West Virginia
College & Career
Readiness Standards

Support for College- and Career-Readiness Standards

MATHEMATICS INTEGRATED PATHWAY

Office of Special Education
Division of Teaching & Learning
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Introduction

Support for College- and Career-Readiness Standards: Mathematics Integrated Pathway is a companion document to the *West Virginia College- and Career-Readiness Standards.*

This document prepares students for study of the grade-level standards through the teaching of prerequisite and enabling skills necessary for mastering each standard. **This allows students to work toward grade-level and course content standards while working at individual ability levels.** By identifying the prerequisite and enabling skills for each standard, teachers may plan instruction **to address the achievement gap.**

Support for College- and Career-Readiness Standards: Mathematics Integrated Pathway provides short-term objectives to help students reach grade-level standards. Educators are encouraged to use the support document to:

- Write annual Individualized Education Program (IEP) goals
- Design targeted interventions
- Write learner objectives
- Develop lesson plans
- Plan for instructional grouping
- Plan for parent communication and conferences
- Prepare students for mastery of state standards

This document helps educators recognize what students are able to do in relation to the grade-level standards in order to help them move toward explicit success criteria.

Educators are reminded that content standards indicate minimum content—what all students should know and be able to do by the end of each grade level or course. Local education agencies (LEAs) may have additional instructional or achievement expectations and may provide instructional guidelines that address content sequence, review, and remediation.

Navigating the Document

The West Virginia College-and Career-Readiness Standards (WV CCRS) will be listed by domain on the left- hand side of each page. The right-hand column labeled *"Can this student…?"* lists suggested supports, or steps, that a student might need to accomplish prior to mastering the grade-level standard.

The bullet at the top of this list is generally the skill closest to the grade-level standard. If a student is unprepared to demonstrate the skill at the top of the list, the teacher should continue to move down the list of suggestions until he or she identifies what that student is able to do. The bulleted list can be thought of as a ladder; starting at the bottom, educators help the student step up each rung until they are ready to demonstrate mastery of the grade-level standard. It is important to remember the pre-requisite skills are **not meant to replace** the grade-level standard nor are they a "break-down" of the standard itself.

For example, if a student has not yet mastered WV CCRS M.6.28, *"Display numerical data in plots on a number line, including dot plots, histograms and plots,"* the teacher should look to the first bullet listed to the right under "Can this student…?" The teacher would ask *"Can this student organize, display, and interpret information in line plot with a horizontal scale in fractional units?"* If not, move to the next bullet: *"Can this student calculate median, mean, and range for a data set?"* Continue down the list until the teacher finds an appropriate starting point for instruction.

Cluster	**Summarize and describe distributions.**	**Can this student…?**
M.6.28	Display numerical data in plots on a number line, including dot plots, histograms and box plots.	• Organize, display, and interpret information in line plots with a horizontal scale in fractional units • Calculate median, mean, and range for a data set • Create number lines using appropriate intervals

*Note for educators writing IEP goals:

Goals should address student's unique needs across the content areas and should link to the West Virginia College- and Career-Readiness Standards so that a student has the foundation or precursor skills and strategies needed to access and progress in the general education curriculum. Keep in mind that the standards themselves are generally not stated in measurable terms and cannot be substituted for individually developed goals. Rather, the annual goal should focus on what is needed for the student to learn and attain the grade-level standard.

Support for Mathematics Standards

High School Mathematics I

All West Virginia teachers are responsible for classroom instruction that integrates content standards and objectives and mathematical habits of mind. Students in this course will focus on six critical units that deepen and extend understanding of linear relationships, in part by contrasting them with exponential phenomena, and in part by applying linear models to data that exhibit a linear trend. Students in Mathematics 1 will use properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades and develop connections between the algebraic and geometric ideas studied. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Relationships between Quantities	Linear and Exponential Relationships
• Solve problems with a wide range of units and solve problems by thinking about units. (e.g., "The Trans Alaska Pipeline System is 800 miles long and cost $8 billion to build. Divide one of these numbers by the other. What is the meaning of the answer?"; "Greenland has a population of 56,700 and a land area of 2,175,600 square kilometers. By what factor is the population density of the United States, 80 persons per square mile, larger than the population density of Greenland?")	• Understand contextual relationships of variables and constants. (e.g., Annie is picking apples with her sister. The number of apples in her basket is described by $n = 22t + 12$, where t is the number of minutes Annie spends picking apples. What do the numbers 22 and 12 tell you about Annie's apple picking?)

Reasoning with Equations	Descriptive Statistics
• Translate between various forms of linear equations. (e.g., The perimeter of a rectangle is given by P = 2W + 2L. Solve for W and restate in words the meaning of this new formula in terms of the meaning of the other variables.) • Explore systems of equations, find and interpret their solutions. (e.g., The high school is putting on the musical Footloose. The auditorium has 300 seats. Student tickets are $3 and adult tickets are $5. The royalty for the musical is $1300. What combination of student and adult tickets do you need to fill the house and pay the royalty? How could you change the price of tickets so more students can go?)	• Use linear regression techniques to describe the relationship between quantities and assess the fit of the model. (e.g., Use the high school and university grades for 250 students to create a model that can be used to predict a student's university GPA based on his high school GPA.)
Congruence, Proof, and Constructions	**Connecting Algebra and Geometry through Coordinates**
• Given a transformation, work backwards to discover the sequence that led to the transformation. • Given two quadrilaterals that are reflections of each other, find the line of that reflection.	• Use a rectangular coordinate system and build on understanding of the Pythagorean Theorem to find distances. (e.g., Find the area and perimeter of a real-world shape using a coordinate grid and Google Earth.) • Analyze the triangles and quadrilaterals on the coordinate plane to determine their properties. (e.g., Determine whether a given quadrilateral is a rectangle.)

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Relationships between Quantities	
Reason quantitatively and use units to solve problems.	Standards 1-3
Interpret the structure of expressions.	Standard 4
Create equations that describe numbers or relationships.	Standards 5-8
Linear and Exponential Relationships	
Represent and solve equations and inequalities graphically.	Standards 9-11
Understand the concept of a function and use function notation.	Standards 12-14
Interpret functions that arise in applications in terms of a context.	Standards 15-17
Analyze functions using different representations.	Standards 18-19
Build a function that models a relationship between two quantities.	Standards 20-21
Build new functions from existing functions.	Standards 22
Construct and compare linear, quadratic, and exponential models and solve problems.	Standards 23-25
Interpret expressions for functions in terms of the situation they model.	Standard 26
Reasoning with Equations	
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 27
Solve equations and inequalities in one variable.	Standard 28
Solve systems of equations.	Standards 29-30
Descriptive Statistics	
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 31-33
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standards 34-35
Interpret linear models.	Standards 36-38

Congruence, Proof, and Constructions	
Experiment with transformations in the plane.	Standards 39-43
Understand congruence in terms of rigid motions.	Standards 44-46
Make geometric constructions.	Standards 47-48
Connecting Algebra and Geometry through Coordinates	
Use coordinates to prove simple geometric theorems algebraically.	Standards 49-51

