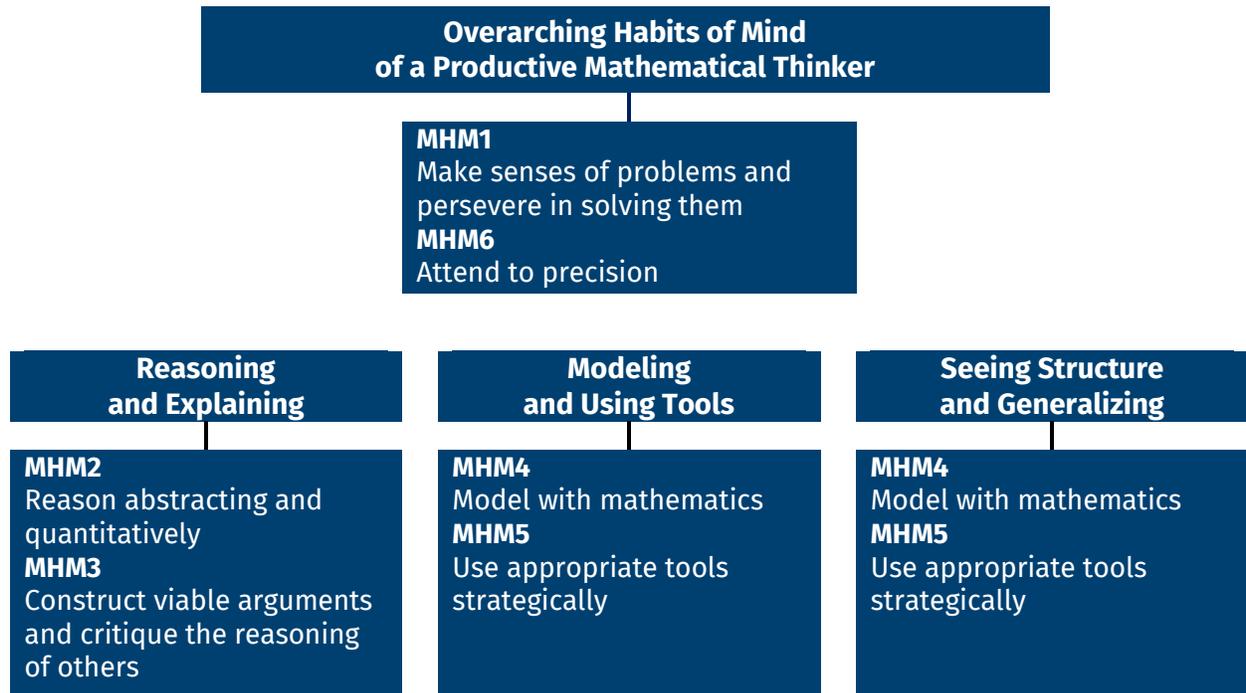# Grade 2

Overview of the West Virginia College- and Career-Readiness Standards for Mathematics

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the grade-level Mathematics Content Standards. These standards address the skills, knowledge, and dispositions that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge – what students need to understand, know, and be able to do. The standards also require that the Mathematical Habits of Mind and the grade-level Mathematics Content Standards be connected. These connections are essential to support the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level Mathematics Content Standards are. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Mathematical Habits of Mind

**Overarching Habits of Mind
of a Productive Mathematical Thinker**

MHM1
Make senses of problems and persevere in solving them
MHM6
Attend to precision

Reasoning and Explaining	**Modeling and Using Tools**	**Seeing Structure and Generalizing**
MHM2 Reason abstracting and quantitatively **MHM3** Construct viable arguments and critique the reasoning of others	**MHM4** Model with mathematics **MHM5** Use appropriate tools strategically	**MHM4** Model with mathematics **MHM5** Use appropriate tools strategically

The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important "processes and proficiencies " with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics' process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding it Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010).

Ideally, several Mathematical Habits of Mind will be evident in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MHM1), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck's research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender a "growth mindset." In Dweck's estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind are able to use these skills not only in mathematics, but across disciplines and into their lives beyond school, college, and career.

Policy 2520.2B
West Virginia College- and Career-Readiness Standards for Mathematics

Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples.

They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 - 3(x - y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y - 2)/(x - 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x - 1)(x + 1)$, $(x - 1)(x^2 + x + 1)$ and $(x - 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Policy 2520.2B
West Virginia College- and Career-Readiness Standards for Mathematics

Grade 2

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in the second grade will focus on four critical areas: (1) extending understanding of base-ten notation; (2) building fluency with addition and subtraction; (3) using standard units of measure; and (4) describing and analyzing shapes. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Continuing the skill progressions from first grade, the following chart represents the mathematical understandings that will be developed in second grade:

Operations and Algebraic Thinking	Number and Operations in Base Ten
• Solve challenging addition and subtraction word problems with one or two steps (e.g., a "one-step" problem would be: "Lucy has 23 fewer apples than Julie. Julie has 47 apples. How many apples does Lucy have?"). • Fluently add with a sum of 20 or less (e.g., 11 + 8); fluently subtract from a number 20 or less (e.g., 16 – 9); and know all sums of one-digit numbers from memory by the end of the year. • Work with equal groups of objects to gain foundations for multiplication.	• Understand what the digits mean in three-digit numbers (place value). • Use an understanding of place value to add and subtract three-digit numbers (e.g., 811 – 367); add and subtract two-digit numbers fluently (e.g., 77 – 28).
Measurement and Data	**Geometry**
• Solve addition and subtraction word problems involving length (e.g., "The pen is 2 cm longer than the pencil. If the pencil is 7 cm long, how long is the pen?"). • Tell time. • Count money.	• Build, draw, and analyze 2-D and 3-D shapes to develop foundations for area, volume, and geometry in later grades. • Divide shapes into equal shares to build the foundations for fractions in later grades.

