West Virginia College & Career Readiness Standards

Support for College- and Career-Readiness Standards

MATHEMATICS TRADITIONAL PATHWAY

Office of Special Education
Division of Teaching & Learning
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Introduction

Support for College- and Career-Readiness Standards: Mathematics Traditional Pathway is a companion document to the *West Virginia College- and Career-Readiness Standards.*

This document prepares students for study of the grade-level standards through the teaching of prerequisite and enabling skills necessary for mastering each standard. **This allows students to work toward grade-level and course content standards while working at individual ability levels.** By identifying the prerequisites and enabling skills for each standard, teachers may plan instruction **to address the achievement gap.**

Support for College- and Career-Readiness Standards: Mathematics Traditional Pathway provides short-term objectives to help students reach grade-level standards. Educators are encouraged to use the support document to:

- Write annual Individualized Education Program (IEP) goals
- Design targeted interventions
- Write learner objectives
- Develop lesson plans
- Plan for instructional grouping
- Plan for parent communication and conferences
- Prepare students for mastery of state standards

This document helps educators recognize what students are able to do in relation to the grade-level standards in order to help them move toward explicit success criteria.

Educators are reminded that content standards indicate minimum content—what all students should know and be able to do by the end of each grade level or course. Local education agencies (LEAs) may have additional instructional or achievement expectations and may provide instructional guidelines that address content sequence, review, and remediation.

Navigating the Document

The West Virginia College-and Career-Readiness Standards (WV CCRS) will be listed by domain on the left-hand side of each page. The right-hand column labeled *"Can this student...?"* lists suggested supports, or steps, that a student might need to accomplish prior to mastering the grade-level standard.

The bullet at the top of this list is generally the skill closest to the grade-level standard. If a student is unprepared to demonstrate the skill at the top of the list, the teacher should continue to move down the list of suggestions until he or she identifies what that student is able to do. The bulleted list can be thought of as a ladder; starting at the bottom, educators help the student step up each rung until they are ready to demonstrate mastery of the grade-level standard. It is important to remember the pre-requisite skills are **not meant to replace** the grade-level standard nor are they a "break-down" of the standard itself.

For example, if a student has not yet mastered WV CCRS M.6.28, *"Display numerical data in plots on a number line, including dot plots, histograms and plots,"* the teacher should look to the first bullet listed to the right under "Can this student...?" The teacher would ask *"Can this student organize, display, and interpret information in line plot with a horizontal scale in fractional units?"* If not, move to the next bullet: *"Can this student calculate median, mean, and range for a data set?"* Continue down the list until the teacher finds an appropriate starting point for instruction.

Cluster	**Summarize and describe distributions.**	**Can this student...?**
M.6.28	Display numerical data in plots on a number line, including dot plots, histograms and box plots.	• Organize, display, and interpret information in line plots with a horizontal scale in fractional units • Calculate median, mean, and range for a data set • Create number lines using appropriate intervals

*Note for educators writing IEP goals:

Goals should address student's unique needs across the content areas and should link to the West Virginia College- and Career-Readiness Standards so that a student has the foundation or precursor skills and strategies needed to access and progress in the general education curriculum. Keep in mind that the standards themselves are generally not stated in measurable terms and cannot be substituted for individually developed goals. Rather, the annual goal should focus on what is needed for the student to learn and attain the grade-level standard.

Support for Mathematics Standard
High School Algebra I

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will focus on five critical units that deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Relationships between Quantities and Reasoning with Equations	**Linear and Exponential Relationships**
• Solve problems with a wide range of units and solve problems by thinking about units. (e.g., The Trans Alaska Pipeline System is 800 miles long and cost $8 billion to build. Divide one of these numbers by the other. What is the meaning of the answer? Greenland has a population of 56,700 and a land area of 2,175,600 square kilometers. By what factor is the population density of the United States, 80 persons per square mile, larger than the population density of Greenland?)	• Understand contextual relationships of variables and constants. (e.g., Annie is picking apples with her sister. The number of apples in her basket is described by $n = 22t + 12$, where t is the number of minutes Annie spends picking apples. What do the numbers 22 and 12 tell you about Annie's apple picking?)
Descriptive Statistics	**Expressions and Equations**
• Use linear regression techniques to describe the relationship between quantities and assess the fit of the model. (e.g., Use the high school and university grades for 250 students to create a model that can be used to predict a student's university GPA based on his high school GPA.)	• Interpret algebraic expressions and transform them purposefully to solve problems. (e.g., In solving a problem about a loan with interest rate r and principal P, seeing the expression $P(1+r)^n$ as a product of P with a factor not depending on P.)
Quadratic Functions and Modeling	
• Solve real-world and mathematical problems by writing and solving nonlinear equations, such as quadratic equations ($ax^2 + bx + c = 0$).	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Relationships between Quantities and Reasoning with Equations	
Reason quantitatively and use units to solve problems.	Standards 1-3
Interpret the structure of expressions.	Standard 4
Create equations that describe numbers or relationships.	Standards 5-8
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 9
Solve equations and inequalities in one variable.	Standard 10
Linear and Exponential Relationships	
Extend the properties of exponents to rational exponents.	Standards 11-12
Solve systems of equations.	Standards 13-14
Represent and solve equations and inequalities graphically.	Standards 15-17
Understand the concept of a function and use function notation.	Standards 18-20
Interpret functions that arise in applications in terms of a context.	Standards 21-23
Analyze functions using different representations.	Standards 24-25
Build a function that models a relationship between two quantities.	Standards 26-27
Build new functions from existing functions.	Standards 28
Construct and compare linear, quadratic, and exponential models and solve problems.	Standards 29-31
Interpret expressions for functions in terms of the situation they model.	Standard 32
Descriptive Statistics	
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 33-35
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standards 36-37
Interpret linear models.	Standards 38-40

Expressions and Equations	
Interpret the structure of equations.	Standards 41-42
Write expressions in equivalent forms to solve problems.	Standard 43
Perform arithmetic operations on polynomials.	Standard 44
Create equations that describe numbers or relationships.	Standards 45-47
Solve equations and inequalities in one variable.	Standard 48
Solve systems of equations.	Standard 49
Quadratic Functions and Modeling	
Use properties of rational and irrational numbers.	Standard 50
Interpret functions that arise in applications in terms of a context.	Standards 51-53
Analyze functions using different representations.	Standards 54-56
Build a function that models a relationship between two quantities.	Standards 57
Build new functions from existing functions.	Standard 58-59
Construct and compare linear, quadratic and exponential models and solve problems.	Standard 60

Relationships between Quantities and Reasoning with Equations

Cluster	**Reason quantitatively and use units to solve problems.**	**Can this student...?**
M.A1HS.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.	• Calculate unit rates in number and word problems • Use ratio reasoning to convert measurement units • Solve unit rate problems • Interpret units consistently in formulae
M.A1HS.2	Define appropriate quantities for the purpose of descriptive modeling. Instructional Note: Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.	• Specify units of measure
M.A1HS.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.	• Round numbers to appropriate levels of accuracy

