Calculus

All West Virginia teachers are responsible for classroom instruction that integrates content standards and objectives and mathematical habits of mind. Students will deepen and extend their understanding of functions, continuity, limits, differentiation, applications of derivatives, integrals, and applications of integration. Students will apply the Rule of Four (Numerical, Analytical, Graphical and Verbal) throughout the course and use available technology to enhance learning. Student will use graphing utilities to investigate concepts and to evaluate derivatives and integrals. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Algebra	Geometry
• A utility company burns coal to generate electricity. The cost C in dollars of removing p% of the air pollutants emissions is $C = \frac{90{,}000p}{100-p}$, $0 \le p < 100$. Find the cost of removing (a) 10%, (b) 25%, and (c) 75% of the pollutants. Find the limit of C as $p \to 100^{-}$. • A management company is planning to build a new apartment complex. Knowing the maximum number of apartments the lot can hold and given a function for the maintenance costs, determine the number of apartments that will minimize the maintenance costs. • The velocity v of the flow of blood at a distance r from the central axis of an artery of radius R is $v = k(R^2 - r^2)$ where k is the constant of proportionality. Find the average rate of flow of blood along a radius of the artery. (Use 0 and R as the limits of integration.)	• The radius of a right circular cylindrical balloon is given by $\sqrt{t+2}$ and its height is $\frac{1}{2}\sqrt{t}$, where t is time in seconds and the dimensions are in inches. Find the rate of change of the volume with respect to time. • Given 50 meters of framing material, construct a window that will let in the most light if the middle of the window is a rectangle and the top and bottom of the window are semi-circles. • The graph of f consists of the three line segments joining the points (0,0), (2,-2), (6,2), and (8,3). The function F is defined as follows $F = \int_0^x f(t)dt$. Find the total enclosed areas generated by f and the x-axis. Determine the points of inflection of F on the interval (0,8).

Data Analysis and Probability	
• The average data entry speeds S (words per minute) of a business student after t weeks of lessons are recorded in the following table. t: 5, 10, 15, 20, 25, 30 S: 28, 56, 79, 90, 93, 94 A model for the data is $S = \frac{100t^2}{65+t}, t > 0$. Do you think that there is a limiting speed? If so, what is the limiting speed? If not, why? • Identify a real life situation that involves quantities that change over time and develop a method to collect and analyze related data. Develop a continuous function to model the data and generalize the results to make a conclusion. • A sheet of typing paper is ruled with parallel lines that are 2 inches apart. A two-inch needle is tossed randomly onto the sheet of paper. The probability that the needle will touch a line is $P = \frac{2}{\pi}\int_0^{\frac{\pi}{2}} \sin\theta \, d\theta$ where θ is the acute angle between the needle and any one of the parallel lines. Find the probability.	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Algebra	
Understand the key concepts, connections and applications of functions, limits, continuity, derivatives, and integrals represented in multiple ways.	Standards 1-19
Geometry	
Apply the key concepts, connections and applications of limits, continuity,	Standards 20-22

derivatives, and integration for a wide variety of regions.	
Data Analysis and Probability	
Apply the key concepts and applications of limits, continuity, derivatives, and integration to analyze functions that represent a collection of data.	Standards 23

Algebra

Cluster	Understand the key concepts, connections and applications of functions, limits, continuity, derivatives, and integrals represented in multiple ways.
M.C.1	Use abstract notation to apply properties of algebraic, trigonometric, exponential, logarithmic and composite functions, as well as their inverses, represented graphically, numerically, analytically, and verbally; and demonstrate an understanding of the connections among these representations.
M.C.2	Demonstrate a conceptual understanding of the definition of a limit via the analysis of continuous and discontinuous functions represented using multiple representations (e.g. graphs and tables).
M.C.3	Use the properties of limits including addition, product, quotient, composition, and squeeze/sandwich theorem to calculate the various forms of limits: one-sided limits, limits at infinity, infinite limits, limits that do not exist, and special limits such as $\lim_{x \to 0} \frac{\sin x}{x} = 1$ and $\lim_{x \to 0} \frac{1 - \cos x}{x} = 0$.
M.C.4	Apply the definition of continuity to determine where a function is continuous or discontinuous including continuity at a point, continuity over an interval, application of the Intermediate Value Theorem, and graphical interpretation of continuity and discontinuity.
M.C.5	Investigate and apply the definition of the derivative graphically, numerically, and analytically at a point, conceptually interpreting the derivative as an instantaneous rate of change and the slope of the tangent line.
M.C.6	Discriminate between the average rate of change and the instantaneous rate of change using real-world problems.
M.C.7	Recognize when the Extreme Value Theorem indicates that function extrema exist.
M.C.8	Quickly recall and apply rules of differentiation including the constant multiple rule, sum rule, the difference rule, the product rule, the quotient rule, the power rule, and the chain rule as applied to algebraic, trigonometric, exponential, logarithmic, and inverse trigonometric functions using techniques of both explicit and implicit differentiation.
M.C.9	Apply Rolle's Theorem and the Mean Value Theorem to real-world problems.

M.C.10	Construct and use mathematical models to solve optimization, related-rates, velocity, and acceleration problems.
M.C.11	Determine antiderivatives that follow from derivatives of basic functions and apply substitution of variables.
M.C.12	Evaluate definite integrals using basic integration properties such as addition, subtraction, constant multipliers, the power rule, substitution, and change of limits.
M.C.13	Characterize the definite integral as the total change of a function over an interval and use this to solve real-world problems.
M.C.14	Apply the Fundamental Theorem of Calculus to evaluate definite integrals and to formulate a cumulative area function and interpret the function as it relates to the integrand.
M.C.15	Use limits to deduce asymptotic behavior of the graph of a function.
M.C.16	Compare and contrast the limit definition (not delta epsilon) of continuity and the graphical interpretation of the continuity of a function at a point; recognize different types of discontinuities.
M.C.17	Develop tangent lines as best linear approximations to functions near specific points; explain this conceptually; and construct these tangent lines; and apply this concept to Newton's Method.
M.C.18	Investigate and explain the relationships among the graphs of a function, its derivative and its second derivative; construct the graph of a function using the first and second derivatives including extrema, points of inflection, and asymptotic behavior.
M.C.19	Approximate areas under a curve using Riemann sums by applying and comparing left, right, and midpoint methods for a finite number of subintervals.

Geometry

Cluster	**Apply the key concepts, connections and applications of limits, continuity, derivatives, and integration for a wide variety of regions.**
M.C.20	Justify why differentiability implies continuity, and classify functional cases when continuity does not imply differentiability.
M.C.21	Calculate a definite integral using Riemann sums by evaluating an infinite limit of a sum using summation notation and rules for summation.
M.C.22	Use integration to solve problems that involve linear displacement, total distance, position, velocity, acceleration and area between curves by looking at both functions of x and functions of y; utilize units to interpret the physical nature of the calculus process.

Data Analysis and Probability

Cluster	**Apply the key concepts and applications of limits, continuity, derivatives, and integration to analyze functions that represent a collection of data.**
M.C.23	Identify a real life situation that involves quantities that change over time; pose a question; make a hypothesis as to the answer; develop, justify, and implement a method to collect, organize, and analyze related data; extend the nature of collected, discrete data to that of a continuous function that describes the known data set; generalize the results to make a conclusion; compare the hypothesis and the conclusion; present the project numerically, analytically, graphically and verbally using the predictive and analytic tools of calculus.