Relationships between Quantities

Cluster	Reason quantitatively and use units to solve problems.	Can this student...?
M.1HS.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	• Calculate unit rates in number and word problems • Use ratio reasoning to convert measurement units • Solve unit rate problems • Interpret units consistently in formulae
M.1HS.2	Define appropriate quantities for the purpose of descriptive modeling. Instructional Note: Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.	• Specify units of measure
M.1HS.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	• Round numbers to appropriate levels of accuracy

Cluster	Interpret the structure of expressions.	Can this student...?
M.1HS.4	Interpret expressions that represent a quantity in terms of its context.* a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Limit to linear expressions and to exponential expressions with integer exponents.	a – b. • Write, read, and evaluate expressions in which letters stand for numbers • Identify important quantities in a practical situation

Cluster	Create equations that describe numbers or relationships.	Can this student...?
M.1HS.5	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions. Instructional Note: Limit to linear and exponential equations and in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.	• Solve one-step equations and inequalities • Use substitution to verify algebraically the accuracy of the solution • Evaluate expressions using exponents • Write, read, and evaluate expressions
M.1HS.6	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Limit to linear and exponential equations and in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.	• Compare representations of data, including graphs, tables, equations and context • Graph single points on a coordinate plane
M.1HS.7	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: Limit to linear equations and inequalities.	• Write equations, inequalities, and systems given a real-world situation
M.1HS.8	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R. Instructional Note: Limit to formulas with a linear focus.	• Solve multi-step equations

Linear and Exponential Relationships

Cluster	Represent and solve equations and inequalities graphically.	Can this student...?
M.1HS.9	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Instructional Note: Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.	• Graph linear equations with and without technology in a variety of representations (e.g., verbal, descriptions, tables, equations) • Graph single points on a coordinate plane
M.1HS.10	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, (e.g., using technology to graph the functions, make tables of values, or find successive approximations). Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value exponential, and logarithmic functions. Instructional Note: Focus on cases where f(x) and g(x) are linear or exponential.	• Understand that the solution to a system is the intersection of the two functions • Graph linear equations with and without technology in a variety of representations (e.g., verbal, descriptions, tables, equations) • Relate function notation to the form y=mx+b
M.1HS.11	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	• Graph single-variable inequalities on a number line using appropriate shading • Test points in the inequality to verify whether the point is a solution to the inequality • Understand the application of inequality symbols as they pertain to graphing

Cluster	Understand the concept of a function and use function notation.	Can this student...?
M.1HS.12	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear and exponential functions.	• Understand that a function is a rule that assigns to each input exactly one output • Generate a set of ordered pairs using a rule which is stated in verbal, algebraic, or table form; generate a sequence given a rule in verbal or algebraic form
M.1HS.13	Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear and exponential functions.	• Interpret the equation y=mx+b as defining a linear function • Solve single and multi-step equations • Complete input-output tables • Substitute values into a function

M.1HS.14	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n)+ f(n-1) for n ≥ 1. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear and exponential functions. Draw connection to M.1HS.21, which requires students to write arithmetic and geometric sequences. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions.	• Determine the rule for a given sequence • Describe visual and numerical patterns

Cluster	**Interpret functions that arise in applications in terms of a context.**	**Can this student...?**
M.1HS.15	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on linear and exponential functions.	• Interpret the equation y=mx+b as defining a linear function • Graph a linear equation with and/or without technology using a variety of representations • Determine the slope (rate of change) of a line from a table, graph, or equation • Analyze graphs, identify situations, or solve problems with varying rates of change • Identify relations as directly proportional, linear, or nonlinear using rules, tables, and graphs
M.1HS.16	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Instructional Note: Focus on linear and exponential functions.	• Understand that a function is a rule that assigns to each input exactly one output • Write, read, and evaluate expressions in which letters stand for numbers

M.1HS.17	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers. Mathematics II and III will address other function types.	• Determine the slope (rate of change) of a line from a graph, table, or equation • Evaluate algebraic expressions in number and word problems • Use a coordinate grid to solve number and word problems

Cluster	**Analyze functions using different representations.**	**Can this student...?**
M.1HS.18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. **Instructional Note:** Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \cdot 2^n$.	a–b. • Interpret the equation y=mx+b as defining a linear function a. • Graph a linear equation with and/or without technology using a variety of representations • Generate a set of ordered pairs using a rule which is stated in verbal, algebraic, or table form • Determine the slope (rate of change) of a line from a graph, table, or equation b. Perform multi-step operations with rational numbers (positive and negative) in number and word problems
M.1HS.19	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \cdot 2^n$.	• Construct a function to model a linear relationship between two quantities • Identify rate of change from a graph, table, equation, or verbal representation • Describe qualitatively the functional relationship between two quantities by analyzing the graph (e.g., increasing or decreasing, linear or nonlinear, etc.)

Cluster	Build a function that models a relationship between two quantities.	Can this student...?
M.1HS.20	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process or steps for calculation from a context. b. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.) Instructional Note: Limit to linear and exponential functions.	a. • Identify a pattern given various representations (visual, pictorial, numerical, and contextual) b. • Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients • Apply the distributive property to produce the equivalent expression
M.1HS.21	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Instructional Note: Limit to linear and exponential functions. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.	• Perform multi-step operations with rational numbers in number and word problems • Find missing terms in sequences

Cluster	Build new functions from existing functions.	Can this student...?
M.1HS.22	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.	• Interpret the equation y=mx+b as defining a linear function

Cluster	Construct and compare linear, quadratic, and exponential models and solve problems.	Can this student...?
M.1HS.23	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	a – c. • Recognize that the slope (rate of change) of a line is the same between any two given points, meaning it is constant • Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is linear or nonlinear) • Interpret rate of change in the context of a real-world situation
M.1HS.24	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).	• Recognize that the slope (rate of a change) of a line is the same between any two given points • Interpret rate of change in the context of a real-world situation

M.1HS.25	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Limit to comparisons between exponential and linear models.	• Read and interpret information from a graph or table

Cluster	**Interpret expressions for functions in terms of the situation they model.**	**Can this student...?**
M.1HS.26	Interpret the parameters in a linear or exponential function in terms of a context. Instructional Note: Limit exponential functions to those of the form $f(x) = b^x + k$.	• Describe qualitatively the functional relationship between two quantities by analyzing a graph

Reasoning with Equations

Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.**	**Can this student...?**
M.1HS.27	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Instructional Note: Students should focus on linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Mathematics III.	• Use the distributive property and the order of operations to create equivalent expressions • Identify when two expressions are equivalent (e.g.: 3y and y+y+y)

Cluster	Solve equations and inequalities in one variable.	Can this student...?
M.1HS.28	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Instructional Note: Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.	• Solve one-step equations and inequalities • Use substitution to algebraically verify the accuracy of the solution • Evaluate expressions that include exponents

