Frameworks for Mathematics

Mathematics II

West Virginia DEPARTMENT OF EDUCATION

High School Mathematics II

In Mathematics II, students extend the laws of exponents to rational exponents and explore distinctions between rational and irrational numbers by considering their decimal representations. Students learn that when quadratic equations do not have real solutions, the number system can be extended so that solutions exist, analogous to the way in which extending whole numbers to negative numbers allows $x + 1 = 0$ to have a solution. Students explore relationships between number systems: whole numbers, integers, rational numbers, real numbers, and complex numbers. The guiding principle is that equations with no solutions in one number system may have solutions in a larger number system.

Students consider quadratic functions, comparing the key characteristics of quadratic functions to those of linear and exponential functions. They select from these functions to model phenomena. Students learn to anticipate the graph of a quadratic function by interpreting various forms of quadratic expressions. In particular, they identify the real solutions of a quadratic equation as the zeros of a related quadratic function. Students also learn that when quadratic equations do not have real solutions, the graph of the related quadratic function does not cross the horizontal axis. Additionally, students expand their experience with functions to include more specialized functions—absolute value, step, and other piecewise-defined functions.

Students in Mathematics II focus on the structure of expressions, writing equivalent expressions to clarify and reveal aspects of the quantities represented. Students create and solve equations, inequalities, and systems of equations involving exponential and quadratic expressions.

Building on probability concepts introduced in the middle grades, students use the language of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events, independent events, and conditional probability. Students use probability to make informed decisions, and they should make use of geometric probability models whenever possible.

Students apply their earlier experience with dilations and proportional reasoning to build a formal understanding of similarity. They identify criteria for similarity of triangles, use similarity to solve problems, and apply similarity in right triangles to understand right-triangle trigonometry, with particular attention to special right triangles and the Pythagorean Theorem. In Mathematics II, students develop facility with geometric proof. They use what they know about congruence and similarity to prove theorems involving lines, angles, triangles, and other polygons. They also explore a variety of formats for writing proofs.

In Mathematics II, students prove basic theorems about circles, chords, secants, tangents, and angle measures. In the Cartesian coordinate system, students use the distance formula to write the equation of a circle when given the radius and the coordinates of its center, and the equation of a parabola with a vertical axis when given an equation of its horizontal directrix and the coordinates of its focus. Given an equation of a circle, students draw the graph in the coordinate plane and apply techniques for solving quadratic equations to determine intersections between lines and circles, between lines and parabolas, and between two circles. Students develop informal arguments to justify common formulas for circumference, area, and volume of geometric objects, especially those related to circles.

Modeling

Standards	Teacher Understandings	Resources	Student Understandings
Although the Modeling category does not include specific standards, the idea of using mathematics to model the world pervades all higher mathematics courses and should hold a significant place in instruction. Modeling is addressed first here to emphasize its importance in the higher mathematics curriculum.	Modeling at the higher mathematics level goes beyond the simple application of previously constructed mathematics and includes real-world problems. True modeling begins with students asking a question about the world around them, and the mathematics is then constructed in the process of attempting to answer the question. Modeling problems have an element of being genuine problems, in the sense that students care about answering the question under consideration. In modeling, mathematics is used as a tool to answer questions that students really want answered.	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. It highlights some necessary foundational skills from previous grade levels.	• When students are presented with a real-world situation and challenged to ask a question, all sorts of new issues arise (e.g., Which of the quantities present in this situation are known, and which are unknown? Can a table of data be made? Is there a functional relationship in this situation?). • Students decide on a solution path that may need to be revised. They make use of tools such as calculators, dynamic geometry software, and spreadsheets. They try to use previously derived models (e.g., linear functions), but may find that a new formula or function will apply.

	Students examine a problem and formulate a *mathematical model* (an equation, table, graph, or the like), compute an answer or rewrite their expression to reveal new information, interpret and validate the results, and report out. This is a new approach for many teachers and may be challenging to implement, but the effort should show students that mathematics is relevant to their lives. From a pedagogical perspective, modeling gives a concrete basis from which to abstract the mathematics and often serves to motivate students to become independent learners. The important ideas surrounding quadratic functions, graphing, solving equations, and rates of change should be explored through the lens of mathematical modeling.	Math TREE Online Education Resources A curated set of aligned internet resources for WV middle and high school math teachers. Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	• Students may see when trying to answer their question that solving an equation arises as a necessity and that the equation often involves the specific instance of knowing the output value of a function at an unknown input value.

	Content by Cluster Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.		

