West Virginia math4life

Frameworks for Mathematics

Mathematics III

West Virginia DEPARTMENT OF EDUCATION

High School Mathematics III

Math III LA course does not include the (+) standards.
Math III STEM course includes standards identified by (+) sign
Math III TR course (Technical Readiness) includes standards identified by (*)
Math IV TR course (Technical Readiness) includes standards identified by (^)

Math III Technical Readiness and Math IV Technical Readiness are course options (for juniors and seniors) built for the mathematics content of Math III through integration of career clusters. These courses integrate academics with hands-on career content. The collaborative teaching model is recommended for our Career and Technical Education (CTE) centers. The involvement of a highly qualified Mathematics teacher and certified CTE teachers will ensure a rich, authentic and respectful environment for delivery of the academics in "real world" scenarios.

In Mathematics III, students understand the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. They connect multiplication of polynomials with multiplication of multi-digit integers and division of polynomials with long division of integers. Students identify zeros of polynomials and make connections between zeros of polynomials and solutions of polynomial equations. Their work on polynomial expressions culminates with the Fundamental Theorem of Algebra. Rational numbers extend the arithmetic of integers by allowing division by all numbers except 0. Similarly, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme of working with rational expressions is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers.

Students synthesize and generalize what they have learned about a variety of function families. They extend their work with exponential functions to include solving exponential equations with logarithms. They explore the effects of transformations on graphs of diverse functions, including functions arising in an application, in order to abstract the general principle that transformations on a graph always have the same effect, regardless of the type of the underlying functions.

Students develop the Laws of Sines and Cosines in order to find missing measures of general (not necessarily right) triangles. They are able to distinguish whether three given measures (angles or sides) define 0, 1, 2, or infinitely many triangles. This discussion of general triangles opens up the idea of trigonometry applied beyond the right triangle—that is, at least to obtuse angles. Students build on this idea to develop the notion of radian measure for angles and extend the domain of the trigonometric functions to all real numbers. They apply this knowledge to model simple periodic phenomena.

Students see how the visual displays and summary statistics they learned in previous grade levels or courses relate to different types of data and to probability distributions. They identify different ways of collecting data—including sample surveys, experiments, and simulations—and recognize the role that randomness and careful design play in the conclusions that may be drawn.

Finally, students in Mathematics III extend their understanding of modeling: they identify appropriate types of functions to model a situation, adjust parameters to improve the model, and compare models by analyzing appropriateness of fit and by making judgments about the domain over which a model is a good fit. The description of modeling as "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions" (National Governors Association Center for Best Practices, Council of Chief State School Officers [NGA/CCSSO] 2010e) is one of the main themes of this course. The discussion about modeling and the diagram of the modeling cycle that appear in this chapter should be considered when students apply knowledge of functions, statistics, and geometry in a modeling context.

Modeling

Standards	Teacher Understandings	Resources	Student Understandings
Although the Modeling category does not include specific standards, the idea of using mathematics to model the world pervades all higher mathematics courses and should hold a significant place in instruction. Modeling is addressed first here to emphasize its importance in the higher mathematics curriculum.	Modeling at the higher mathematics level goes beyond the simple application of previously constructed mathematics and includes real-world problems. True modeling begins with students asking a question about the world around them, and mathematics is then constructed in the process of attempting to answer the question. Students may see when trying to answer their question that solving an	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding,	• When students are presented with a real-world situation and challenged to ask a question, new issues arise (e.g., Which of the quantities present in this situation are known, and which are unknown?). • Students decide on a solution path that may need to be revised. They make use of tools such as calculators, dynamic geometry software, and spreadsheets. They try to

	equation arises as a necessity and that the equation often involves the specific instance of knowing the output value of a function at an unknown input value. Modeling problems have an element of being genuine problems in the sense that students care about answering the question under consideration. In modeling, mathematics is used as a tool to answer questions that students really want answered. Students examine a problem and formulate a *mathematical model* (an equation, table, graph, or the like), compute an answer or rewrite their expression to reveal new information, interpret and validate the results, and report out. This is a new approach for many teachers and may be challenging to implement, but the effort should show students that mathematics is relevant to their lives. From a	procedural skill and fluency, and application. It highlights some necessary foundational skills from previous grade levels. [Math TREE Online Education Resources](#) A curated set of aligned, internet resources for WV middle and high school math teachers. [Quantile Teacher Assistant](#) This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	use previously derived models (e.g., linear functions), but may find that a new formula or function will apply.

	pedagogical perspective, modeling gives a concrete basis from which to abstract the mathematics and often serves to motivate students to become independent learners. The important ideas surrounding rational functions, graphing, solving equations, and rates of change should be explored through the lens of mathematical modeling. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to		

	address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.		