Cluster	Solve systems of equations.	Can this student...?
M.1HS.29	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. Instructional Note: Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to M.1HS.50, which requires students to prove the slope criteria for parallel lines.	• Graph a system of equations with or without technology • Apply the distributive property to an expression
M.1HS.30	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Instructional Note: Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to M.1HS.50, which requires students to prove the slope criteria for parallel lines.	• Graph linear equations with and without technology in a variety of representations (e.g., verbal, descriptions, tables, equations

Descriptive Statistics

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.	Can this student...?
M.1HS.31	Represent data with plots on the real number line (dot plots, histograms, and box plots).	• Determine the quartiles or interquartile range for a set of data • Interpret information on a graph • Locate points on a number line
M.1HS.32	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Instructional Note: In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.	• Choose a measure of central tendency based on the shape of the data distribution • Describe data using or selecting the appropriate measure of central tendency • Calculate median, mean, and interquartile range
M.1HS.33	Interpret differences in shape, center and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Instructional Note: In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.	• Choose a measure of central tendency based on the shape of the data distribution • Describe data using or selecting the appropriate measure of central tendency • Describe patterns such as clustering, outliers, positive or negative association, and linear and nonlinear association

Cluster	Summarize, represent, and interpret data on two categorical and quantitative variables.	Can this student...?
M.1HS.34	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data.	• Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table • Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects • Use relative frequencies calculated for rows or columns to describe possible associations between two variables • Calculate with percentages • Convert from a fraction to a percent
M.1HS.35	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. (Focus should be on situations for which linear models are appropriate.) c. Fit a linear function for scatter plots that suggest a linear association. Instructional Note: Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.	a—c. • Organize, display, and interpret information in various graphs (e.g. scatter plots and line graphs) a. and c. • Sketch a line of best fit for a set of data points • Determine if data has positive association, negative association, or no association • Describe patterns of clustering • Identify possible outliers in a given set of data • Graph single points on a coordinate plane •

Cluster	Interpret linear models.	Can this student...?
M.1HS.36	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Instructional Note: Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.	• Calculate slope (rate of change) from a variety of representations • Interpret the equation y=mx+b as defining a linear function • Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities
M.1HS.37	Compute (using technology) and interpret the correlation coefficient of a linear fit. Instructional Note: Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.	• Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities • Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept •
M.1HS.38	Distinguish between correlation and causation. Instructional Note: The important distinction between a statistical relationship and a cause-and-effect relationship arises here.	• Construct and interpret scatter plots for bivariate measurement data to investigate patters of association between two quantities

Congruence, Proof, and Constructions

Cluster	Experiment with transformations in the plane.	Can this student...?
M.1HS.39	Know precise definitions of angle, circle, perpendicular line, parallel line and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	• Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure • Draw and identify perpendicular and parallel lines

M.1HS.40	Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).	• Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates • Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections and translations; given two congruent figures, describe a sequence that exhibits the congruence between them
M.1HS.41	Given a rectangle, parallelogram, trapezoid or regular polygon, describe the rotations and reflections that carry it onto itself. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).	• Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates
M.1HS.42	Develop definitions of rotations, reflections and translations in terms of angles, circles, perpendicular lines, parallel lines and line segments. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).	• Define parallel lines, perpendicular bisector, and concentric circles • Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates

M.1HS.43	Given a geometric figure and a rotation, reflection or translation draw the transformed figure using, e.g., graph paper, tracing paper or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).	• Define parallel lines, perpendicular bisector, and concentric circles • Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates

Cluster	**Understand congruence in terms of rigid motions.**	**Can this student...?**
M.1HS.44	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.	• Understand the properties of similarity and congruence in geometric shapes • Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates

M.1HS.45	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.	• Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them
M.1HS.46	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.	• Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two- dimensional figures, describe a sequence that exhibits the similarity between them

Cluster	Make geometric constructions.	Can this student...?
M.1HS.47	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.	• Define segment, angle, angle bisector, segment bisector, perpendicular lines, and parallel lines
M.1HS.48	Construct an equilateral triangle, a square and a regular hexagon inscribed in a circle. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and defend how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.	• Draw geometric shapes with given conditions • Define equilateral triangle, square, regular hexagon, and inscribe

Connecting Algebra and Geometry through Coordinates

Cluster	Use coordinates to prove simple geometric theorems algebraically.	Can this student...?
M.1HS.49	Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).) Instructional Note: Reasoning with triangles in this unit is limited to right triangles (e.g., derive the equation for a line through two points using similar right triangles).	• Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions • Apply the Pythagorean Theorem to find the distance between two points in a coordinate system
M.1HS.50	Prove the slope criteria for parallel and perpendicular lines; use them to solve geometric problems. (e.g., Find the equation of a line parallel or perpendicular to a given line that passes through a given point.) Instructional Note: Reasoning with triangles in this unit is limited to right triangles (e.g., derive the equation for a line through two points using similar right triangles). Relate work on parallel lines to work on M.1HS.29 involving systems of equations having no solution or infinitely many solutions.	• Derive the equation y=mx+b given a line passing through two points • Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane
M.1HS.51	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, (e.g., using the distance formula). Instructional Note: Reasoning with triangles in this unit is limited to right triangles (e.g., derive the equation for a line through two points using similar right triangles). This standard provides practice with the distance formula and its connection with the Pythagorean theorem.	• Apply the Pythagorean Theorem to find the distance between two points in the coordinate plane • Understand and apply concepts of perimeter and area in two-dimensional shapes • Graph single points on a coordinate plane

Support for Mathematics Standards

High School Mathematics II

All West Virginia teachers are responsible for classroom instruction that integrates content standards and objectives and mathematical habits of mind. Students in this course will focus on the need to extend the set of rational numbers, introducing real and complex numbers so that all quadratic equations can be solved. Students will explore the link between probability and data through conditional probability and counting methods, including their use in making and evaluating decisions. The study of similarity will lead students in Mathematics II to an understanding of right triangle trigonometry and connections to quadratics through Pythagorean relationships. Students will explore circles, with their quadratic algebraic representations. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Extending the Number System	Quadratic Functions and Modeling
• Apply and reinforce laws of exponents to convert between radical notation and rational exponent notation; extend the properties of integer exponents to rational exponents and use them to simplify expressions. (e.g., ; (.)	• Find an explicit algebraic expression or series of steps to model the context with mathematical representations. (e.g., The total revenue for a company is found by multiplying the price per unit by the number of units sold minus the production cost. The price per unit is modeled by $p(n) = -0.5n^2 + 6$. The number of units sold is n. Production cost is modeled by $c(n) = 3n + 7$. Write the revenue function.)
Expressions and Equations	**Applications of Probability**
• Solve a system consisting of a linear equation and a quadratic equation in two variables. (e.g., Find the intersection of the circle with a radius of 1 centered at the origin and the line $y = -3(x - 2)$. Show your work both graphically and algebraically.)	• Work with probability and using ideas from probability in everyday situations. (e.g., Compare the chance that a person who smokes will develop lung cancer to the chance that a person who develops lung cancer smokes.)