Functions

Standards	Teacher Understandings	Resources	Student Understandings
QUADRATIC FUNCTIONS AND MODELING **Interpret functions that arise in applications in terms of a context.** **M.2HS.7** For a function that models a	The standards of the Functions conceptual category can serve as motivation for the study of standards in the other	**Educators' Guide** Organized by conceptual categories, this document provides	• Students develop an understanding of the concept of a function, interpret characteristics of functions in context,

relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in Mathematics I. **M.2HS.8** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in Mathematics I. **M.2HS.9** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a	Mathematics II conceptual categories. Students have already worked with equations in which they have to "solve for x" as a search for the input of a function f that gives a specified output; solving the equation amounts to undoing the work of the function. The types of functions that students encounter in Mathematics II have new properties. For example, while linear functions show constant additive change and exponential functions show constant multiplicative change, quadratic functions exhibit a different change and can be used to model new situations. New techniques for solving equations need to be constructed carefully, as extraneous solutions may arise or no real-number solutions may exist. In general, functions describe how two quantities are related in a precise way and can be used to make	exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. Math TREE Online Education Resources A curated set of aligned internet resources for WV middle and high school math teachers. Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills	and represent functions in different ways. • Students represent functions with graphs and identify key features of the graph. They represent the same function algebraically in different forms and interpret these differences in terms of the graph or context. • Students work with linear, exponential, and quadratic functions and develop fluency with these types of functions, including the ability to graph them by hand. • Students work with functions that model data and with choosing an appropriate model function by considering the context that produced the data. • Students' ability to recognize rates of change, growth and decay, end behavior, roots, and other characteristics of

specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in Mathematics I. **Analyze functions using different representations.** **M.2HS.10** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root and piecewise-defined functions, including step functions and absolute value functions. Instructional Note: Compare and contrast absolute value, step and piecewise defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range and usefulness when examining piecewise-defined functions. Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots and	predictions and generalizations, keeping true to the emphasis on modeling that occurs in higher mathematics. The core question when students investigate functions is, “Does each element of the domain correspond to exactly one element of the range?” Students extend their previous work with linear and exponential expressions, equations, and systems of equations and inequalities to quadratic relationships. A parallel extension occurs from linear and exponential functions to quadratic functions: students begin to analyze functions in terms of transformations. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content	most relevant to standards.	functions becomes more sophisticated; they use this expanding repertoire of families of functions to inform their choices for models. • Students focus on how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. • Students develop models for more complex or sophisticated situations because the types of functions available to them have expanded. • Students develop an understanding of a *family of functions* and characterize such function families based on the properties of those families. • Students develop an understanding of the effect on the output of a function under transformations. Students see the effect of

that once roots are known, a quadratic equation can be factored. **M.2HS.11** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values and symmetry of the graph and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. (e.g., Identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.) Instructional Note: This unit and, in particular, this standard extends the work begun in Mathematics I on exponential functions with integer exponents. Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots and that once roots are known, a quadratic equation can be factored. **M.2HS.12** Compare properties of two functions	standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.		transformations on the graph of a function and, in particular, they comprehend that the effect on the graph is the opposite to the transformation on the variable. • Students learn that some functions have the property that an input can be recovered from a given output—as with the equation $f(x) = c$, which can be solved for x given that c lies in the range of f. This provides students a contextually appropriate way to find the expression for an inverse function, in contrast with the practice of simply swapping x and y in an equation and solving for y. • Students continue their investigation of exponential functions by comparing them with linear and quadratic functions, observing that exponential functions will

each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum). Instructional Note: Focus on expanding the types of functions considered to include, linear, exponential and quadratic. Extend work with quadratics to include the relationship between coefficients and roots and that once roots are known, a quadratic equation can be factored. **Build a function that models a relationship between two quantities.** **M.2HS.13** Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process or steps for calculation from a context. b. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Instructional Note: Focus on situations that exhibit a quadratic or exponential relationship.			always grow larger than any polynomial function. • Students use the Pythagorean identity to find the output of a trigonometric function at given angle θ when the output of another trigonometric function is known.

Build new functions from existing functions. **M.2HS.14** Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on quadratic functions and consider including absolute value functions. **M.2HS.15** Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x)= 2 x^3 or f(x) = (x+1)/(x-1) for x ≠ 1. Instructional Note: Focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as f(x) = x^2, x > 0. **Construct and compare linear, quadratic, and exponential models and**			

solve problems. **M.2HS.16** Using graphs and tables, observe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically; or (more generally) as a polynomial function. Instructional Note: Compare linear and exponential growth studied in Mathematics I to quadratic growth. **SIMILARITY, RIGHT TRIANGLE TRIGONOMETRY, AND PROOF** **Prove and apply trigonometric identities.** **M.2HS.51** Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, and the quadrant of the angle. Instructional Note: Limit θ to angles between 0 and 90 degrees. Connect with the Pythagorean theorem and the distance formula. Extension of trigonometric functions to other angles through the unit circle is included in Mathematics III.			