Cluster	**Interpret the structure of expressions.**	**Can this student...?**
M.A1HS.4	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. (e.g., Interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Limit to linear expressions and to exponential expressions with integer exponents.	a – b. • Write, read, and evaluate expressions in which letters stand for numbers • Identify important quantities in a practical situation

Cluster	**Create equations that describe numbers or relationships.**	**Can this student...?**
M.A1HS.5	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.	• Solve one-step equations and inequalities • Use substitution to verify algebraically the accuracy of the solution • Evaluate expressions using exponents • Write, read, and evaluate expressions in which letters stand for numbers
M.A1HS.6	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.	• Compare representations of data, including graphs, tables, equations and context • Graph single points on a coordinate plane

M.A1HS.7	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: Limit to linear equations and inequalities.	• Write equations, inequalities, and systems given a real-world situation
M.A1HS.8	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Limit to formulas with a linear focus.	• Solve multi-step equations

Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.**	**Can this student…?**
M.A1HS.9	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Instructional Note: Students should focus on and master linear equations and be able to extend and apply their reasoning to other types of equations in future courses. Students will solve exponential equations with logarithms in Algebra II.	• Use the distributive property and the order of operations to create equivalent expressions • Identify when two expressions are equivalent (e.g.: 3y and y+y+y)

Cluster	Solve equations and inequalities in one variable.	Can this student...?
M.A1HS.10	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Instructional Note: Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.	• Solve one-step equations and inequalities • Use substitution to algebraically verify the accuracy of the solution • Evaluate expressions that include exponents

Linear and Exponential Relationships

Cluster	Extend the properties of exponents to rational exponents.	Can this student...?
M.A1HS.11	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. (e.g., We define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5.) Instructional Note: Address this standard before discussing exponential functions with continuous domains.	• Apply properties of exponents to integer exponents • Calculate square roots and cube roots of perfect squares and perfect cubes • Understand and define rational numbers
M.A1HS.12	Rewrite expressions involving radicals and rational exponents using the properties of exponents. Instructional Note: Address this standard before discussing exponential functions with continuous domains.	• Apply the properties of exponents to integer exponents

Cluster	Solve systems of equations.	Can this student...?
M.A1HS.13	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions.	• Graph a system of equations with or without technology • Apply the distributive property to an expression

M.A1HS.14	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Instructional Note: Build on student experiences graphing and solving systems of linear equations from middle school to focus on justification of the methods used. Include cases where the two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution); connect to standards in Geometry which require students to prove the slope criteria for parallel lines.	• Graph linear equations with and without technology in a variety of representations (e.g., verbal, descriptions, tables, equations)

Cluster	**Represent and solve equations and inequalities graphically.**	**Can this student…?**
M.A1HS.15	Recognize that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Instructional Note: Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.	• Graph linear equations with and without technology in a variety of representations (e.g., verbal, descriptions, tables, equations) • Graph single points on a coordinate plane
M.A1HS.16	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations). Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential and logarithmic functions. Instructional Note: Focus on cases where f(x) and g(x) are linear or exponential.	• Understand that the solution to a system is the intersection of the two functions • Graph linear equations with and without technology in a variety of representations (e.g., verbal, descriptions, tables, equations) • Relate function notation to the form y=mx+b

M.A1HS.17	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.	• Graph single-variable inequalities on a number line using appropriate shading • Test points in the inequality to verify whether the point is a solution to the inequality • Understand the application of inequality symbols as they pertain to graphing

Cluster	**Understand the concept of a function and use function notation.**	**Can this student...?**
M.A1HS.18	Recognize that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear functions and exponential functions having integral domains.	• Understand that a function is a rule that assigns to each input exactly one output • Generate a set of ordered pairs using a rule which is stated in verbal, algebraic, or table form; generate a sequence given a rule in verbal or algebraic form
M.A1HS.19	Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear functions and exponential functions having integral domains.	• Interpret the equation y=mx+b as defining a linear function • Solve single and multi-step equations • Complete input-output tables • Substitute values into a function

M.A1HS.20	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. (e.g., The Fibonacci sequence is defined recursively by $f(0) = f(1) = 1$, $f(n+1) = f(n)+ f(n-1)$ for $n \geq 1$. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Draw examples from linear functions and exponential functions having integral domains. Draw connection to M.A1HS.27, which requires students to write arithmetic and geometric sequences. Emphasize arithmetic and geometric sequences as examples of linear and exponential functions.	• Determine the rule for a given sequence • Describe visual and numerical patterns

Cluster	**Interpret functions that arise in applications in terms of a context.**	**Can this student...?**
M.A1HS.21	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on linear and exponential functions.	• Interpret the equation $y=mx+b$ as defining a linear function • Graph a linear equation with and/or without technology using a variety of representations • Determine the slope (rate of change) of a line from a table, graph, or equation • Analyze graphs, identify situations, or solve problems with varying rates of change • Identify relations as directly proportional, linear, or nonlinear using rules, tables, and graphs

M.A1HS.22	Relate the domain of a function to its graph and where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Instructional Note: Focus on linear and exponential functions.	• Understand that a function is a rule that assigns to each input exactly one output • Write, read, and evaluate expressions in which letters stand for numbers
M.A1HS.23	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on linear functions and exponential functions whose domain is a subset of the integers. The Unit on Quadratic Functions and Modeling in this course and the Algebra II course address other types of functions.	• Determine the slope (rate of change) of a line from a graph, table, or equation • Evaluate algebraic expressions in number and word problems • Use a coordinate grid as a tool to solve number and word problems

Cluster	**Analyze functions using different representations.**	**Can this student...?**
M.A1HS.24	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph exponential and logarithmic functions, showing intercepts and end behavior and trigonometric functions, showing period, midline and amplitude. Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100^{2n}$)	a–b. • Interpret the equation y=mx+b as defining a linear function a. • Graph a linear equation with and/or without technology using a variety of representations • Generate a set of ordered pairs using a rule which is stated in verbal, algebraic, or table form • Determine the slope (rate of change) of a line from a graph, table, or equation b. • Perform multi-step operations with rational numbers (positive and negative) in number and word problems

M.A1HS.25	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100^{2n}$)	• Construct a function to model a linear relationship between two quantities • Identify rate of change from a graph, table, equation, or verbal representation • Describe qualitatively the functional relationship between two quantities by analyzing the graph (e.g., increasing or decreasing, linear or nonlinear, etc.)