Similarity, Right Triangle Trigonometry, and Proof	Circles With and Without Coordinates
• Apply knowledge of trigonometric ratios and the Pythagorean Theorem to determine distances in realistic situations. (e.g., Determine heights of inaccessible objects using various instruments, such as clinometers, hypsometers, transits, etc.)	• Use coordinates and equations to describe geometric properties algebraically. (e.g., Write the equation for a circle in the plane with specified center and radius.)

Numbering of Standards

The following Mathematics Standards are numbered continuously. The following ranges relate to the clusters found within Mathematics:

Extending the Number System	
Extend the properties of exponents to rational exponents.	Standards 1-2
Use properties of rational and irrational numbers.	Standard 3
Perform arithmetic operations with complex numbers.	Standards 4-5
Perform arithmetic operations on polynomials.	Standard 6
Quadratic Functions and Modeling	
Interpret functions that arise in applications in terms of a context.	Standards 7-9
Analyze functions using different representations.	Standards 10-12
Build a function that models a relationship between two quantities.	Standard 13
Build new functions from existing functions.	Standards 14-15
Construct and compare linear, quadratic, and exponential models and solve problems.	Standard 16
Expressions and Equations	
Interpret the structure of expressions.	Standards 17-18
Write expressions in equivalent forms to solve problems.	Standard 19
Create equations that describe numbers or relationships.	Standards 20-22
Solve equations and inequalities in one variable.	Standard 23
Use complex numbers in polynomial identities and equations.	Standards 24-26
Solve systems of equations.	Standard 27

Applications of Probability	
Understand independence and conditional probability and use them to interpret data.	Standards 28-32
Use the rules of probability to compute probabilities of compound events in a uniform probability model.	Standards 33-36
Use probability to evaluate outcomes of decisions.	Standards 37-38
Similarity, Right Triangle Trigonometry, and Proof	
Understand similarity in terms of similarity transformations.	Standards 39-41
Prove geometric theorems.	Standards 42-44
Prove theorems involving similarity.	Standards 45-46
Use coordinates to prove simple geometric theorems algebraically.	Standard 47
Define trigonometric ratios and solve problems involving right triangles.	Standards 48-50
Prove and apply trigonometric identities.	Standard 51
Circles With and Without Coordinates	
Understand and apply theorems about circles.	Standards 52-55
Find arc lengths and areas of sectors of circles.	Standard 56
Translate between the geometric description and the equation for a conic section.	Standards 57-58
Use coordinates to prove simple geometric theorems algebraically.	Standard 59
Explain volume formulas and use them to solve problems.	Standards 60-61

Relationships between Quantities

Cluster	Extend the properties of exponents to rational exponents.	Can this student...?
M.2HS.1	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. (e.g., We define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.)	• Apply properties of exponents to integer exponents • Calculate square roots and cube roots of perfect squares and perfect cubes • Understand and define rational numbers

M.2HS.2	Rewrite expressions involving radicals and rational exponents using the properties of exponents.	• Apply the properties of exponents to integer exponents

Cluster	**Use properties of rational and irrational numbers.**	**Can this student...?**
M.2HS.3	Explain why sums and products of rational numbers are rational, that the sum of a rational number and an irrational number is irrational and that the product of a nonzero rational number and an irrational number is irrational. Instructional Note: Connect to physical situations, e.g., finding the perimeter of a square of area 2.	• Use rational approximations of irrational numbers to compare size of irrational numbers, locate them approximately on a number line, and estimate the value of expressions • Differentiate between rational and irrational numbers

Cluster	**Perform arithmetic operations with complex numbers.**	**Can this student...?**
M.2HS.4	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.	• Identify the constant and the coefficient in an expression
M.2HS.5	Use the relation $i^2 = -1$ and the commutative, associative and distributive properties to add, subtract and multiply complex numbers. Instructional Note: Limit to multiplications that involve i^2 as the highest power of i	• Apply properties of operations to produce equivalent expressions with variables • Apply the commutative, associative, and distributive properties to algebraic expressions

Cluster	**Perform arithmetic operations on polynomials.**	**Can this student...?**
M.2HS.6	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract and multiply polynomials. Instructional Note: Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.	• Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients • Apply the distributive property in algebraic expressions

Quadratic Functions and Modeling

Cluster	Interpret functions that arise in applications in terms of a context.	Can this student...?
M.2HS.7	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in Mathematics I.	• Construct a function to model a relationship between two quantities • Describe qualitatively the functional relationship between two quantities by analyzing a graph
M.2HS.8	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in Mathematics I.	• Compare properties of two functions each represented in a different way • Describe qualitatively the functional relationship between two quantities by analyzing a graph
M.2HS.9	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in Mathematics I.	• Determine the slope (rate of change) of a line from a graph, table, or equation • Interpret the meaning of slope in the context of a real-world problem

Cluster	Analyze functions using different representations.	Can this student...?
M.2HS.10	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root and piecewise-defined functions, including step functions and absolute value functions. Instructional Note: Compare and contrast absolute value, step and piecewise defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range and usefulness when examining piecewise-defined functions. Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots and that once roots are known, a quadratic equation can be factored.	a. • Graph a linear equation with or without technology using a variety of representations • Determine the slope (rate of change) of a line from a graph, table, or equation • Identify and interpret the intercepts of a linear relation in number and word problems b. • Use substitution of input values to determine output values of square root, cube root, and absolute value equations a – b. • Scale a graph appropriately • Identify and interpret the intercepts of a linear relation in number and word problems

M.2HS.11	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values and symmetry of the graph and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. (e.g., Identify percent rate of change in functions such as $y = (1.02)^{t,}$ $y = (0.97)^{t}$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay.) Instructional Note: This unit and, in particular, this standard extends the work begun in Mathematics I on exponential functions with integer exponents. Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots and that once roots are known, a quadratic equation can be factored.	a—b. • Evaluate algebraic expressions in number and word problems a. • Apply properties of operation as strategies to add, subtract, factor, and expand linear expressions with rational coefficients b. • Know and apply the properties of integer exponents
M.2HS.12	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum). Instructional Note: Focus on expanding the types of functions considered to include linear, exponential and quadratic. Extend work with quadratics to include the relationship between coefficients and roots and that once roots are known, a quadratic equation can be factored.	• Identify rate of change from a graph, table, equation, or verbal representation • Identify and interpret the intercepts of a linear relation in number and word problems • Describe qualitatively the function relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing)

Cluster	Build a function that models a relationship between two quantities.	Can this student...?
M.2HS.13	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process or steps for calculation from a context. b. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Instructional Note: Focus on situations that exhibit a quadratic or exponential relationship.	a. • Identify a pattern given various representations (visual, pictorial, numerical, and contextual) b. • Apply properties of operation as strategies to add, subtract, factor, and expand linear expressions with rational coefficients • Apply the distributive property to algebraic expressions