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Operations and Algebraic Thinking	
Represent and solve problems involving addition and subtraction.	Standard 1
Add and subtract within 20.	Standard 2
Work with equal groups of objects to gain foundations for multiplication.	Standards 3-4
Number and Operations in Base Ten	
Understand place value.	Standard 5-8
Use place value understanding and properties of operations to add and subtract.	Standards 9-13
Measurement and Data	
Measure and estimate lengths in standard units.	Standards 14-17
Relate addition and subtraction to length.	Standards 18-19
Work with time and money.	Standards 20-21
Represent and interpret data.	Standards 22-23
Geometry	
Reason with shapes and their attributes.	Standards 24-26

Operations and Algebraic Thinking

Cluster	Represent and solve problems involving addition and subtraction.
M.2.1	Use addition and subtraction within 100 to solve one- and two-step word problems involving situations of adding to, taking from, putting together, taking apart, and comparing, with unknowns in all positions (e.g. by using drawings and equations with a symbol for the unknown number to represent the problem).

Cluster	Add and subtract within 20.
M.2.2	Fluently add and subtract within 20 using mental strategies and by end of Grade 2, know from memory all sums of two one-digit numbers.

Cluster	Work with equal groups of objects to gain foundations for multiplication.
M.2.3	Determine whether a group of objects (up to 20) has an odd or even number of members, e.g. by pairing objects or counting them by 2s; write an equation to express an even number as a sum of two equal addends.
M.2.4	Use addition to find the total number of objects arranged in rectangular arrays with up to 5 rows and up to 5 columns; write an equation to express the total as a sum of equal addends.

Number and Operations in Base Ten

Cluster	Understand place value.
M.2.5	Understand that the three digits of a three-digit number represent amounts of hundreds, tens and ones (e.g., 706 equals 7 hundreds, 0 tens and 6 ones). Understand the following as special cases: a. 100 can be thought of as a bundle of ten tens – called a "hundred." b. Numbers 100, 200, 300, 400, 500, 600, 700, 800, 900 refer to one, two, three, four, five, six, seven, eight or nine hundreds, and 0 tens and 0 ones.
M.2.6	Count within 1000 and skip-count by 5s, 10s and 100s.
M.2.7	Read and write numbers to 1000 using base-ten numerals, number names and expanded form.
M.2.8	Compare two three-digit numbers based on meanings of the hundreds, tens and ones digits, using >, = and < symbols to record the results of comparisons.

Cluster	Use place value understanding and properties of operations to add and subtract.
M.2.9	Fluently add and subtract within 100 using strategies based on place value, properties of operations and/or the relationship between addition and subtraction.
M.2.10	Add up to four two-digit numbers using strategies based on place value and properties of operations.
M.2.11	Add and subtract within 1000, using concrete models or drawings and strategies based on place value, properties of operations and/or the relationship between addition and subtraction; relate the strategy to a written method. Understand that in adding or subtracting three-digit numbers, one adds or subtracts hundreds and hundreds, tens and tens, ones and ones and sometimes it is necessary to compose or decompose tens or hundreds.
M.2.12	Mentally add 10 or 100 to a given number 100-900 and mentally subtract 10 or 100 from a given number 100-900.
M.2.13	Explain why addition and subtraction strategies work, using place value and the properties of operations. Instructional Note: Explanations may be supported by drawing or objects.

Measurement and Data

Cluster	Measure and estimate lengths in standard units.
M.2.14	Measure the length of an object by selecting and using appropriate tools such as rulers, yardsticks, meter sticks, and measuring tapes.
M.2.15	Measure the length of an object twice, using length units of different lengths for the two measurements, describe how the two measurements relate to the size of the unit chosen.
M.2.16	Estimate lengths using units of inches, feet, centimeters, and meters.
M.2.17	Measure to determine how much longer one object is than another, expressing the length difference in terms of a standard length unit.

Cluster	Relate addition and subtraction to length.
M.2.18	Use addition and subtraction within 100 to solve word problems involving lengths that are given in the same units (e.g., by using drawings, such as drawings of rulers), and equations with a symbol for the unknown number to represent the problem.
M.2.19	Represent whole numbers as lengths from 0 on a number line diagram with equally spaced points corresponding to the numbers 0, 1, 2… and represent whole-number sums and differences within 100 on a number line diagram.

Cluster	Work with time and money.
M.2.20	Tell and write time from analog and digital clocks to the nearest five minutes, using a.m. and p.m.
M.2.21	Solve word problems involving dollar bills, quarters, dimes, nickels, and pennies, using $ and ¢ symbols appropriately (e.g., If you have 2 dimes and 3 pennies, how many cents do you have?).

Cluster	Represent and interpret data.
M.2.22	Generate measurement data by measuring lengths of several objects to the nearest whole unit or by making repeated measurements of the same object. Show the measurements by making a line plot, where the horizontal scale is marked off in whole-number units.
M.2.23	Draw a picture graph and a bar graph (with single-unit scale) to represent a data set with up to four categories. Solve simple put-together, take-apart, and compare problems using information presented in a bar graph.

Geometry

Cluster	Reason with shapes and their attributes
M.2.24	Recognize and draw shapes having specified attributes, such as a given number of angles or a given number of equal faces (sizes are compared directly or visually, not compared by measuring). Identify triangles, quadrilaterals, pentagons, hexagons, and cubes.
M.2.25	Partition a rectangle into rows and columns of same-size squares and count to find the total number of them.
M.2.26	Partition circles and rectangles into two, three, or four equal shares, describe the shares using the words halves, thirds, half of, a third of, etc., describe the whole as two halves, three thirds, four fourths. Recognize that equal shares of identical wholes need not have the same shape.