Cluster	**Build a function that models a relationship between two quantities.**	**Can this student...?**
M.A1HS.26	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.) Instructional Note: Limit to linear and exponential functions.	a. • Identify a pattern given various representations (visual, pictorial, numerical, and contextual) b. • Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients • Apply the distributive property to produce an equivalent expression
M.A1HS.27	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Instructional Note: Limit to linear and exponential functions. Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.	• Perform multi-step operations with rational numbers in number and word problems • Find missing terms in sequences

Cluster	**Build new functions from existing functions.**	**Can this student…?**
M.A1HS.28	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.	• Interpret the equation y=mx+b as defining a linear function

Cluster	**Construct and compare linear, quadratic, and exponential models and solve problems.**	**Can this student…?**
M.A1HS.29	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.	a – c. • Recognize that the slope (rate of change) of a line is the same between any two given points, meaning it is constant • Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is linear or nonlinear) • Interpret rate of change in the context of a real-world situation

M.A1HS.30	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship or two input-output pairs (include reading these from a table). Instructional Note: In constructing linear functions, draw on and consolidate previous work in Grade 8 on finding equations for lines and linear functions.	• Recognize that the slope (rate of a change) of a line is the same between any two given points • Interpret rate of change in the context of a real-world situation
M.A1HS.31	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Limit to comparisons between exponential and linear models.	• Read and interpret information from a graph or table

Cluster	**Interpret expressions for functions in terms of the situation they model.**	**Can this student…?**
M.A1HS.32	Interpret the parameters in a linear or exponential function in terms of a context. Instructional Note: Limit exponential functions to those of the form $f(x) = b^x + k$.	• Describe qualitatively the functional relationship between two quantities by analyzing a graph

Descriptive Statistics

Cluster	**Summarize, represent, and interpret data on a single count or measurement variable.**	**Can this student…?**
M.A1HS.33	Represent data with plots on the real number line (dot plots, histograms, and box plots).	• Determine the quartiles or interquartile range for a set of data • Interpret information on a graph • Locate points on a number line

M.A1HS.34	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Instructional Note: In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.	• Choose a measure of central tendency based on the shape of the data distribution • Describe data using or selecting the appropriate measure of central tendency • Calculate median, mean, and interquartile range
M.A1HS.35	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Instructional Note: In grades 6 – 8, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.	• Choose a measure of central tendency based on the shape of the data distribution • Describe data using or selecting the appropriate measure of central tendency • Describe patterns such as clustering, outliers, positive or negative association, and linear and nonlinear association

Cluster	**Summarize, represent, and interpret data on two categorical and quantitative variables.**	**Can this student...?**
M.A1HS.36	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data.	• Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table • Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects • Use relative frequencies calculated for rows or columns to describe possible association between two variables • Calculate with percentages • Convert from a fraction to a percent

M.A1HS.37	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. Instructional Note: Focus should be on situations for which linear models are appropriate. c. Fit a linear function for scatter plots that suggest a linear association. Instructional Note: Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.	a—c. • Organize, display, and interpret information in various graphs (e.g. scatter plots and line graphs) a. and c. • Sketch a line of best fit for a set of data points • Determine if data has positive association, negative association, or no association • Describe patterns of clustering • Identify possible outliers in a given set of data • Graph single points on a coordinate plane •

Cluster	**Interpret linear models.**	**Can this student...?**
M.A1HS.38	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Instructional Note: Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.	• Calculate slope (rate of change) from a variety of representations • Interpret the equation y=mx+b as defining a linear function • Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities

M.A1HS.39	Compute (using technology) and interpret the correlation coefficient of a linear fit. Instructional Note: Build on students' work with linear relationships in eighth grade and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.	• Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities • Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept
M.A1HS.40	Distinguish between correlation and causation. Instructional Note: The important distinction between a statistical relationship and a cause-and-effect relationship is the focus.	• Construct and interpret scatter plots for bivariate measurement data to investigate patters of association between two quantities

Expressions and Equations

Cluster	Interpret the structure of equations.	Can this student...?
M.A1HS.41	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Exponents are extended from the integer exponents found in the unit on Relationships between Quantities and Reasoning with Equations to rational exponents focusing on those that represent square or cube roots. Instructional Note: Focus on quadratic and exponential expressions.	a – b. • Identify the terms, factors, and coefficients of an expression • Calculate square roots and cube roots of perfect squares and perfect cubes
M.A1HS.42	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. Instructional Note: Focus on quadratic and exponential expressions.	• Know and apply the properties of integer exponents to generate equivalent numerical expressions • Find factors, common factors, and the greatest common factors of expressions

Cluster	Write expressions in equivalent forms to solve problems.	Can this student...?
M.A1HS.43	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. Instructional Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.	a. • Evaluate algebraic expressions in number and word problems c. • Know and apply properties of exponents to integer exponents

Cluster	Perform arithmetic operations on polynomials.	Can this student...?
M.A1HS.44	Recognize that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Instructional Note: Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.	• Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients • Apply the distributive property in algebraic expressions

Cluster	Create equations that describe numbers or relationships.	Can this student...?
M.A1HS.45	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Extend work on linear and exponential equations in the Relationships between Quantities and Reasoning with Equations unit to quadratic equations.	• Identify important quantities in a practical situation
M.A1HS.46	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Extend work on linear and exponential equations in the Relationships between Quantities and Reasoning with Equations unit to quadratic equations.	• Compare representations of data including graphs, tables, equations, and context
M.A1HS.47	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R. Instructional Note: Extend work on linear and exponential equations in the Relationships between Quantities and Reasoning with Equations unit to quadratic equations. Extend this standard to formulas involving squared variables.	• Solve multi-step equations • Solve simple equations with squared variables

Cluster	Solve equations and inequalities in one variable.	Can this student...?
M.A1HS.48	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x – p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. Instructional Note: Students should learn of the existence of the complex number system, but will not solve quadratics with complex solutions until Algebra II.	b. • Evaluate square roots of perfect square numbers

Cluster	Solve systems of equations.	Can this student...?
M.A1HS.49	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. For example, find the points of intersection between the line $y = –3x$ and the circle $x^2 + y^2 = 3$. Instructional Note: Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between $x^2 + y^2 = 1$ and $y = (x+1)/2$ leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple $3^2 + 4^2 = 5^2$.	• Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously • Solve a system of linear equations

Quadratic Functions and Modeling

Cluster	Use properties of rational and irrational numbers.	Can this student...?
M.A1HS.50	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. Instructional Note: Connect to physical situations (e.g., finding the perimeter of a square of area 2).	• Use rational approximations of irrational numbers to compare size of irrational numbers, locate them approximately on a number line, and estimate the value of expressions • Differentiate between rational and irrational numbers

Cluster	Interpret functions that arise in applications in terms of a context.	Can this student...?
M.A1HS.51	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in the Unit on Linear and Exponential Relationships.	• Construct a function to model a relationship between two quantities • Describe qualitatively the functional relationship between two quantities by analyzing a graph
M.A1HS.52	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in the Unit on Linear and Exponential Relationships.	• Compare properties of two functions each represented in a different way • Describe qualitatively the functional relationship between two quantities by analyzing a graph

M.A1HS.53	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studied in the Unit on Linear and Exponential Relationships.	• Determine the slope (rate of change) of a line from a graph, table, or equation • Interpret the meaning of slope in the context of a real-world problem