Cluster	Build new functions from existing functions.	Can this student...?
M.2HS.14	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on quadratic functions and consider including absolute value functions.	• Construct a function to model a linear relationship between two quantities • Interpret the meaning of each part (intercepts, slope, etc.) of a function
M.2HS.15	Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x)= 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$. Instructional Note: Focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2$, $x > 0$.	• Evaluate algebraic expressions in number and word problems

Cluster	Construct and compare linear, quadratic, and exponential models and solve problems.	Can this student...?
M.2HS.16	Using graphs and tables, observe that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically; or (more generally) as a polynomial function. Instructional Note: Compare linear and exponential growth studied in Mathematics I to quadratic growth.	• Graph a linear or exponential function • Read and interpret information from a graph or table

Expressions and Equations

Cluster	Interpret the structure of expressions.	Can this student...?
M.2HS.17	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Focus on quadratic and exponential expressions. Exponents are extended from the integer exponents found in Mathematics I to rational exponents focusing on those that represent square or cube roots.	a – b. • Identify the terms, factors, and coefficients of an expression • Calculate square roots and cube roots of perfect squares and perfect cubes
M.2HS.18	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. Instructional Note: Focus on quadratic and exponential expressions.	• Know and apply the properties of integer exponents to generate equivalent numerical expressions • Find factors, common factors, and the greatest common factors of expressions

Cluster	Write expressions in equivalent forms to solve problems.	Can this student...?
M.2HS.19	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. Instructional Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.	a. • Evaluate algebraic expressions in number and word problems c. • Know and apply properties of exponents to integer exponents

Cluster	Create equations that describe numbers or relationships.	Can this student...?
M.2HS.20	Create equations and inequalities in one variable and use them to solve problems. Instructional Note: Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Extend work on linear and exponential equations in Mathematics I to quadratic equations.	• Identify important quantities in a practical situation

M.2HS.21	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Extend work on linear and exponential equations in Mathematics I to quadratic equations.	• Compare representations of data including graphs, tables, equations, and context
M.2HS.22	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Extend to formulas involving squared variables. Extend work on linear and exponential equations in Mathematics I to quadratic equations.	• Solve multi-step equations • Solve simple equations with squared variables

Cluster	**Solve equations and inequalities in one variable.**	**Can this student...?**
M.2HS.23	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. Instructional Note: Extend to solving any quadratic equation with real coefficients, including those with complex solutions.	b. • Evaluate square roots of perfect square numbers

Cluster	Use complex numbers in polynomial identities and equations.	Can this student...?
M.2HS.24	Solve quadratic equations with real coefficients that have complex solutions. Instructional Note: Limit to quadratics with real coefficients.	• Solve quadratic equations with real coefficients that have real solutions
M.2HS.25(+)	Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$. Instructional Note: Limit to quadratics with real coefficients.	(+) Standard indicates this standard only applies to accelerated mathematics courses. • Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2 = q$ that has the same solutions • Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring
M.2HS.26(+)	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Instructional Note: Limit to quadratics with real coefficients.	(+) Standard indicates this standard only applies to accelerated mathematics courses. • Determine the degree of a polynomial and indicate the coefficients, constants, and number of terms in the polynomial • Use graphs and/or by-hand calculations to demonstrate why all quadratics have two roots

Cluster	Solve systems of equations.	Can this student...?
M.2HS.27	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. (e.g., Find the points of intersection between the line $y = –3x$ and the circle $x^2 + y^2 = 3$.) Instructional Note: Include systems that lead to work with fractions. (e.g., Finding the intersections between $x^2 + y^2 = 1$ and $y = (x+1)/2$ leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple $3^2 + 4^2 = 5^2$.)	• Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously • Solve a system of linear equations

Applications of Probability

Cluster	Understand independence and conditional probability and use them to interpret data.	Can this student...?
M.2HS.28	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes or as unions, intersections or complements of other events ("or," "and," "not").	• Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring • Find probabilities of events using organized lists, tables, tree diagrams, and simulation
M.2HS.29	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities and use this characterization to determine if they are independent.	• Determine the probability of a simple event • Compute with rational numbers
M.2HS.30	Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.	• Compute with rational numbers
M.2HS.31	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. (e.g., Collect data from a random sample of students in your school on their favorite subject among math, science and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results.) Instructional Note: Build on work with two-way tables from Mathematics I to develop understanding of conditional probability and independence.	• Solve problems that involve finding the whole, given a part and a percent • Use proportional relationships to solve multistep percent problems • Find a percent of a quantity as a rate per 100

M.2HS.32	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. (e.g., Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.)	• Define conditional probability and independence

Cluster	**Use the rules of probability to compute probabilities of compound events in a uniform probability model.**	**Can this student...?**
M.2HS.33	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A and interpret the answer in terms of the model.	• Define conditional probability and independence • Compute with rational numbers
M.2HS.34	Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.	• Compute with rational numbers
M.2HS.35(+)	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A)P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.	(+) Standard indicates this standard only applies to accelerated mathematics courses. • Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations • Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects
M.2HS.36(+)	Use permutations and combinations to compute probabilities of compound events and solve problems.	(+) Standard indicates this standard only applies to accelerated mathematics courses. • Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations • Know and use factorial notation

Cluster	Use probability to evaluate outcomes of decisions.	Can this student...?
M.2HS.37(+)	Use probabilities to make fair decisions (e.g., drawing by lots or using a random number generator).	(+) Standard indicates this standard only applies to accelerated mathematics courses. • Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability
M.2HS.38(+)	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game). Instructional Note: This unit sets the stage for work in Mathematics III, where the ideas of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e., incomplete information) requires an understanding of probability concepts.	(+) Standard indicates this standard only applies to accelerated mathematics courses. • Make decisions based on probability and connect these concepts with long-run frequency • Analyze both costs and benefits in different contextual situations

Similarity, Right Triangle Trigonometry, and Proof

Cluster	Understand similarity in terms of similarity transformations	Can this student...?
M.2HS.39	Verify experimentally the properties of dilations given by a center and a scale factor. a. A dilation takes a line not passing through the center of the dilation to a parallel line and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	a – b. • Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two- dimensional figures, describe a sequence that exhibits the similarity between them • Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates • Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale
M.2HS.40	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	• Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two- dimensional figures, describe a sequence that exhibits the similarity between them
M.2HS.41	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	• Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two- dimensional figures, describe a sequence that exhibits the similarity between them

Cluster	Prove geometric theorems.	Can this student...?
M.2HS.42	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. Implementation may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for M.2HS.C.3. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.	• Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure • Define perpendicular bisector, angle bisector, and equidistant
M.2HS.43	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of this standard may be extended to include concurrence of perpendicular bisectors and angle bisectors in preparation for the unit on Circles With and Without Coordinates.	• Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure • Recognize the attributes of various types of triangles • Notice the conditions which determine a unique triangle, more than one triangle, or no triangle