Number and Quantity

Standards	Teacher Understandings	Resources	Student Understandings
EXTENDING THE NUMBER SYSTEM **Extend the properties of exponents to rational exponents.** **M.2HS.1** Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. (e.g., We define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.) **M.2HS.2** Rewrite expressions involving radicals and rational exponents using the properties of exponents. **Use properties of rational and irrational numbers.** **M.2HS.3** Explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational and that the product of a nonzero rational number and an irrational number is irrational. Instructional Note: Connect	In grade eight, students encounter some examples of irrational numbers, such as π and $\sqrt{2}$ (or $\sqrt{P}$ for p as a non-square number). In Mathematics II, students extend this understanding beyond the fact that there are numbers that are not rational; they begin to understand that rational numbers form a closed system. Students have witnessed that, with each extension of number, the meanings of addition, subtraction, multiplication, and division are extended. In each new number system—integers, rational numbers, and real numbers—the distributive law continues to hold, and the commutative and associative laws are still valid for both addition and multiplication. However, in Mathematics II, students go further along this path.	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. Math TREE Online Education Resources A curated set of aligned internet resources for WV middle and high school math teachers.	• Students make meaning of the representation of radicals with rational exponents. • Students have an understanding of the basic properties of exponents and use these properties to explain the meaning of rational exponents. • Students extend their understanding of radical and rational exponents to variable expressions. • Students work with examples of quadratic functions and solve quadratic equations, encountering situations in which a resulting equation does not have a solution that is a real number. • Students expand their extension of the concept of *number* to include complex numbers so that

to physical situations, e.g., finding the perimeter of a square of area 2. **Perform arithmetic operations with complex numbers.** **M.2HS.4** Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real. **M.2HS.5** Use the relation $i^2 = -1$ and the commutative, associative and distributive properties to add, subtract and multiply complex numbers. Instructional Note: Limit to multiplications that involve i^2 as the highest power of i. **EXPRESSIONS AND EQUATIONS** **Use complex numbers in polynomial identities and equations.** **M.2HS.24** Solve quadratic equations with real coefficients that have complex solutions. Instructional Note: Limit to quadratics with real coefficients. **M.2HS.25(+)** Extend polynomial identities to the complex numbers. For example, rewrite x2 + 4 as (x + 2i)(x – 2i). Instructional	Students may explain that the sum or product of two rational numbers is rational by arguing that the sum of two fractions with integer numerator and denominator is also a fraction of the same type, showing that the rational numbers are *closed* under the operations of addition and multiplication. Students argue that the sum of a rational and an irrational is irrational, and the product of a non-zero rational and an irrational is still irrational, showing that irrational numbers are truly an additional set of numbers that, along with rational numbers, form a larger *system*: real numbers. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and		an equation that does not have a solution that is a real number can be solved. • By exploring polynomials that can be factored with real and complex roots, students develop an understanding of the Fundamental Theorem of Algebra.

Note: Limit to quadratics with real coefficients. **M.2HS.26(+)** Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Instructional Note: Limit to quadratics with real coefficients.	understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.		

Algebra

Standards	Teacher Understandings	Resources	Student Understandings
EXPRESSIONS AND EQUATIONS **Interpret the structure of expressions.** **M.2HS.17** Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.	Students begin their work with expressions and equations in grades six through eight and extend their work to more complex expressions in Mathematics I. In Mathematics II, students encounter quadratic expressions for the first time and learn a new set of strategies for working with these expressions. As in Mathematics I, the Algebra conceptual category is	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and	• Students extend their work with expressions to include examples of more complicated expressions, such as those that involve multiple variables and exponents. • Students factor second-degree polynomials and simple third-degree polynomials by making use of special forms and using factoring