Cluster	Analyze functions using different representations.	Can this student...?
M.A1HS.54	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Instructional Note: Compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.	a – b. • Identify and interpret the intercepts of a linear relation in number and word problems a. • Graph a linear equation with and/or without technology using a variety of representations • Determine the slope (rate of change) of a line from a graph, table, or equation • Identify and interpret the intercepts of a linear relation in number and word problems b. • Use substitution of input values to determine output values of square root, cube root, and absolute value equations

M.A1HS.55	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay. Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored. This standard extends the work begun in the Linear and Exponential Relationships unit on exponential functions with integer exponents.	a—b. • Evaluate algebraic expressions in number and word problems a. • Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients b. • Know and apply the properties of integer exponents
M.A1HS.56	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum. Instructional Note: Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic equation can be factored.	• Identify rate of change from a graph, table, equation, or verbal representation • Identify and interpret the intercepts of a linear relation in number and word problems • Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing)

Cluster	**Build a function that models a relationship between two quantities.**	**Can this student...?**
M.A1HS.57	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Instructional Note: Focus on situations that exhibit a quadratic relationship.	a. • Identify a pattern given various representations (visual, pictorial, numerical, and contextual) b. • Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients • Apply the distributive property to algebraic expressions

Cluster	**Build new functions from existing functions.**	**Can this student...?**
M.A1HS.58	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on quadratic functions, and consider including absolute value functions.	• Construct a function to model a linear relationship between two quantities • Interpret the meaning of each part (intercepts, slope, etc.) of a function
M.A1HS.59	Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or f(x) = (x+1)/(x-1) for x ≠ 1. Instructional Note: Focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2$, x > 0.	• Evaluate algebraic expressions in number and word problems

Cluster	Construct and compare linear, quadratic and exponential models and solve problems.	Can this student…?
M.A1HS.60	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Compare linear and exponential growth to quadratic growth.	• Construct a function to model a linear relationship between two quantities • Read and interpret information from a graph or table

Support for Mathematics Standards

High School Geometry

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will explore more complex geometric situations and deepen their explanations of geometric relationships, moving towards formal mathematical arguments. Important differences exist between this Geometry course and the historical approach taken in Geometry classes. For example, transformations are emphasized early in this course. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Congruence, Proof, and Constructions	**Similarity, Proof, and Trigonometry**
• Prove theorems about triangles and other figures (e.g., that the sum of the measures of the angles in a triangle is 180°). • Given a transformation, work backwards to discover the sequence that led to the transformation. • Given two quadrilaterals that are reflections of each other, find the line of that reflection.	• Apply knowledge of trigonometric ratios and the Pythagorean Theorem to determine distances in realistic situations. (e.g., Determine determine heights of inaccessible objects using various instruments, such as clinometers, hypsometers, transits, etc.)
Extending to Three Dimensions	**Connecting Algebra and Geometry Through Coordinates**
• Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.	• Use a rectangular coordinate system and build on understanding of the Pythagorean Theorem to find distances. (e.g., Find the area and perimeter of a real-world shape using a coordinate grid and Google Earth.) • Analyze the triangles and quadrilaterals on the coordinate plane to determine their properties. (e.g., Determine whether a given quadrilateral is a rectangle).

Circles With and Without Coordinates	Applications of Probability
• Use coordinates and equations to describe geometric properties algebraically. (e.g., Write the equation for a circle in the plane with specified center and radius.)	• Work with probability and using ideas from probability in everyday situations. (e.g., Compare the chance that a person who smokes will develop lung cancer to the chance that a person who develops lung cancer smokes.)
Modeling with Geometry	
• Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision (e.g., estimate water and food needs in a disaster area, or use volume formulas and graphs to find an optimal size for an industrial package).	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Congruence, Proof, and Constructions	
Experiment with transformations in the plane.	Standards 1-5
Understand congruence in terms of rigid motions.	Standards 6-8
Prove geometric theorems.	Standards 9-11
Make geometric constructions.	Standards 12-13
Similarity, Proof, and Trigonometry	
Understand similarity in terms of similarity transformations.	Standards 14-16
Prove theorems involving similarity.	Standards 17-18
Define trigonometric ratios and solve problems involving right triangles.	Standards 19-21
Apply trigonometry to general triangles.	Standards 22-24
Extending to Three Dimensions	
Explain volume formulas and use them to solve problems.	Standards 25-26
Visualize the relation between two dimensional and three-dimensional objects.	Standard 27
Apply geometric concepts in modeling situations.	Standard 28

Connecting Algebra and Geometry Through Coordinates	
Use coordinates to prove simple geometric theorems algebraically.	Standards 29-32
Translate between the geometric description and the equation for a conic section.	Standard 33
Circles With and Without Coordinates	
Understand and apply theorems about circles.	Standards 34-37
Find arc lengths and areas of sectors of circles.	Standard 38
Translate between the geometric description and the equation for a conic section.	Standard 39
Use coordinates to prove simple geometric theorems algebraically.	Standard 40
Apply geometric concepts in modeling situations.	Standard 41
Applications of Probability	
Understand independence and conditional probability and use them to interpret data.	Standards 42-46
Use the rules of probability to compute probabilities of compound events in a uniform probability model.	Standards 47-50
Use probability to evaluate outcomes of decisions.	Standards 51-52
Modeling with Geometry	
Visualize relationships between two dimensional and three-dimensional objects and apply geometric concepts in modeling situations.	Standards 53-55

Congruence, Proof and Constructions

Cluster	Experiment with transformations in the plane.	Can this student...?
M.GHS.1	Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.	• Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure • Draw and identify perpendicular and parallel lines

M.GHS.2	Represent transformations in the plane using, for example, transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch). Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).	• Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates • Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them
M.GHS.3	Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).	• Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates
M.GHS.4	Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle).	• Define parallel lines, perpendicular bisector, and concentric circles • Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates

M.GHS.5	Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, for example, graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another. Instructional Note: Build on student experience with rigid motions from earlier grades. Point out the basis of rigid motions in geometric concepts, (e.g., translations move points a specified distance along a line parallel to a specified line; rotations move objects along a circular arc with a specified center through a specified angle)	• Define parallel lines, perpendicular bisector, and concentric circles • Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates

Cluster	**Understand congruence in terms of rigid motions.**	**Can this student...?**
M.GHS.6	Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.	• Understand the properties of similarity and congruence in geometric shapes • Describe the effect of dilations, translations, rotations and reflections on two-dimensional figures using coordinates

M.GHS.7	Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.	• Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them
M.GHS.8	Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions. Instructional Note: Rigid motions are at the foundation of the definition of congruence. Students reason from the basic properties of rigid motions (that they preserve distance and angle), which are assumed without proof. Rigid motions and their assumed properties can be used to establish the usual triangle congruence criteria, which can then be used to prove other theorems.	• Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them

