREE Online Education Resources A curated set of aligned internet resources for WV middle and high school math teachers.</p>	<ul style="list-style-type: none"> Students refine the commonly held (but imprecise) definition that shapes are congruent when they “have the same size and shape” by a more mathematically precise one: <i>Two shapes are congruent if there is a sequence of rigid motions in the plane that takes one shape exactly onto the other.</i> While this definition is explored intuitively in the grade-eight standards, in Mathematics I, it is investigated more closely. Students develop more precise definitions for the <i>rigid motions</i> (rotation, reflection, and translation) that were introduced in grades seven and eight. Students base their understanding of these definitions on



<p>specified center through a specified angle).</p> <p>M.1HS.41 Given a rectangle, parallelogram, trapezoid or regular polygon, describe the rotations and reflections that carry it onto itself. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).</p> <p>M.1HS.42 Develop definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).</p>	<p>grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.</p>	<p>Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.</p>	<p>their experience with transforming figures using patty paper, transparencies, or geometry software.</p> <ul style="list-style-type: none"> • As students refine their definition of <i>congruence</i> in terms of rigid motions, they are better able to apply this concept to any shape in the plane. (Previously, congruence seemed to depend on criteria that were specific only to particular shapes.) • Students use coordinates to prove simple geometric theorems. • Students use relationships between slopes of parallel and perpendicular lines to solve problems and to justify why these relationships are true.
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<p>M.1HS.43 Given a geometric figure and a rotation, reflection or translation draw the transformed figure using, e.g., graph paper, tracing paper or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).</p> <p>Understand congruence in terms of rigid motions.</p> <p>M.1HS.44 Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle),</p>			
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which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.

M.1HS.45

Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.

M.1HS.46

Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that



they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.

Make geometric constructions.

M.1HS.47

Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.

M.1HS.48

Construct an equilateral triangle, a



<p>square and a regular hexagon inscribed in a circle. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.</p>			
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Statistics and Probability

Standards	Teacher Understandings	Resources	Student Understandings
<p>Summarize, represent, and interpret data on a single count or measurement variable. M.1HS.31 Represent data with plots on the real number line (dot plots, histograms, and box plots). M.1HS.32 Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Instructional Note: In grades 6 – 8, students describe center and spread in a data distribution. Here</p>	<p>Students in Mathematics I build on their understanding of key ideas for describing distributions— shape, center, and spread—presented in the standards for grades six through eight. This enhanced understanding allows students to give more precise answers to deeper questions, often involving comparisons of data sets.</p> <p>Content by Cluster Teachers must provide</p>	<p>Educators’ Guide Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding,</p>	<ul style="list-style-type: none"> • Students use the shape of a distribution and question(s) to be answered to decide on the median or mean as the more appropriate measure of center and to justify their choice through statistical reasoning. • Students use parallel box plots or histograms to compare differences in the shape, center, and spread of comparable data sets.



<p>they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.</p> <p>M.1HS.33 Interpret differences in shape, center and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Instructional Note: In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.</p> <p>Summarize, represent, and interpret data on two categorical and quantitative variables.</p> <p>M.1HS.34 Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data.</p> <p>M.1HS.35 Represent data on two quantitative</p>	<p>students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students’ skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.</p>	<p>procedural skill and fluency, and application.</p> <p>Math TREE Online Education Resources A curated set of aligned internet resources for WV middle and high school math teachers.</p> <p>Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.</p>	<ul style="list-style-type: none"> • Students take a deeper look at bivariate data, using their knowledge of proportions to describe categorical associations and using their knowledge of functions to fit models to quantitative data. • Students extend their knowledge of scatter plots to fit mathematical models that capture key elements of the relationship between two variables and to explain what the model indicates about the relationship. • Students examine residuals to learn more about the behavior of data. • <i>Students compute correlation coefficients using technology and interpret the value of the coefficient.</i> • Students explore situations where correlation and causation are mistakenly interchanged; students
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<p>variables on a scatter plot, and describe how the variables are related.</p> <ol style="list-style-type: none"> Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. Informally assess the fit of a function by plotting and analyzing residuals. (Focus should be on situations for which linear models are appropriate.) Fit a linear function for scatter plots that suggest a linear association. <p>Instructional Note: Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.</p> <p>Interpret linear models. M.1HS.36 Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Instructional Note: Build on students' work with linear relationships in eighth grade and introduce the correlation</p>			<p>carefully examine the story that data and computed statistics try to tell.</p>
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<p>coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.</p> <p>M.1HS.37 Compute (using technology) and interpret the correlation coefficient of a linear fit. Instructional Note: Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.</p> <p>M.1HS.38 Distinguish between correlation and causation. Instructional Note: The important distinction between a statistical relationship and a cause-and-effect relationship arises here.</p>			
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