Instructional Note: Focus on quadratic and exponential expressions. Exponents are extended from the integer exponents found in Mathematics I to rational exponents focusing on those that represent square or cube roots. **M.2HS.18** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. Instructional Note: Focus on quadratic and exponential expressions. **Write expressions in equivalent forms to solve problems.** **M.2HS.19** Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for	closely tied to the Functions conceptual category, linking the writing of equivalent expressions, solving equations, and graphing to concepts involving functions. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.	fluency, and application. Math TREE Online Education Resources A curated set of aligned internet resources for WV middle and high school math teachers. Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	techniques based on properties of operations. • Students employ purposeful transformation of expressions into equivalent forms that are suitable for the purpose at hand. • Students use different forms of the same expression to reveal important characteristics of the expressions. • To perform operations with polynomials meaningfully, students draw parallels between the set of integers and the set of all polynomials with real coefficients. • Students work with all available types of functions to create equations, including quadratic functions, absolute value functions, and simple rational and exponential functions. • Students extend their work with exponents to include quadratic

exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. Instructional Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal. **Create equations that describe numbers or relationships.** **M.2HS.20** Create equations and inequalities in one variable and use them to solve problems. Instructional Note: Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Extend work on linear and exponential equations in Mathematics I to quadratic equations. **M.2HS.21** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and			functions and equations. To extend their understanding of quadratic expressions and the functions defined by such expressions, students investigate properties of quadratics and their graphs in the conceptual category, functions. • In Mathematics I, students investigate how to “undo” linear and simple exponential functions; student now do so for quadratic functions and discover that the process is more complex. • Students solve quadratic equations by using the zero product property, completing the square, and the quadratic formula.

scales. Instructional Note: Extend work on linear and exponential equations in Mathematics I to quadratic equations. **M.2HS.22** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Extend to formulas involving squared variables. Extend work on linear and exponential equations in Mathematics I to quadratic equations. **Solve equations and inequalities in one variable.** **M.2HS.23** Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic			

formula gives complex solutions and write them as a ± bi for real numbers a and b. Instructional Note: Extend to solving any quadratic equation with real coefficients, including those with complex solutions. **Solve systems of equations.** **M.2HS.27** Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. (e.g., Find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.) Instructional Note: Include systems that lead to work with fractions. (e.g., Finding the intersections between $x^2 + y^2 = 1$ and $y = (x+1)/2$ leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple $3^2 + 4^2 = 5^2$.) **EXTENDING THE NUMBER SYSTEM** **Perform arithmetic operations on polynomials** **M.2HS.6** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract and multiply polynomials. Instructional			

Note: Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.			

Geometry

Standards	Teacher Understandings	Resources	Student Understandings
SIMILARITY, RIGHT TRIANGLE TRIGONOMETRY, AND PROOF **Understand similarity in terms of similarity transformations** **M.2HS.39** Verify experimentally the properties of dilations given by a center and a scale factor. a. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. **b.** The dilation of a line segment is longer or shorter in the ratio given by the scale factor. **M.2HS.40** Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of	In Mathematics I, students begin to formalize their understanding of geometry by defining congruence in terms of well-defined rigid motions of the plane. They find that congruence can be deduced in certain cases by investigating other relationships (e.g., that for triangles, the ASA, SAS, and SSS congruence criteria held). In Mathematics II, students further enrich their ability to reason deductively and begin to write more formal proofs of various geometric results. They also apply to triangles their knowledge of	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. Math TREE Online Education Resources A curated set of aligned internet resources for WV middle and high school math teachers.	• Students prove the congruence criteria for triangles (ASA, SAS, and SSS) with the more basic concept of congruence by rigid motions. Students develop the reasoning involved in connecting one step in the logical argument to the next. • Students make conjectures based on experimentation and justify their conjectures, communicating their reasoning to their peers. • Students focus on right triangles and triangle relationships. • Students worked with dilations as a transformation in Grade

similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides. **M.2HS.41** Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. **Prove geometric theorems.** **M.2HS.42** Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. Implementation may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for M.2HS.C.3. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying	similarity and discover powerful relationships in right triangles, leading to the discovery of trigonometric functions. Finally, students' understanding of the Pythagorean relationship and their work with quadratics leads to algebraic representations of circles and more complex proofs of results in the plane. Building on their work with transformations, students produce increasingly formal arguments about geometric relationships, particularly around the concept of similarity. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students'	[Quantile Teacher Assistant](#) This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	8. Students now explore the properties of dilations in more detail and develop an understanding of the concept of scale factor. • Students develop a more precise mathematical definition of similarity. Students understand that two objects are similar if there is a sequence of transformations that maps one object onto the other. • Student expand and apply their understanding of the angle-angle similarity criterion for triangles to right triangles and develop and understanding of the trigonometric functions as relationships completely determined by angles. • Students investigate the relationships between sine, cosine, and tangent. • Students explore the relationship between the

reasoning while exploring a variety of formats for expressing that reasoning. **M.2HS.43** Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of this standard may be extended to include concurrence of perpendicular bisectors and angle bisectors in preparation for the unit on Circles With and Without Coordinates. **M.2HS.44** Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a	skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.		sine and cosine of complementary angles. • Students investigate properties of circles and relationships among angles, radii, and chords. • Students extend their understanding of the concept of similarity to develop a definition of radian measure. • Students develop connections between algebraic and geometric concepts by representing two-dimensional shapes on a coordinate system and describing the connections by using algebraic equations and inequalities. • Students use the Pythagorean Theorem and the definition of a circle to derive the equation of a circle. • Students derive the equation of a parabola given the focus and directrix.