Cluster	Prove geometric theorems.	Can this student...?
M.GHS.9	Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.	• Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure • Define perpendicular bisector, angle bisector, and equidistant
M.GHS.10	Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning. Implementation of this standard may be extended to include concurrence of perpendicular bisectors and angle bisectors as preparation for M.GHS.36.	• Use facts about supplementary, complementary, vertical, and adjacent angles in a multi-step problem to write and solve simple equations for an unknown angle in a figure • Recognize the attributes of various types of triangles • Notice the conditions which determine a unique triangle, more than one triangle, or no triangle

M.GHS.11	Prove theorems about parallelograms. Theorems include: opposite sides are congruent, opposite angles are congruent, the diagonals of a parallelogram bisect each other, and conversely, rectangles are parallelograms with congruent diagonals. Instructional Note: Encourage multiple ways of writing proofs, such as in narrative paragraphs, using flow diagrams, in two-column format, and using diagrams without words. Students should be encouraged to focus on the validity of the underlying reasoning while exploring a variety of formats for expressing that reasoning.	• Understand the attributes of parallelograms • Understand the shared attributes of quadrilaterals

Cluster	**Make geometric constructions.**	**Can this student...?**
M.GHS.12	Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.	• Define segment, angle, angle bisector, segment bisector, perpendicular lines, and parallel lines

M.GHS.13	Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle. Instructional Note: Build on prior student experience with simple constructions. Emphasize the ability to formalize and explain how these constructions result in the desired objects. Some of these constructions are closely related to previous standards and can be introduced in conjunction with them.	• Draw geometric shapes with given conditions • Define equilateral triangle, square, regular hexagon, and inscribe

Similarity, Proof, and Trigonometry

Cluster	Understand similarity in terms of similarity transformations.	Can this student…?
M.GHS.14	Verify experimentally the properties of dilations given by a center and a scale factor. a. A dilation takes a line not passing through the center of the dilation to a parallel line, and leaves a line passing through the center unchanged. b. The dilation of a line segment is longer or shorter in the ratio given by the scale factor.	a – b. • Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two- dimensional figures, describe a sequence that exhibits the similarity between them • Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates • Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale
M.GHS.15	Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.	• Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them

M.GHS.16	Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar.	• Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them

Cluster	**Prove theorems involving similarity.**	**Can this student...?**
M.GHS.17	Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.	• Explain a proof of the Pythagorean Theorem and its converse • Identify and define parallel lines • Use proportional relationships to write and solve multistep ratio equations
M.GHS.18	Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.	• Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two dimensional figures, describe a sequence that exhibits the similarity between them • Recognize the attributes of various types of triangles

Cluster	**Define trigonometric ratios and solve problems involving right triangles.**	**Can this student...?**
M.GHS.19	Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.	• Solve for missing measures in similar triangles
M.GHS.20	Explain and use the relationship between the sine and cosine of complementary angles.	• Use facts about complementary angles to write and solve simple equations for an unknown angle in a figure
M.GHS.21	Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.	• Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions • Identify opposite and adjacent sides to an angle

Cluster	Apply trigonometry to general triangles.	Can this student...?
M.GHS.22	Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.	• Calculate and understand the area of a triangle using the formula of A=1/2bh
M.GHS.23	Prove the Laws of Sines and Cosines and use them to solve problems. Instructional Note: With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.	• Identify opposite and adjacent sides in a triangle from a given angle • Solve multi-step equations
M.GHS.24	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles. Instructional Note: With respect to the general case of the Laws of Sines and Cosines, the definitions of sine and cosine must be extended to obtuse angles.	• Use informal arguments to establish facts about the angle sum in triangles

Extending to Three Dimensions

Cluster	Explain volume formulas and use them to solve problems.	Can this student...?
M.GHS.25	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments. Instructional Note: Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. Similarly, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.	• Calculate the circumference and area of a circle • Calculate the volume of a cylinder, pyramid, and cone

M.GHS.26	Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems. Instructional Note: Informal arguments for area and volume formulas can make use of the way in which area and volume scale under similarity transformations: when one figure in the plane results from another by applying a similarity transformation with scale factor k, its area is k^2 times the area of the first. Similarly, volumes of solid figures scale by k^3 under a similarity transformation with scale factor k.	• Calculate the volume of a cylinder, pyramid, cone, and sphere

Cluster	**Visualize the relation between two dimensional and three-dimensional objects.**	**Can this student...?**
M.GHS.27	Identify the shapes of two-dimensional cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.	• Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates • Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids

Cluster	**Apply geometric concepts in modeling situations.**	**Can this student...?**
M.GHS.28	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). Instructional Note: Focus on situations that require relating two- and three-dimensional objects, determining and using volume, and the trigonometry of general triangles.	• Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects

Connecting Algebra and Geometry Through Coordinates

(This unit has a close connection with the unit, Circles With and Without Coordinates. Reasoning with triangles in this unit is limited to right triangles; e.g., derive the equation for a line through two points using similar right triangles. Relate work on parallel lines to work in High School Algebra I involving systems of equations having no solution or infinitely many solutions. M.GHS.32 provides practice with the distance formula and its connection with the Pythagorean Theorem.)

Cluster	Use coordinates to prove simple geometric theorems algebraically.	Can this student…?
M.GHS.29	Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).	• Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions • Apply the Pythagorean Theorem to find the distance between two points in a coordinate system
M.GHS.30	Prove the slope criteria for parallel and perpendicular lines and uses them to solve geometric problems. (e.g., Find the equation of a line parallel or perpendicular to a given line that passes through a given point.) Instructional Note: Relate work on parallel lines to work in High School Algebra I involving systems of equations having no solution or infinitely many solutions.	• Derive the equation y=mx+b given a line passing through two points • Use similar triangles to explain why the slope m is the same between any two distinct points on a non-vertical line in the coordinate plane
M.GHS.31	Find the point on a directed line segment between two given points that partitions the segment in a given ratio.	• Apply the Pythagorean Theorem to find the distance between two points in a coordinate plane • Understand and apply concepts of perimeter and area in two-dimensional shapes • Graph single points on a coordinate plane
M.GHS.32	Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula. This standard provides practice with the distance formula and its connection with the Pythagorean theorem.	• Apply the Pythagorean Theorem to find the distance between two points in a coordinate plane • Understand and apply concepts of perimeter and area in two-dimensional shapes • Graph single points on a coordinate plane

Cluster	Translate between the geometric description and the equation for a conic section.	Can this student...?
M.GHS.33	Derive the equation of a parabola given a focus and directrix. Instructional Note: The directrix should be parallel to a coordinate axis.	• Graph linear and quadratic functions and show intercepts, maxima, and minima • Apply the Pythagorean Theorem to find the distance between two points in a coordinate system • Find distances between points with the same first coordinate or the same second coordinate • Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines

Circles With and Without Coordinates

Cluster	Understand and apply theorems about circles.	Can this student...?
M.GHS.34	Prove that all circles are similar.	• Understand the properties of similar figures • Solve problems involving scale drawings
M.GHS.35	Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is perpendicular to the tangent where the radius intersects the circle.	• Define inscribed angle, circumscribed angle, radius, diameter, and chord of a circle
M.GHS.36	Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.	• List the properties of a parallelogram (e.g., sides are parallel, the angle sum is 360°, etc.)
M.GHS.37	Construct a tangent line from a point outside a given circle to the circle.	• Graph points on a coordinate plane