M.2HS.44	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other and conversely, rectangles are parallelograms with congruent diagonals. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.	• Understand the attributes of parallelograms • Understand the shared attributes of quadrilaterals

Cluster	**Prove theorems involving similarity.**	**Can this student...?**
M.2HS.45	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally and conversely; the Pythagorean Theorem proved using triangle similarity.	• Explain a proof of the Pythagorean Theorem and its converse • Identify and define parallel lines • Use proportional relationships to write and solve multistep ratio equations
M.2HS.46	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	• Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two- dimensional figures, describe a sequence that exhibits the similarity between them • Recognize the attributes of various types of triangles

Cluster	**Use coordinates to prove simple geometric theorems algebraically.**	**Can this student...?**
M.2HS.47	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	• Apply the Pythagorean Theorem to find the distance between two points in the coordinate plane • Understand and apply concepts of perimeter and area in two-dimensional shapes • Graph single points on a coordinate plane

Cluster	Define trigonometric ratios and solve problems involving right triangles.	Can this student...?
M.2HS.48	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	• Solve for missing measures in similar triangles
M.2HS.49	Explain and use the relationship between the sine and cosine of complementary angles.	• Use facts about complementary angles to write and solve simple equations for an unknown angle in a figure
M.2HS.50	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	• Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions • Identify opposite and adjacent sides to an angle

Cluster	Prove and apply trigonometric identities.	Can this student...?
M.2HS.51	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, given $\sin(\theta)$, $\cos(\theta)$, or $\tan(\theta)$, and the quadrant of the angle. Instructional Note: Limit θ to angles between 0 and 90 degrees. Connect with the Pythagorean theorem and the distance formula. Extension of trigonometric functions to other angles through the unit circle is included in Mathematics III.	• Apply the Pythagorean Theorem to solve right triangles • Use the distance formula to compute the lengths

Circles With and Without Coordinates

Cluster	Understand and apply theorems about circles.	Can this student...?
M.2HS.52	Prove that all circles are similar.	• Understand the properties of similar figures • Solve problems involving scale drawings

M.2HS.53	Identify and describe relationships among inscribed angles, radii and chords. Include the relationship between central, inscribed and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	• Define inscribed angle, circumscribed angle, radius, diameter, and chord of a circle
M.2HS.54	Construct the inscribed and circumscribed circles of a triangle and prove properties of angles for a quadrilateral inscribed in a circle.	• List the properties of a parallelogram (e.g., sides are parallel, the angle sum is 360°, etc.)
M.2HS.55(+)	Construct a tangent line from a point outside a given circle to the circle.	(+) Standard indicates this standard only applies to accelerated mathematics courses. • Graph points on a coordinate plane

Cluster	**Find arc lengths and areas of sectors of circles.**	**Can this student...?**
M.2HS.56	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Instructional Note: Emphasize the similarity of all circles. Note that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.	• Calculate the area of a circle • Use proportional relationships to solve multistep ratio problems

Cluster	Translate between the geometric description and the equation for a conic section.	Can this student...?
M.2HS.57	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation. Instructional Note: Connect the equations of circles and parabolas to prior work with quadratic equations.	• Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions • Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines
M.2HS.58	Derive the equation of a parabola given the focus and directrix. Instructional Note: The directrix should be parallel to a coordinate axis.	• Graph linear and quadratic functions and show intercepts, maxima, and minima • Apply the Pythagorean Theorem to find the distance between two points in a coordinate system • Find distances between points with the same first coordinate or the same second coordinate • Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines

Cluster	Use coordinates to prove simple geometric theorems algebraically.	Can this student...?
M.2HS.59	Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, √3) lies on the circle centered at the origin and containing the point (0, 2).) Instructional Note: Include simple proofs involving circles.	• Apply the Pythagorean Theorem to right triangles • Apply the Pythagorean Theorem to find the distance between two points in a coordinate plane

Cluster	Explain volume formulas and use them to solve problems.	Can this student...?
M.2HS.60	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle and informal limit arguments. Instructional Note: Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first.	• Calculate the circumference and area of a circle • Calculate the volume of a cylinder, pyramid, and cone
M.2HS.61	Use volume formulas for cylinders, pyramids, cones and spheres to solve problems. Volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.	• Calculate the volume of a cylinder, pyramid, cone, and sphere

Support for Mathematics Standards

High School Mathematics III (LA and STEM)

Math III LA course does not include the (+) standards.

Math III STEM course includes standards identified by (+) sign

Math III TR course (Technical Readiness) includes standards identified by (*)

Math IV TR course (Technical Readiness) includes standards identified by (^)

Math III Technical Readiness and Math IV Technical Readiness are course options (for juniors and seniors) built for the mathematics content of Math III through integration of career clusters. These courses integrate academics with hands-on career content. The collaborative teaching model is recommended based at our Career and Technical Education (CTE) centers. The involvement of a highly qualified Mathematics teacher and certified CTE teachers will ensure a rich, authentic and respectful environment for delivery of the academics in "real world" scenarios.

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will make connections and applications the accumulation of learning that they have from their previous courses, with content grouped into four critical units. Students will apply methods from probability and statistics to draw inferences and conclusions from data. They will expand their repertoire of functions to include polynomial, rational, and radical functions and their study of right triangle trigonometry to include general triangles. Students will bring together their experiences with functions and geometry to create models and solve contextual problems. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Inferences and Conclusions from Data	Polynomials, Rational, and Radical Relationships
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies. Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	• Derive the formula for the sum of a geometric series, and use the formula to solve problems. (e.g., Calculate mortgage payments.)

Trigonometry of General Triangles and Trigonometric Functions	Mathematical Modeling
• Apply knowledge of the Law of Sines and the Law of Cosines to determine distances in realistic situations. (e.g., Determine heights of inaccessible objects.)	• Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision. (e.g., Estimate water and food needs in a disaster area, or use volume formulas and graphs to find an optimal size for an industrial package.)