parallelogram bisect each other and conversely, rectangles are parallelograms with congruent diagonals. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. **Prove theorems involving similarity.** **M.2HS.45** Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally and conversely; the Pythagorean Theorem proved using triangle similarity. **M.2HS.46** Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures. **Use coordinates to prove simple geometric theorems algebraically.** **M.2HS.47** Find the point on a directed line			• Students explain volume formulas and use them to solve problems.

segment between two given points that partitions the segment in a given ratio. **Define trigonometric ratios and solve problems involving right triangles.** **M.2HS.48** Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles. **M.2HS.49** Explain and use the relationship between the sine and cosine of complementary angles. **M.2HS.50** Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems. **CIRCLES WITH AND WITHOUT COORDINATES** **Understand and apply theorems about circles.** **M.2HS.52** Prove that all circles are similar. **M.2HS.53** Identify and describe relationships among inscribed angles, radii and chords. Include the relationship between central, inscribed and			

circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle. **M.2HS.54** Construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle. **M.2HS.55(+)** Construct a tangent line from a point outside a given circle to the circle. **Find arc lengths and areas of sectors of circles.** **M.2HS.56** Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Instructional Note: Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be			

applied to the development of circular trigonometry in this course. **Translate between the geometric description and the equation for a conic section.** **M.2HS.57** Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. Instructional Note: Connect the equations of circles and parabolas to prior work with quadratic equations. **M.2HS.58** Derive the equation of a parabola given the focus and directrix. Instructional Note: The directrix should be parallel to a coordinate axis. **Use coordinates to prove simple geometric theorems algebraically.** **M.2HS.59** Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.)			

Instructional Note: Include simple proofs involving circles. **Explain volume formulas and use them to solve problems.** **M.2HS.60** Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle and informal limit arguments. Instructional Note: Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. **M.2HS.61** Use volume formulas for cylinders, pyramids, cones and spheres to solve problems. Volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.			

Statistics and Probability

Standards	Teacher Understandings	Resources	Student Understandings
APPLICATIONS OF PROBABILITY **Understand independence and conditional probability and use them to interpret data.** **M.2HS.28** Verify experimentally the properties outcomes) using characteristics (or categories) of the outcomes or as unions, intersections or complements of other events (“or,” “and,” “not”). **M.2HS.29** Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent. **M.2HS.30** Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	In grades seven and eight, students learn some basics of probability, including chance processes, probability models, and sample spaces. In higher mathematics, the relative frequency approach to probability is extended to conditional probability and independence, rules of probability and their use in finding probabilities of compound events, and the use of probability distributions to solve problems involving expected value. Building on probability concepts that develop in grades six through eight, students use the language of set theory to expand their ability to compute and interpret theoretical and experimental probabilities for compound events, attending to mutually exclusive events,	**Educators’ Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. Math TREE Online Education Resources A curated set of aligned internet resources for WV middle and high school math teachers.	• Students develop an understanding of conditional probability, including those in which the uniform probabilities attached to outcomes lead to independence of the outcomes and those in which they do not. • Students find probabilities of compound events by using the Addition Rule and the general Multiplication Rule. • Students use probability models and probability experiments to make decisions.

M.2HS.31 Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. (e.g., Collect data from a random sample of students in your school on their favorite subject among math, science and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.) Instructional Note: Build on work with two-way tables from Mathematics I to develop understanding of conditional probability and independence. **M.2HS.32** Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. (e.g., Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have	independent events, and conditional probability. Students should make use of geometric probability models wherever possible. They use probability to make informed decisions. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.	Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	

lung cancer.) **Use the rules of probability to compute probabilities of compound events in a uniform probability model.** **M.2HS.33** Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model. **M.2HS.34** Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model. **M.2HS.35(+)** Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model. **M.2HS.36(+)** Use permutations and combinations to compute probabilities of compound events and solve problems. **Use probability to evaluate outcomes of decisions.** **M.2HS.37(+)** Use probabilities to make fair			

decisions (e.g., drawing by lots or using a random number generator). **M.2HS.38(+)** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game). Instructional Note: This unit sets the stage for work in Mathematics III, where the ideas of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e., incomplete information) requires an understanding of probability concepts. .			

Steven L. Paine, Ed.D.
West Virginia Superintendent of Schools