Cluster	Find arc lengths and areas of sectors of circles.	Can this student...?
M.GHS.38	Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector. Instructional Note: Emphasize the similarity of all circles. Reason that by similarity of sectors with the same central angle, arc lengths are proportional to the radius. Use this as a basis for introducing radian as a unit of measure. It is not intended that it be applied to the development of circular trigonometry in this course.	• Calculate the area of a circle • Use proportional relationships to solve multistep ratio problems

Cluster	Translate between the geometric description and the equation for a conic section.	Can this student...?
M.GHS.39	Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.	• Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions • Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines

Cluster	Use coordinates to prove simple geometric theorems algebraically.	Can this student...?
M.GHS.40	Use coordinates to prove simple geometric theorems algebraically. (e.g., Prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$) lies on the circle centered at the origin and containing the point (0, 2).) Instructional Note: Include simple proofs involving circles.	• Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions • Apply the Pythagorean Theorem to find the distance between two points in a coordinate plane

Cluster	Apply geometric concepts in modeling situations.	Can this student...?
M.GHS.41	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder). Instructional Note: Focus on situations in which the analysis of circles is required.	• Use geometric shapes to model real-world situations

Applications of Probability

Cluster	Understand independence and conditional probability and use them to interpret data.	Can this student...?
M.GHS.42	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).	• Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring • Find probabilities of events using organized lists, tables, tree diagrams, and simulations
M.GHS.43	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.	• Determine the probability of a simple event • Compute with rational numbers
M.GHS.44	Recognize the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B. Instructional Note: Build on work with two-way tables from Algebra I to develop understanding of conditional probability and independence.	• Compute with rational numbers

M.GHS.45	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. Instructional Note: Build on work with two-way tables from Algebra I to develop understanding of conditional probability and independence.	• Solve problems that involve finding the whole, given a part and a percent • Use proportional relationships to solve multistep percent problems • Find a percent of a quantity as a rate per 100
M.GHS.46	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.	• Define conditional probability and independence

Cluster	**Use the rules of probability to compute probabilities of compound events in a uniform probability model.**	**Can this student...?**
M.GHS.47	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.	• Define conditional probability and independence • Compute with rational numbers
M.GHS.48	Apply the Addition Rule, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$, and interpret the answer in terms of the model.	• Compute with rational numbers
M.GHS.49	Apply the general Multiplication Rule in a uniform probability model, $P(A \text{ and } B) = P(A)P(B\|A) = P(B)P(A\|B)$, and interpret the answer in terms of the model.	• Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations • Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects

M.GHS.50	Use permutations and combinations to compute probabilities of compound events and solve problems.	• Find probabilities of compound events using organized lists, tables, tree diagrams, and simulations • Know and use factorial notation

Cluster	**Use probability to evaluate outcomes of decisions.** Instructional Note: This unit sets the stage for work in Algebra II, where the ideas of statistical inference are introduced. Evaluating the risks associated with conclusions drawn from sample data (i.e. incomplete information) requires an understanding of probability concepts.	**Can this student...?**
M.GHS.51	Use probabilities to make fair decisions (e.g., drawing by lots and/or using a random number generator).	• Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability
M.GHS.52	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game).	• Make decisions based on probability and connect these ideas with long-run frequency • Analyze both costs and benefits in different contextual situations

Modeling with Geometry

Cluster	**Visualize relationships between two dimensional and three-dimensional objects and apply geometric concepts in modeling situations.**	**Can this student...?**
M.GHS.53	Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).	• Calculate volume and surface area of shapes • Identify basic three-dimensional shapes
M.GHS.54	Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).	• Calculate area and volume of shapes • Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units

M.GHS.55	Apply geometric methods to solve design problems (e.g., designing an object or structure to satisfy physical constraints or minimize cost; working with typographic grid systems based on ratios).	• Calculate area, volume, and surface area of various shapes • Extend the application of information from a real-world situation to solve a given problem about the situation

Support for Mathematics Standards
High School Algebra II

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will build on their work with linear, quadratic, and exponential functions and extend their repertoire of functions to include polynomial, rational, and radical functions. (In this course rational functions are limited to those whose numerators are of degree at most 1 and denominators of degree at most 2; radical functions are limited to square roots or cube roots of at most quadratic polynomials.) Students will work closely with the expressions that define the functions, and continue to expand and hone their abilities to model situations and to solve equations, including solving quadratic equations over the set of complex numbers and solving exponential equations using the properties of logarithms. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Polynomial, Rational, and Radical Relationships	Trigonometric Functions
• Derive the formula for the sum of a geometric series, and use the formula to solve problems. (e.g., Calculate mortgage payments.)	• Apply knowledge of trigonometric functions to determine distances in realistic situations. (e.g., Determine heights of inaccessible objects.)
Modeling with Functions	**Inferences and Conclusions from Data**
• Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision. (e.g., Estimate water and food needs in a disaster area, or use volume formulas and graphs to find an optimal size for an industrial package.) • Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, pulling a hockey goalie at the end of a game).	• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Polynomial, Rational, and Radical Relationships	
Perform arithmetic operations with complex numbers.	Standards 1-2
Use complex numbers in polynomial identities and equations.	Standards 3-5
Interpret the structure of expressions.	Standard 6-7
Write expressions in equivalent forms to solve problems.	Standard 8
Perform arithmetic operations on polynomials.	Standard 9
Understand the relationship between zeros and factors of polynomials.	Standard 10-11
Use polynomial identities to solve problems.	Standards 12-13
Rewrite rational expressions.	Standards 14-15
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 16
Represent and solve equations and inequalities graphically.	Standard 17
Analyze functions using different representations.	Standard 18
Trigonometric Functions	
Extend the domain of trigonometric functions using the unit circle.	Standards 19-20
Model periodic phenomena with trigonometric functions.	Standard 21
Prove and apply trigonometric identities.	Standard 22
Modeling with Functions	
Create equations that describe numbers or relationships.	Standards 23-26
Interpret functions that arise in applications in terms of a context.	Standards 27-29
Analyze functions using different representations.	Standards 30-32
Build a function that models a relationship between two quantities.	Standard 33
Build new functions from existing functions.	Standards 34-35
Construct and compare linear, quadratic, and exponential models and solve problems.	Standard 36

Inferences and Conclusions from Data	
Summarize, represent, and interpret data on a single count or measurement variable.	Standard 37
Understand and evaluate random processes underlying statistical experiments.	Standard 38-39
Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	Standards 40-43
Use probability to evaluate outcomes of decisions.	Standards 44-45

