Mathematics - High School Geometry

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning.   Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Congruence, Proof, and Constructions	Similarity, Proof, and Trigonometry
- Prove theorems about triangles and other figures (e.g., that the sum of the measures of the angles in a triangle is 180°). - Given a transformation, work backwards to discover the sequence that led to the transformation. - Given two quadrilaterals that are reflections of each other, find the line of that reflection.	Apply knowledge of trigonometric ratios and the Pythagorean Theorem to determine distances in realistic situations. (e.g., Determine heights of inaccessible objects using various instruments, such as clinometers, hypsometers, transits, etc.)
Extending to Three Dimensions	**Connecting Algebra and Geometry Through Coordinates**
Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	- Use a rectangular coordinate system and build on understanding of the Pythagorean Theorem to find distances. (e.g., Find the area and perimeter of a real-world shape using a coordinate grid and Google Earth.) - Analyze the triangles and quadrilaterals on the coordinate plane to determine their properties. (e.g., Determine whether a given quadrilateral is a rectangle).

Circles With and Without Coordinates	Applications of Probability
• Use coordinates and equations to describe geometric properties algebraically. (e.g., Write the equation for a circle in the plane with specified center and radius.)	• Work with probability and using ideas from probability in everyday situations. (e.g., Compare the chance that a person who smokes will develop lung cancer to the chance that a person who develops lung cancer smokes.)
Modeling with Geometry	
• Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision (e.g., estimate water and food needs in a disaster area, or use volume formulas and graphs to find an optimal size for an industrial package).	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Congruence, Proof, and Constructions	
Experiment with transformations in the plane.	Standards 1-5
Understand congruence in terms of rigid motions.	Standards 6-8
Prove geometric theorems.	Standards 9-11
Make geometric constructions.	Standards 12-13
Similarity, Proof, and Trigonometry	
Understand similarity in terms of similarity transformations.	Standards 14-16
Prove theorems involving similarity.	Standards 17-18
Define trigonometric ratios and solve problems involving right triangles.	Standards 19-21
Apply trigonometry to general triangles.	Standards 22-24
Extending to Three Dimensions	
Explain volume formulas and use them to solve problems.	Standards 25-26
Visualize the relation between two dimensional and three-dimensional objects.	Standard 27

Apply geometric concepts in modeling situations.	Standard 28
Connecting Algebra and Geometry Through Coordinates	
Use coordinates to prove simple geometric theorems algebraically.	Standards 29-32
Translate between the geometric description and the equation for a conic section.	Standard 33
Circles With and Without Coordinates	
Understand and apply theorems about circles.	Standards 34-37
Find arc lengths and areas of sectors of circles.	Standard 38
Translate between the geometric description and the equation for a conic section.	Standard 39
Use coordinates to prove simple geometric theorems algebraically.	Standard 40
Apply geometric concepts in modeling situations.	Standard 41
Applications of Probability	
Understand independence and conditional probability and use them to interpret data.	Standards 42-46
Use the rules of probability to compute probabilities of compound events in a uniform probability model.	Standards 47-50
Use probability to evaluate outcomes of decisions.	Standards 51-52
Modeling with Geometry	
Visualize relationships between two dimensional and three-dimensional objects and apply geometric concepts in modeling situations.	Standards 53-55

Congruence, Proof and Constructions

Cluster	Experiment with transformations in the plane.
M.GHS.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.
M.GHS.2	Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take

	points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.GHS.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.GHS.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).
M.GHS.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, for example, graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle)

Cluster	Understand congruence in terms of rigid motions.
M.GHS.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.
M.GHS.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. Instructional Note: Rigid

	motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.
M.GHS.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.

Cluster	Prove geometric theorems.
M.GHS.9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.
M.GHS.10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of this standard may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for M.GHS.36.
M.GHS.11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.

Cluster	Make geometric constructions.
M.GHS.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.
M.GHS.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.

Similarity, Proof, and Trigonometry

Cluster	Understand similarity in terms of similarity transformations.

M.GHS.14	Verify experimentally the properties of dilations given by a center and a scale factor.
	    a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged.
	    b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.

M.GHS.15	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.
M.GHS.16	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.

Cluster	Prove theorems involving similarity.
M.GHS.17	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

M.GHS.18	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

Cluster	**Define trigonometric ratios and solve problems involving right triangles.**
M.GHS.19	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
M.GHS.20	Explain and use the relationship between the sine and cosine of complementary angles.
M.GHS.21	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.