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Inferences and Conclusions from Data	
Summarize, represent, and interpret data on single count or measurement variable.	Standard 1
Understand and evaluate random processes underlying statistical experiments.	Standards 2-3
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	Standards 4-7
Use probability to evaluate outcomes of decisions.	Standards 8-9
Polynomials, Rational, and Radical Relationships	
Use complex numbers in polynomial identities and equations.	Standards 10-11
Interpret the structure of expressions.	Standards 12-13
Write expressions in equivalent forms to solve problems.	Standard 14
Perform arithmetic operations on polynomials.	Standard 15
Understand the relationship between zeros and factors of polynomials.	Standards 16-17
Use polynomial identities to solve problems.	Standards 18-19
Rewrite rational expressions.	Standards 20-21
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 22
Represent and solve equations and inequalities graphically.	Standard 23
Analyze functions using different representations.	Standard 24

Trigonometry of General Triangles and Trigonometric Functions	
Apply trigonometry to general triangles.	Standards 25-27
Extend the domain of trigonometric functions using the unit circle.	Standards 28-29
Model periodic phenomena with trigonometric functions.	Standard 30
Mathematical Modeling	
Create equations that describe numbers or relationships.	Standards 31-34
Interpret functions that arise in applications in terms of a context.	Standards 35-37
Analyze functions using different representations.	Standards 38-40
Build a function that models a relationship between two quantities.	Standard 41
Build new functions from existing functions.	Standards 42-43
Construct and compare linear, quadratic, and exponential models and solve problems.	Standard 44
Visualize relationships between two dimensional and three-dimensional objects.	Standard 45
Apply geometric concepts in modeling situations.	Standards 46-48

Inferences and Conclusions from Data

Cluster	Summarize, represent, and interpret data on single count or measurement variable.	Can this student...?
M.3HS.1(*)	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets and tables to estimate areas under the normal curve. Instructional Note: While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution.	• Calculate mean and standard deviation of a set of data • Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets • Interpret differences in shape, center and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers)

Cluster	Understand and evaluate random processes underlying statistical experiments.	Can this student...?
M.3HS.2(*)	Understand that statistics allows inferences to be made about population parameters based on a random sample from that population.	• Use data from a random sample to draw inferences about a population with an unknown characteristic of interest • Understand that random sampling tends to produce representative samples and support valid inferences
M.3HS.3(*)	Decide if a specified model is consistent with results from a given data-generating process, for example, using simulation. (e.g., A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?) Instructional Note: Include comparing theoretical and empirical results to evaluate the effectiveness of a treatment.	• Calculate experimental and theoretical probability

Cluster	Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	Can this student...?
M.3HS.4 (*,^)	Recognize the purposes of and differences among sample surveys, experiments and observational studies; explain how randomization relates to each. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.	• Represent data with plots on the real number line (dot plots, histograms, and box plots)
M.3HS.5 (*,^)	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. Instructional Note: Focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.	• Understand that statistics can be used to gain information about a population by examining a sample of the population

M.3HS.6 (*,^)	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. Instructional Note: Focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.	• Understand that statistics can be used to gain information about a population by examining a sample of the population • Display numerical data (e.g., plots on a number line, dot plots, histograms, and box plots)
M.3HS.7 (*,^)	Evaluate reports based on data. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.	• Understand that statistics can be used to gain information about a population by examining a sample of the population • Display numerical data (e.g., plots on a number line, dot plots, histograms, and box plots)

Cluster	**Use probability to evaluate outcomes of decisions.**	**Can this student...?**
M.3HS.8 (+, ^)	Use probabilities to make fair decisions (e.g., drawing by lots or using a random number generator).	• Develop a probability model and use it to find probabilities of events • Calculate and compare with rational numbers
M.3HS.9 (+, ^)	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game). Instructional Note: Extend to more complex probability models. Include situations such as those involving quality control or diagnostic tests that yields both false positive and false negative results.	• Develop a probability model and use it to find probabilities of events

Polynomials, Rational, and Radical Relationships

Cluster	Use complex numbers in polynomial identities and equations.	Can this student...?
M.3HS.10 (+)	Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$. Instructional Note: Build on work with quadratics equations in Mathematics II. Limit to polynomials with real coefficients.	(+) Standard indicates this standard only applies to accelerated mathematics courses. • Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2 = q$ that has the same solutions • Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring
M.3HS.11 (+)	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.	(+) Standard indicates this standard only applies to accelerated mathematics courses. • Determine the degree of a polynomial and indicate the coefficients, constants, and number of terms in the polynomial • Use graphs and/or by-hand calculations to demonstrate why all quadratics have two roots

Cluster	Interpret the structure of expressions.	Can this student...?
M.3HS.12 (*)	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. (e.g., Interpret $P(1 + r)^n$ as the product of P and a factor not depending on P.) Instructional Note: Extend to polynomial and rational expressions.	a – b. • Identify the terms, factors, and coefficients of an expression

M.3HS.13 (*)	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. Instructional Note: Extend to polynomial and rational expressions.	• Solve quadratic and exponential expressions • Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents • Know and apply the properties of integer exponents to generate equivalent numerical expressions • Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring

Cluster	**Write expressions in equivalent forms to solve problems.**	**Can this student...?**
M.3HS.14 (^)	Derive the formula for the sum of a geometric series (when the common ratio is not 1), and use the formula to solve problems. (e.g., Calculate mortgage payments.) Instructional Note: Consider extending to infinite geometric series in curricular implementations of this course description.	• Find the common ratio of a geometric sequence • Substitute values into a function

Cluster	**Perform arithmetic operations on polynomials.**	**Can this student...?**
M.3HS.15(*)	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction and multiplication; add, subtract and multiply polynomials. Instructional Note: Extend beyond the quadratic polynomials found in Mathematics II.	• Add, subtract, and multiply linear and/or quadratic polynomials

Cluster	**Understand the relationship between zeros and factors of polynomials.**	**Can this student...?**
M.3HS.16 (*)	Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x – a is p(a), so p(a) = 0 if and only if (x – a) is a factor of p(x).	• Substitute values into a function • Fluently divide multi-digit numbers using the standard algorithm

M.3HS.17 (*)	Identify zeros of polynomials when suitable factorizations are available and use the zeros to construct a rough graph of the function defined by the polynomial.	• Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring • Solve one-step and multi-step equations

Cluster	**Use polynomial identities to solve problems.**	**Can this student...?**
M.3HS.18 (^)	Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. Instructional Note: This cluster has many possibilities for optional enrichment, such as relating the example in M.A2HS.10 to the solution of the system $u^2 + v^2 = 1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x + y)^{n+1} = (x + y)(x + y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction.	• Define identity • Know and apply the properties of integer exponents to generate equivalent numerical expressions • Factor higher order expressions (e.g., greatest common factor, grouping, difference of squares, difference of cubes, sum of cubes, etc.) • Reason from expressions with numerical coefficients to those with variable coefficients, understanding the properties of one apply to the other
M.3HS.19 (+,^)	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. Instructional Note: This cluster has many possibilities for optional enrichment, such as relating the example in M.A2HS.10 to the solution of the system $u^2 + v^2 = 1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x + y)^{n+1} = (x + y)(x + y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction.	• Use the Fundamental Counting Principle to develop and use formulas for combinations and permutations

Cluster	Rewrite rational expressions	Can this student...?
M.3HS.20 (*)	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. Instructional Note: The limitations on rational functions apply to the rational expressions.	• Find sums, differences, products, and quotients of rational algebraic expressions • Rewrite or simplify algebraic expressions including the use of the commutative, associative, and distributive properties, and inverses and identities in number and word problems • Determine the degree of a polynomial and indicate the coefficients, constants, and number of terms in the polynomial
M.3HS.21 (+)	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply and divide rational expressions. Instructional Note: Requires the general division algorithm for polynomials.	(+) Standard indicates this standard only applies to accelerated mathematics courses. • Relate rational number arithmetic to rational expression arithmetic