Polynomial, Rational, and Radical Relationships

Cluster	**Perform arithmetic operations with complex numbers.**	**Can this student…?**
M.A2HS.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.	• Identify the constant and the coefficient in an expression
M.A2HS.2	Use the relation $i^2 = -1$ and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.	• Apply properties of operations to produce equivalent expressions with variables • Apply the commutative, associative, and distributive properties to algebraic expressions

Cluster	**Use complex numbers in polynomial identities and equations.**	**Can this student…?**
M.A2HS.3	Solve quadratic equations with real coefficients that have complex solutions. Instructional Note: Limit to polynomials with real coefficients.	• Solve quadratic equations with real coefficients that have real solutions
M.A2HS.4	Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$. Instructional Note: Limit to polynomials with real coefficients.	• Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x-p)^2 = q$ that has the same solutions • Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring

M.A2HS.5	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials. Instructional Note: Limit to polynomials with real coefficients.	• Determine the degree of a polynomial and indicate the coefficients, constants, and number of terms in the polynomial • Use graphs and/or by-hand calculations to demonstrate why all quadratics have two roots

Cluster	**Interpret the structure of expressions.**	**Can this student...?**
M.A2HS.6	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Extend to polynomial and rational expressions.	a – b. • Identify the terms, factors, and coefficients of an expression
M.A2HS.7	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$. Instructional Note: Extend to polynomial and rational expressions.	• Solve quadratic and exponential equations • Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents • Know and apply the properties of integer exponents to generate equivalent numerical expressions • Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring

Cluster	Write expressions in equivalent forms to solve problems.	Can this student...?
M.A2HS.8	Derive the formula for the sum of a finite geometric series (when the common ratio is not 1), and use the formula to solve problems. For example, calculate mortgage payments. Instructional Note: Consider extending this standard to infinite geometric series in curricular implementations of this course description.	• Find the common ratio of a geometric sequence • Substitute values into a function

Cluster	Perform arithmetic operations on polynomials.	Can this student...?
M.A2HS.9	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Instructional Note: Extend beyond the quadratic polynomials found in Algebra I.	• Add, subtract, and multiply linear and/or quadratic polynomials

Cluster	Understand the relationship between zeros and factors of polynomials.	Can this student...?
M.A2HS.10	Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by $x – a$ is $p(a)$, so $p(a) = 0$ if and only if $(x – a)$ is a factor of $p(x)$.	• Substitute values into a function • Fluently divide multi-digit numbers using the standard algorithm
M.A2HS.11	Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.	• Solve quadratic equations by inspection, taking square roots, completing the square, the quadratic formula, and factoring • Solve one-step and multi-step equations

Cluster	Use polynomial identities to solve problems.	Can this student...?
M.A2HS.12	Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity $(x^2 + y^2)^2 = (x^2 - y^2)^2 + (2xy)^2$ can be used to generate Pythagorean triples. Instructional Note: This cluster has many possibilities for optional enrichment, such as relating the example in M.A2HS.10 to the solution of the system $u^2 + v^2 = 1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x + y)^{n+1} = (x + y)(x + y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction.	• Define identity • Know and apply the properties of integer exponents to generate equivalent numerical expressions • Factor higher order expressions (e.g., greatest common factor, grouping, difference of squares, difference of cubes, sum of cubes, etc.) • Reason from expressions with numerical coefficients to those with variable coefficients, understanding the properties of one apply to the other
M.A2HS.13	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle. Instructional Note: This cluster has many possibilities for optional enrichment, such as relating the example in M.A2HS.10 to the solution of the system $u^2 + v^2 = 1$, $v = t(u+1)$, relating the Pascal triangle property of binomial coefficients to $(x + y)^{n+1} = (x + y)(x + y)^n$, deriving explicit formulas for the coefficients, or proving the binomial theorem by induction.	• Use the Fundamental Counting Principle to develop and use formulas for combinations and permutations

Cluster	Rewrite rational expressions.	Can this student...?
M.A2HS.14	Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system. Instructional Note: The limitations on rational functions apply to the rational expressions.	• Find sums, differences, products, and quotients of rational algebraic expressionsRewrite or simplify algebraic expressions including the use of the commutative, associative, and distributive properties, and inverses and identities in number and word problems • Determine the degree of a polynomial and indicate the coefficients, constants, and number of terms in the polynomial

M.A2HS.15	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions. Instructional Note: This standard requires the general division algorithm for polynomials.	• Relate rational number arithmetic to rational expression arithmetic

Cluster	**Understand solving equations as a process of reasoning and explain the reasoning.**	**Can this student…?**
M.A2HS.16	Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise. Instructional Note: Extend to simple rational and radical equations.	• Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number • Compute fluently with radicals expressions

Cluster	**Represent and solve equations and inequalities graphically.**	**Can this student…?**
M.A2HS.17	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Instructional Note: Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions. Instructional Note: Include combinations of linear, polynomial, rational, radical, absolute value, and exponential functions.	• Graph a linear equation with or without technology • Graph absolute value equations with or without technology • Graph a quadratic equation with or without technology • Graph higher order equations with or without technology • Solve a linear-quadratic system of equations • Solve a system of linear equations

Cluster	Analyze functions using different representations.	Can this student...?
M.A2HS.18	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior. Instructional Note: Relate this standard to the relationship between zeros of quadratic functions and their factored forms.	• Graph quadratic functions • Factor a quadratic expression to reveal the zeros of the function it defines

Trigonometric Functions

Cluster	Extend the domain of trigonometric functions using the unit circle.	Can this student...?
M.A2HS.19	Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.	• Use properties of circles to solve number and word problems involving arcs formed by central angles or inscribed angles
M.A2HS.20	Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.	• Use properties of right triangles to solve problems using the relationships in special right triangles • Derive the equation of a circle of given center and radius using the Pythagorean Theore

Cluster	Model periodic phenomena with trigonometric functions.	Can this student...?
M.A2HS.21	Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.	• Identify amplitude, frequency, and midline

Cluster	Prove and apply trigonometric identities.	Can this student...?
M.A2HS.22	Prove the Pythagorean identity $\sin^2(\theta) + \cos^2(\theta) = 1$ and use it to find sin (θ), cos (θ), or tan (θ), given sin (θ), cos (θ), or tan (θ), and the quadrant of the angle. Instructional Note: An Algebra II course with an additional focus on trigonometry could include the standard "Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems." This could be limited to acute angles in Algebra II.	• Apply the Pythagorean Theorem to solve right triangles • Use the distance formula to compute the lengths

Modeling with Functions

Cluster	Create equations that describe numbers or relationships.	Can this student...?
M.A2HS.23	Create equations and inequalities in one variable and use them to solve problems. Instructional Note: Include equations arising from linear and quadratic functions, and simple rational and exponential functions.	• Solve linear and quadratic equations
M.A2HS.24	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: While functions will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. (e.g., Finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line).	• Create an equation, inequality, or system to model a situation with two variables • Choose correctly among linear functions, exponential functions, and others, as appropriate for modeling a situation