Cluster	**Apply trigonometry to general triangles.**
M.GHS.22	Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
M.GHS.23	Prove the Laws of Sines and Cosines and use them to solve problems. Instructional Note: With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.
M.GHS.24	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles. Instructional Note: With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.

Extending to Three Dimensions

Cluster	**Explain volume formulas and use them to solve problems.**
M.GHS.25	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. Instructional Note: Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. Similarly, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.
M.GHS.26	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. Instructional Note: Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times

	the area of the first. Similarly, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.

Cluster	**Visualize the relation between two dimensional and three-dimensional objects.**
M.GHS.27	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

Cluster	**Apply geometric concepts in modeling situations.**
M.GHS.28	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). Instructional Note: Focus on situations that require relating two- and three-dimensional objects, determining and using volume, and the trigonometry of general triangles.

Connecting Algebra and Geometry Through Coordinates
(This unit has a close connection with the unit, Circles With and Without Coordinates. Reasoning with triangles in this unit is limited to right triangles; e.g., derive the equation for a line through two points using similar right triangles. Relate work on parallel lines to work in High School Algebra I involving systems of equations having no solution or infinitely many solutions. M.GHS.32 provides practice with the distance formula and its connection with the Pythagorean Theorem.)

Cluster	**Use coordinates to prove simple geometric theorems algebraically.**
M.GHS.29	Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).
M.GHS.30	Prove the slope criteria for parallel and perpendicular lines and uses them to solve geometric problems. (e.g., Find the equation of a line parallel or perpendicular to a given line that passes through a given point.) Instructional Note: Relate work on parallel lines to work in High School Algebra I involving systems of equations having no solution or infinitely many solutions.
M.GHS.31	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
M.GHS.32	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. This standard provides practice with the distance formula and its connection with the Pythagorean theorem.

Cluster	Translate between the geometric description and the equation for a conic section.
M.GHS.33	Derive the equation of a parabola given a focus and directrix. Instructional Note: The directrix should be parallel to a coordinate axis.

Circles With and Without Coordinates

Cluster	Understand and apply theorems about circles.
M.GHS.34	Prove that all circles are similar.
M.GHS.35	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.
M.GHS.36	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
M.GHS.37	Construct a tangent line from a point outside a given circle to the circle.

Cluster	Find arc lengths and areas of sectors of circles.
M.GHS.38	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Instructional Note: Emphasize the similarity of all circles. Reason that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.

Cluster	Translate between the geometric description and the equation for a conic section.
M.GHS.39	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.

Cluster	Use coordinates to prove simple geometric theorems algebraically.
M.GHS.40	Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point $(0, 2)$.) Instructional Note: Include simple proofs involving circles.

Cluster	Apply geometric concepts in modeling situations.
M.GHS.41	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). Instructional Note: Focus on situations in which the analysis of circles is required.

Applications of Probability

Cluster	Understand independence and conditional probability and use them to interpret data.
M.GHS.42	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
M.GHS.43	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
M.GHS.44	Recognize the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. Instructional Note: Build on work with two-way tables from Algebra I to develop understanding of conditional probability and independence.
M.GHS.45	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. Instructional Note: Build on work with two-way tables from Algebra I to develop understanding of conditional probability and independence.
M.GHS.46	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.

Cluster	Use the rules of probability to compute probabilities of compound events in a uniform probability model.
M.GHS.47	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.

M.GHS.48	Apply the Addition Rule, P(A or B) = P(A) + P(B) − P(A and B), and interpret the answer in terms of the model.
M.GHS.49	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
M.GHS.50	Use permutations and combinations to compute probabilities of compound events and solve problems.

Cluster	**Use probability to evaluate outcomes of decisions.** Instructional Note: This unit sets the stage for work in Algebra II, where the ideas of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e. incomplete information) requires an understanding of probability concepts.
M.GHS.51	Use probabilities to make fair decisions (e.g., drawing by lots and/or using a random number generator).
M.GHS.52	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game).

Modeling with Geometry

Cluster	**Visualize relationships between two dimensional and three-dimensional objects and apply geometric concepts in modeling situations.**
M.GHS.53	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).
M.GHS.54	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).
M.GHS.55	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).