Functions

Standards	Teacher Understandings	Resources	Student Understandings
POLYNOMIAL, RATIONAL, AND RADICAL RELATIONSHIPS **Analyze functions using different representations.** **M.3HS.24 (*,^)** Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior. Instructional Note: Relate to the relationship between zeros of quadratic functions and their factored forms. **TRIGONOMETRY OF GENERAL TRIANGLES AND TRIGONOMETRIC FUNCTIONS** **Extend the domain of trigonometric functions using the unit circle.**	The standards in the Functions conceptual category can serve as motivation for the study of standards in the other Mathematics III conceptual categories. Students have already worked with equations in which they have to "solve for x" as a search for the input of a function f that gives a specified output; solving the equation amounts to undoing the work of the function. The types of functions that students encounter in Mathematics III have new properties. Students previously learned that quadratic functions exhibit different behavior	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. Math TREE Online Education Resources	• Students extend their knowledge of linear, exponential, and quadratic functions to include a much broader range of classes of functions. • As in Mathematics II, students work with functions that model data and choose an appropriate model function by considering the context that produced the data. Students' ability to recognize rates of change, growth and decay, end behavior, roots, and other characteristics of functions becomes more

M.3HS.28 (*) Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle. **M.3HS.29 (*)** plain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle. **Model periodic phenomena with trigonometric functions.** **M.3HS.30 (*)** Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline. **MATHEMATICAL MODELING** **Interpret functions that arise in applications in terms of a context.** **M.3HS.35 (*)** For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function	from linear and exponential functions; now they investigate polynomial, rational, and trigonometric functions in greater generality. As in the Mathematics II course, students must discover new techniques for solving the equations they encounter. Students see how rational functions can model real-world phenomena, in particular in instances of inverse variation ($x \bullet y = k$, k a constant), and how trigonometric functions can model periodic phenomena. In general, functions describe how two quantities are related in a precise way and can be used to make predictions and generalizations, keeping true to the emphasis on modeling that occurs in higher mathematics. As stated in the University of Arizona (UA) Progressions Documents, "students should develop ways of thinking that are general and allow them to	A curated set of aligned, internet resources for WV middle and high school math teachers. Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	sophisticated; they use this expanding repertoire of families of functions to inform their choices of models. • Students determine how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. • In grade eight, students learn that the *rate of change* of a linear function is equal to the slope of the graph of that function. And because the slope of a line is constant, the phrase "rate of change" is clear for linear functions. For non-linear functions, however, rates of change are not constant, and thus average rates of change over an interval are used. • Students in Mathematics III develop models for more complex or sophisticated situations

is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Emphasize the selection of a model function based on behavior of data and context. **M.3HS.36 (*)** Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Instructional Note: Emphasize the selection of a model function based on behavior of data and context. **M.3HS.37 (*)** Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Emphasize the selection of a model function based on behavior of data and context. **Analyze functions using different representations.** **M.3HS.38 (*,^)**	approach any function, work with it, and understand how it behaves, rather than see each function as a completely different animal in the bestiary." This document describes the skills and understandings necessary for a student to be prepared for grade seven mathematics. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and		than in previous courses, due to the expansion of the types of functions available to them. • Students learn that some functions have the property that an input can be recovered from a given output. Students understand that this is an attempt to "undo" the function, or to "go backwards." Tables and graphs can support student understanding. • Students extend their work with exponential models from Mathematics II and deduce that the exponential function $f(x) = b^x$ has an inverse—which is called *the logarithm to the base* b. • Students solve problems involving exponential functions and logarithms and express their answers by using logarithm notation.

Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph square root, cube root and piecewise-defined functions, including step functions and absolute value functions. b. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline and amplitude. Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. **M.3HS.39 (*,^)** Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. **M.3HS.40 (*,^)** Compare properties of two functions each represented in a different way	strengthen understandings throughout the school year.		• In general, students understand logarithms as functions that *undo* their corresponding exponential functions. • Students expand their understanding of trigonometric functions, initially developed in Mathematics II. Through exploration of the unit circle, students develop the Pythagorean Identity; this basic identity yields other identities through algebraic manipulation and allows students to find values of other trigonometric functions for a given if one value is known. • Students explore the graphs of trigonometric functions, focusing on the connection between the unit-circle representation of the trigonometric functions and their properties—for example, to illustrate the periodicity of the functions, the

(algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate. **Build a function that models a relationship between two quantities.** **M.3HS.41 (*)** Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.) Instructional Note: Develop models for more complex or sophisticated situations than in previous courses. **Build new functions from existing functions.** **M.3HS.42 (*)** Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k			relationship between the maximums and minimums of the sine and cosine graphs, zeros, etc. • Students use trigonometric functions to model periodic phenomena.

(both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Use transformations of functions to find more optimum models as students consider increasingly more complex situations. Note the effect of multiple transformations on a single function and the common effect of each transformation across function types. Include functions defined only by graph. **M.3HS.43 (*)** Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. (e.g., f(x) = 2 x^3 or f(x) = (x+1)/(x-1) for x ≠ 1.) Instructional Note: Extend this standard to simple rational, simple radical, and simple exponential functions. **Construct and compare linear, quadratic, and exponential models and solve problems.** **M.3HS.44 (*)**			

For exponential models, express as a logarithm the solution to a $b^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. Instructional Note: Consider extending this unit to include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that log xy = log x + log y.			

Number and Quantity

Standards	Teacher Understandings	Resources	Student Understandings
POLYNOMIAL, RATIONAL, AND RADICAL RELATIONSHIPS **Use complex numbers in polynomial identities and equations.** **M.3HS.10 (+)** Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x - 2i)$. Instructional Note: Build on work with quadratics equations in Mathematics II. Limit to polynomials with real coefficients. **M.3HS.11 (+)** Know the Fundamental Theorem of	Students continue to expand their understanding of the number system by finding complex-number roots when solving quadratic equations. Complex numbers have a practical application, and many phenomena involving real numbers become simpler when real numbers are viewed as a subsystem of complex numbers. As an example, complex solutions of differential equations can present a clear picture of the	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual	• Students continue working with complex numbers as solutions to polynomial equations. For example, students can draw upon the Mathematics III algebra standards and find roots of equations such as $x^3 + 5x^2 + 8x + 6 = 0$ using the remainder theorem, polynomial long division, or other factoring techniques.

Algebra; show that it is true for quadratic polynomials.	behavior of real solutions. Students are introduced to this when they study complex solutions of quadratic equations—and when complex numbers are involved, each quadratic polynomial can be expressed as a product of linear factors. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.	understanding, procedural skill and fluency, and application. Math TREE Online Education Resources A curated set of aligned, internet resources for WV middle and high school math teachers.	• Students develop an understanding of the Fundamental Theorem of Algebra.

Algebra

Standards	Teacher Understandings	Resources	Student Understandings
POLYNOMIAL, RATIONAL, AND RADICAL RELATIONSHIPS **Interpret the structure of expressions.** **M.3HS.12 (*)** Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. (e.g., Interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.) Instructional Note: Extend to polynomial and rational expressions. **M.3HS.13 (*)** Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 – y^4$ as $(x^2)^2 – (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 – y^2)(x^2 + y^2)$. Instructional Note: Extend to polynomial and rational expressions. **Write expressions in equivalent forms to solve problems.**	Along with the standards in the conceptual category, number and quantity, the standards from the conceptual category, algebra, of the Mathematics III course develop the structural similarities between the system of polynomials and the system of integers. Students draw on analogies between polynomial arithmetic and base-ten computation, focusing on properties of operations, particularly the distributive property. Students connect multiplication of polynomials with multiplication of multi-digit integers and connect division of polynomials with long division of integers. Similar to the way that rational numbers extend the arithmetic of integers by allowing	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. Math TREE Online Education Resources A curated set of aligned, internet resources for WV middle and high school math teachers. Quantile Teacher Assistant This tool is aligned to WV standards and is	• Students continue to pay attention to the meaning of expressions in context and interpret the parts of an expression by "chunking"—that is, by viewing parts of an expression as a single entity. For example, their facility with using special cases of polynomial factoring allows them to fully factor more complicated polynomials. • Students' ability to factor expressions expands with their new understanding of complex numbers. • Students see polynomials as a system analogous to the integers that they can add, subtract, and multiply. • Students extend this understanding of polynomials to rational expressions, which are