Cluster	Understand solving equations as a process of reasoning and explain the reasoning.	Can this student...?
M.3HS.22 (*)	Solve simple rational and radical equations in one variable and give examples showing how extraneous solutions may arise. Instructional Note: Extend to simple rational and radical equations.	• Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number • Compute fluently with radical expressions

Cluster	Represent and solve equations and inequalities graphically.	Can this student...?
M.3HS.23 (*,^)	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential and logarithmic functions. Instructional Note: Include combinations of linear, polynomial, rational, radical, absolute value, and exponential functions.	• Graph a linear equation with or without technology • Graph absolute value equations with or without technology • Graph a quadratic equation with or without technology • Graph higher order equations with or without technology • Solve a linear-quadratic system of equations • Solve a system of linear equations

Cluster	Analyze functions using different representations.	Can this student...?
M.3HS.24 (*,^)	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available and showing end behavior. Instructional Note: Relate to the relationship between zeros of quadratic functions and their factored forms.	• Graph quadratic functions • Factor a quadratic expression to reveal the zeros of the function it defines

Trigonometry of General Triangles and Trigonometric Functions

Cluster	Apply trigonometry to general triangles.	Can this student...?
M.3HS.25 (+,^)	Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	• Calculate and understand the area of a triangle using the formula of A=1/2bh

M.3HS.26 (+, ^)	Prove the Laws of Sines and Cosines and use them to solve problems. Instructional Note: With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.	• Identify opposite and adjacent sides in a triangle from a given angle • Solve multi-step equations
M.3HS.27 (+,^)	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems and/or resultant forces).	• Use informal arguments to establish facts about the angle sum in triangles

Cluster	**Extend the domain of trigonometric functions using the unit circle.**	**Can this student...?**
M.3HS.28 (*)	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	• Use properties of circles to solve number and word problems involving arcs formed by central angles or inscribed angles
M.3HS.29 (*)	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	• Use properties of right triangles to solve problems

Cluster	**Model periodic phenomena with trigonometric functions.**	**Can this student...?**
M.3HS.30 (*)	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	• Identify amplitude, frequency, and midline

Mathematical Modeling

Cluster	Create equations that describe numbers or relationships.	Can this student...?
M.3HS.31 (*,^)	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Use all available types of functions to create such equations, including root functions, but constrain to simple cases.	• Solve various types of equations (e.g., linear, quadratic, square root, etc.)
M.3HS.32 (*,^)	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: While functions will often be linear, exponential or quadratic the types of problems should draw from more complex situations than those addressed in Mathematics I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line.	• Create an equation, inequality, or system to model a situation with two variables • Choose correctly among linear functions, exponential functions, and others, as appropriate for modeling a situation
M.3HS.33 (*,^)	Represent constraints by equations or inequalities and by systems of equations and/or inequalities and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.)	• Write equations, inequalities, and systems given a real-world situation
M.3HS.34 (*,^)	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: The example given applies to earlier instances of this standard, not to the current course.	• Solve multi-step equations

Cluster	Interpret functions that arise in applications in terms of a context.	Can this student...?
M.3HS.35 (*)	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Emphasize the selection of a model function based on behavior of data and context.	• Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases • Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior
M.3HS.36 (*)	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Instructional Note: Emphasize the selection of a model function based on behavior of data and context.	• Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range • Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context
M.3HS.37 (*)	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Emphasize the selection of a model function based on behavior of data and context.	• Determine the slope (rate of change) of a line from a graph, table, or equation

Cluster	Analyze functions using different representations.	Can this student...?
M.3HS.38 (*,^)	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph square root, cube root and piecewise-defined functions, including step functions and absolute value functions. b. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline and amplitude. Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.	a—b. • Use substitution of input values to determine output values of square root, cube root, piecewise, and absolute value equations • Graph a linear or quadratic equation with or without technology using a variety of representations • Determine the slope (rate of change) of a line from a graph, table, or equation • Determine if the slope (rate of change) is positive or negative over a given interval • Identify a function that models a relationship between two quantities and interpret key features of graphs
M.3HS.39 (*,^)	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.	• Understand that factoring is the reversal of the distributive property • Factor the greatest common factor from an expression • Know and apply the properties of integer exponents to generate equivalent numerical expressions • Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another

M.3HS.40 (*,^)	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.	• Identify rate of change from a graph, table, equation, or verbal representation • Identify a function that models a relationship between two quantities and interpret key features of graphs (e.g., x- and y-intercepts) • Identify whether a function is increasing or decreasing over a given interval

Cluster	**Build a function that models a relationship between two quantities.**	**Can this student...?**
M.3HS.41 (*)	Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.) Instructional Note: Develop models for more complex or sophisticated situations than in previous courses.	• Identify a pattern given various representations (visual, pictorial, numerical, and contextual) • Rewrite or simplify algebraic expressions including the use of the commutative, associative, and distributive properties, and inverses and identities in number and word problems • Apply the properties of operations (e.g., the distributive property) to generate equivalent expressions

Cluster	Build new functions from existing functions.	Can this student...?
M.3HS.42 (*)	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Use transformations of functions to find more optimum models as students consider increasingly more complex situations. Note the effect of multiple transformations on a single function and the common effect of each transformation across function types. Include functions defined only by graph.	• Identify and interpret intercepts of the graph of a function • Graph functions with technology • Recognize horizontal and vertical shifts from an equation • Recognize horizontal and vertical compressions and stretches from an equation
M.3HS.43 (*)	Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. (e.g., $f(x) = 2x^3$ or f(x) = (x+1)/(x-1) for x ≠ 1.) Instructional Note: Extend this standard to simple rational, simple radical, and simple exponential functions.	• Solve equations for a given variable • Solve rational, radical, and exponential equations

Cluster	Construct and compare linear, quadratic, and exponential models and solve problems.	Can this student...?
M.3HS.44 (*)	For exponential models, express as a logarithm the solution to a $b^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. Instructional Note: Consider extending this unit to include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that log xy = log x + log y.	• Use rules of exponents to simplify numeric and algebraic expressions

Cluster	Visualize relationships between two dimensional and three-dimensional objects.	Can this student...?
M.3HS.45 (*,^)	Identify the shapes of two-dimensional cross-sections of three dimensional objects and identify three-dimensional objects generated by rotations of two-dimensional objects.	• Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates • Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids

Cluster	Apply geometric concepts in modeling situations.	Can this student...?
M.3HS.46 (*,^)	Use geometric shapes, their measures and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	• Calculate volume and surface area of shapes • Identify basic three-dimensional shapes
M.3HS.47 (*,^)	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile or BTUs per cubic foot).	• Calculate area and volume of shapes • Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units

M.3HS.48 (*,^)	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost and/or working with typographic grid systems based on ratios).	• Calculate area, volume, and surface area of various shapes • Extend the application of information from a real-world situation to solve a given problem about the situation

Steven L. Paine, Ed.D.
West Virginia Superintendent of Schools