M.A2HS.25	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: While functions will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line.	• Write equations, inequalities, and systems given a real-world situation
M.A2HS.26	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) While functions will often be linear, exponential, or quadratic the types of problems should draw from more complex situations than those addressed in Algebra I. For example, finding the equation of a line through a given point perpendicular to another line allows one to find the distance from a point to a line. This example applies to earlier instances of this standard, not to the current course.	• Solve multi-step equations

Cluster	Interpret functions that arise in applications in terms of a context.	Can this student...?
M.A2HS.27	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Emphasize the selection of a model function based on behavior of data and context.	• Graph functions expressed symbolically and show key features of the graph by hand in simple cases and using technology for more complicated cases • Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior
M.A2HS.28	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. (e.g., If the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function.) Note: Emphasize the selection of a model function based on behavior of data and context.	• Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range • Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context
M.A2HS.29	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Note: Emphasize the selection of a model function based on behavior of data and context.	• Determine the slope (rate of change) of a line from a graph, table, or equation

Cluster	Analyze functions using different representations.	Can this student...?
M.A2HS.30	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. b. Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude. Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.	a—b. • Use substitution of input values to determine output values of square root, cube root, piecewise, and absolute value equations • Graph a linear or quadratic equation with or without technology using a variety of representations • Determine the slope (rate of change) of a line from a graph, table, or equation • Determine if the slope (rate of change) is positive or negative over a given interval • Identify a function that models a relationship between two quantities and interpret key features of graphs
M.A2HS.31	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.	• Understand that factoring is the reversal of the distributive property • Factor the greatest common factor from an expression • Know and apply the properties of integer exponents to generate equivalent numerical expressions • Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another
M.A2HS.32	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on applications and how key features relate to characteristics of a situation, making selection of a particular type of function model appropriate.	• Identify rate of change from a graph, table, equation, or verbal representation • Identify a function that models a relationship between two quantities and interpret key features of graphs (e.g., x- and y-intercepts) • Identify whether a function is increasing or decreasing over a given interval

Cluster	Build a function that models a relationship between two quantities.	Can this student...?
M.A2HS.33	Write a function that describes a relationship between two quantities. Combine standard function types using arithmetic operations. (e.g., Build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.) Instructional Note: Develop models for more complex or sophisticated situations than in previous courses.	• Identify a pattern given various representations (visual, pictorial, numerical, and contextual) • Rewrite or simplify algebraic expressions including the use of the commutative, associative, and distributive properties, and inverses and identities in number and word problems • Apply the properties of operations (e.g., the distributive property) to generate equivalent expressions

Cluster	Build new functions from existing functions.	Can this student...?
M.A2HS.34	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Use transformations of functions to find models as students consider increasingly more complex situations. Observe the effect of multiple transformations on a single graph and the common effect of each transformation across function types.	• Identify and interpret intercepts of the graph of a function • Graph functions with technology • Recognize horizontal and vertical shifts from an equation • Recognize horizontal and vertical compressions and stretches from an equation

M.A2HS.35	Find inverse functions. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. (e.g., $f(x) = 2x^3$ or $f(x) = (x+1)/(x-1)$ for $x \neq 1$.) Instructional Note: Use transformations of functions to find models as students consider increasingly more complex situations. Extend this standard to simple rational, simple radical, and simple exponential functions; connect this standard to M.A2HS.34.	• Solve equations for a given variable • Solve rational, radical, and exponential equations

Cluster	**Construct and compare linear, quadratic, and exponential models and solve problems.**	**Can this student…?**
M.A2HS.36	For exponential models, express as a logarithm the solution to $a\,b^{ct} = d$ where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology. Instructional Note: Consider extending this unit to include the relationship between properties of logarithms and properties of exponents, such as the connection between the properties of exponents and the basic logarithm property that $\log xy = \log x + \log y$.	• Use rules of exponents to simplify numeric and algebraic expressions

Inferences and Conclusions from Data

Cluster	Summarize, represent, and interpret data on a single count or measurement variable.	Can this student...?
M.A2HS.37	Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve. Instructional Note: While students may have heard of the normal distribution, it is unlikely that they will have prior experience using it to make specific estimates. Build on students' understanding of data distributions to help them see how the normal distribution uses area to make estimates of frequencies (which can be expressed as probabilities). Emphasize that only some data are well described by a normal distribution.	• Calculate mean and standard deviation of a set of data • Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets • Interpret differences in shape, center and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers)

Cluster	Understand and evaluate random processes underlying statistical experiments.	Can this student...?
M.A2HS.38	Understand statistics as a process for making inferences about population parameters based on a random sample from that population. Instructional Note: Include comparing theoretical and empirical results to evaluate the effectiveness of a treatment.	• Use data from a random sample to draw inferences about a population with an unknown characteristic of interest • Understand that random sampling tends to produce representative samples and support valid inferences
M.A2HS.39	Decide if a specified model is consistent with results from a given data-generating process, e.g., using simulation. (e.g., A model says a spinning coin falls heads up with probability 0.5. Would a result of 5 tails in a row cause you to question the model?)	• Calculate experimental and theoretical probability

Cluster	Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	Can this student...?
M.A2HS.40	Recognize the purposes of and differences among sample surveys, experiments, and observational studies; explain how randomization relates to each. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.	• Represent data with plots on the real number line (dot plots, histograms, and box plots)
M.A2HS.41	Use data from a sample survey to estimate a population mean or proportion; develop a margin of error through the use of simulation models for random sampling. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment. Focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.	• Understand that statistics can be used to gain information about a population by examining a sample of the population

M.A2HS.42	Use data from a randomized experiment to compare two treatments; use simulations to decide if differences between parameters are significant. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment. Focus on the variability of results from experiments—that is, focus on statistics as a way of dealing with, not eliminating, inherent randomness.	• Understand that statistics can be used to gain information about a population by examining a sample of the population • Display numerical data (e.g., plots on a number line, dot plots, histograms, and box plots)
M.A2HS.43	Evaluate reports based on data. Instructional Note: In earlier grades, students are introduced to different ways of collecting data and use graphical displays and summary statistics to make comparisons. These ideas are revisited with a focus on how the way in which data is collected determines the scope and nature of the conclusions that can be drawn from that data. The concept of statistical significance is developed informally through simulation as meaning a result that is unlikely to have occurred solely as a result of random selection in sampling or random assignment in an experiment.	• Understand that statistics can be used to gain information about a population by examining a sample of the population • Display numerical data (e.g., plots on a number line, dot plots, histograms, and box plots)

Cluster	**Use probability to evaluate outcomes of decisions.**	**Can this student...?**
M.A2HS.44	Use probabilities to make fair decisions (e.g., drawing by lots or using a random number generator). Instructional Note: Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results.	• Develop a probability model and use it to find probabilities of events • Calculate and compare with rational numbers
M.A2HS.45	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game). Instructional Note: Extend to more complex probability models. Include situations such as those involving quality control, or diagnostic tests that yield both false positive and false negative results.	• Develop a probability model and use it to find probabilities of events

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