M.3HS.14 (^) Derive the formula for the sum of a geometric series (when the common ratio is not 1), and use the formula to solve problems. (e.g., Calculate mortgage payments.) Instructional Note: Consider extending to infinite geometric series in curricular implementations of this course description. **Perform arithmetic operations on polynomials.** **M.3HS.15(*)** Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction and multiplication; add, subtract and multiply polynomials. Instructional Note: Extend beyond the quadratic polynomials found in Mathematics II. **Understand the relationship between zeros and factors of polynomials.** **M.3HS.16 (*)** Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x). **M.3HS.17 (*)** Identify zeros of polynomials when	division by all numbers except 0, rational expressions extend the arithmetic of polynomials by allowing division by all polynomials except the zero polynomial. A central theme that arises is that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content	designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	analogous to rational numbers. • Students continue to develop their understanding of the set of polynomials as a system analogous to the set of integers that exhibits certain properties, and they explore the relationship between the factorization of polynomials and the roots of a polynomial. • Students use the zeros of a polynomial to create a rough sketch of its graph and connect the results to their understanding of polynomials as functions. • Students explore rational functions as a system analogous to the rational numbers. They see rational functions as useful for describing many real-world situations. • Students are able to rewrite rational expressions to highlight characteristics, such as

suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial. **Use polynomial identities to solve problems.** **M.3HS.18 (^)** Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 – y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. Instructional Note: This cluster has many possibilities for optional enrichment, such as relating the example in M.A2HS.10 to the solution of the system $u^2 + v^2 = 1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x + y)^{n+1} = (x + y)(x + y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction. **M.3HS.19 (+,^)** Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal’s Triangle. Instructional Note: This cluster has many possibilities for	standards and to revisit and practice skills and strengthen understandings throughout the school year.		rates of growth, approximation, roots, axis intersections, asymptotes, end behavior, etc. • Students work with all available types of functions, including root functions, to create equations and address more complex situations than those addressed in Mathematics I and Mathematics II. • Students extend their equation-solving skills to those involving rational expressions and radical equations, and they make sense of extraneous solutions that may arise.

optional enrichment, such as relating the example in M.A2HS.10 to the solution of the system $u^2 + v^2 = 1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x + y)^{n+1} = (x + y)(x + y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction. **Rewrite rational expressions.** **M.3HS.20 (*)** Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. Instructional Note: The limitations on rational functions apply to the rational expressions. **M.3HS.21 (+)** Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply and divide rational expressions. Instructional Note: Requires the			

general division algorithm for polynomials. **REASONING WITH EQUATIONS AND INEQUALITIES** **Understand solving equations as a process of reasoning and explain the reasoning.** **M.3HS.22 (*)** Solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise. Instructional Note: Extend to simple rational and radical equations. **Represent and solve equations and inequalities graphically.** **M.3HS.23 (*,^)** Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential and logarithmic functions. Instructional Note: Include combinations of linear, polynomial,			

rational, radical, absolute value, and exponential functions. **MATHEMATICAL MODELING** **Create equations that describe numbers or relationships.** **M.3HS.31 (*,^)** Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Use all available types of functions to create such equations, including root functions, but constrain to simple cases. **M.3HS.32 (*,^)** Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: While functions will often be linear, exponential or quadratic the types of problems should draw from more complex situations than those addressed in Mathematics I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line.			

M.3HS.33 (*,^) Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) **M.3HS.34 (*,^)** Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: The example given applies to earlier instances of this standard, not to the current course.			

Geometry

Standards	Teacher Understandings	Resources	Student Understandings
TRIGONOMETRY OF GENERAL TRIANGLES AND TRIGONOMETRIC FUNCTIONS **Apply trigonometry to general triangles.** **M.3HS.25 (+,^)** Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an	In Mathematics III, students extend their understanding of the relationship between algebra and geometry as they explore the equations for circles and parabolas. They also expand their understanding of	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight	• Students advance their knowledge of right-triangle trigonometry by applying trigonometric ratios in non-right triangles.

auxiliary line from a vertex perpendicular to the opposite side. **M.3HS.26 (+, ^)** Prove the Laws of Sines and Cosines and use them to solve problems. Instructional Note: With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles. **M.3HS.27 (+,^)** Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems and/or resultant forces). **MATHEMATICAL MODELING** **Visualize relationships between two dimensional and three-dimensional objects.** **M.3HS.45 (*,^)** Identify the shapes of two-dimensional cross-sections of three dimensional objects and identify three-dimensional objects generated by rotations of two-dimensional objects. **Apply geometric concepts in modeling situations.** **M.3HS.46 (*,^)**	trigonometry to include finding unknown measurements in non-right triangles. The Geometry standards included in the Mathematics III course offer many rich opportunities for students to practice mathematical modeling **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and	connections to the Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. [Math TREE Online Education Resources] A curated set of aligned, internet resources for WV middle and high school math teachers. [Quantile Teacher Assistant] This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	• Students use reasoning about similarity and trigonometric identities to derive the Laws of Sines and Cosines in acute triangles, and they use these and other relationships to solve problems. • Students use geometric shapes, their measures, and their properties to describe objects. Student work with two- and three-dimensional shapes is not relegated to simple applications of formulas. • Students solve design problems by modeling with geometry. • Students further their understanding of the connection between algebra and geometry by applying the definition of circles and parabolas to derive equations and then deciding whether a given quadratic equation of the form $ax^2 + by^2 +$

Use geometric shapes, their measures and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). **M.3HS.47 (*,^)** Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile or BTUs per cubic foot). **M.3HS.48 (*,^)** Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost and/or working with typographic grid systems based on ratios).	strengthen understandings throughout the school year.		$cx + dy + e = 0$ represents a circle or a parabola.

Statistics and Probability

Standards	Teacher Understandings	Resources	Student Understandings
INFERENCES AND CONCLUSIONS FROM DATA **Summarize, represent, and interpret data on single count or measurement variable.** **M.3HS.1(*)** Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate	In Mathematics III, students develop a more formal and precise understanding of statistical inference, which requires a deeper understanding of probability. They explore the conditions that meet *random sampling* of a population and that	**Educators' Guide** Organized by conceptual categories, this document provides exemplars to explain the content standards, highlight connections to the	• Students examine the role of randomization in statistical design. • Students build on their understanding of data distributions to see how to use the area under the normal distribution to make estimates of

population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve. Instructional Note: While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution. **Understand and evaluate random processes underlying statistical experiments.** **M.3HS.2(*)** Understand that statistics allows inferences to be made about population parameters based on a random sample from that population. **M.3HS.3(*)** Decide if a specified model is consistent with results from a given data-generating process, for example, using simulation. (e.g., A model says a spinning coin falls heads up with	allow for generalization of results to that population. Students also learn to use significant differences to make inferences about data gathered during the course of experiments. **Content by Cluster** Teachers must provide students opportunity to master each content standard. It is important to understand that neglecting grade-level content standards will leave gaps in students' skills and understandings and will leave students unprepared for the challenges they face in later grades. A content plan must demonstrate a means by which students can be provided opportunity to address all grade-level content standards and to revisit and practice skills and strengthen understandings throughout the school year.	Mathematical Habits of Mind, and demonstrate the importance of developing conceptual understanding, procedural skill and fluency, and application. Math TREE Online Education Resources A curated set of aligned, internet resources for WV middle and high school math teachers. Quantile Teacher Assistant This tool is aligned to WV standards and is designed to help educators locate resources that can support instruction and identify skills most relevant to standards.	frequencies (which can be expressed as probabilities). • Students move beyond analysis of data to make sound statistical decisions based on probability models. • In earlier grade levels, students were introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. Students revisit these concepts with a focus on how the way in which data are collected determines the scope and nature of the conclusions that can be drawn from those data. Through simulation, students develop the concept of *statistical significance*. • Students' prior work in applying probability models to make and analyze decisions is extended to more

probability 0.5. Would a result of 5 tails in a row cause you to question the model?) Instructional Note: Include comparing theoretical and empirical results to evaluate the effectiveness of a treatment. **Make inferences and justify conclusions from sample surveys, experiments, and observational studies.** **M.3HS.4 (*,^)** Recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.			complex probability models.

M.3HS.5 (*,^) Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. Instructional Note: Focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness. **M.3HS.6 (*,^)** Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. Instructional Note: Focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness. **M.3HS.7 (*,^)** Evaluate reports based on data. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of			

the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment. **Use probability to evaluate outcomes of decisions.** **M.3HS.8 (+, ^)** Use probabilities to make fair decisions (e.g., drawing by lots or using a random number generator). **M.3HS.9 (+, ^)** Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game). Instructional Note: Extend to more complex probability models. Include situations such as those involving quality control or diagnostic tests that yields both false positive and false negative results.			

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