## $8^{\text {th }}$ Grade High School Mathematics I

## Overview of the West Virginia College- and Career-Readiness Standards for Mathematics

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the gradelevel Mathematics Content Standards. These standards address the skills, knowledge, and dispositions that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge - what students need to understand, know, and be able to do. The standards also require that the Mathematical Habits of Mind and the grade-level Mathematics Content Standards be connected. These connections are essential to support the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level Mathematics Content Standards are. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Mathematical Habits of Mind


| Reasoning <br> and Explaining |
| :--- |
| MHM2 |
| Reason abstracting and |
| quantitatively |
| MHM3 |
| Construct viable arguments |
| and critique the reasoning |
| of others |



The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important "processes and proficiencies " with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics' process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding it Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010).

Ideally, several Mathematical Habits of Mind will be evident in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MHM1), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck's research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender a "growth mindset." In Dweck's estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind are able to use these skills not only in mathematics, but across disciplines and into their lives beyond school, college, and career.

## Policy 2520.2B

## West Virginia College- and Career-Readiness Standards for Mathematics

## Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

## MHM1. Make sense of problems and persevere in solving them.

Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

## MHM2. Reason abstractly and quantitatively.

Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize-to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents-and the ability to contextualize to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

## MHM3. Construct viable arguments and critique the reasoning of others.

Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples.

They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and-if there is a flaw in an argument-explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

## MHM4. Model with mathematics.

Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

## MHM5. Use appropriate tools strategically.

Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

## MHM6. Attend to precision.

Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

## MHM7. Look for and make use of structure.

Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see $7 \times 8$ equals the well-remembered $7 \times 5+7 \times 3$, in preparation for learning about the distributive property. In the expression $x^{2}+9 x+14$, older students can see the 14 as $2 \times 7$ and the 9 as $2+7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5-3(x-y)^{2}$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers $x$ and $y$.

## MHM8. Look for and express regularity in repeated reasoning.

Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through $(1,2)$ with slope 3 , middle school students might abstract the equation $(y-2) /(x-1)=3$. Noticing the regularity in the way terms cancel when expanding $(x-1)(x+1),(x-1)\left(x^{2}+x+1\right)$ and $(x-1)\left(x^{3}+x^{2}+x+1\right)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

## Policy 2520.2B

## West Virginia College- and Career-Readiness Standards for Mathematics

## $8^{\text {th }}$ Grade High School Mathematics I

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will focus on six critical units that deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend. Students explore the role of rigid motions in congruence and similarity, are introduced to the Pythagorean Theorem, and examine volume relationships of cones, cylinders and spheres. Students in 8th Grade Mathematics 1 use properties and theorems involving congruent figures to deepen and extend understanding of geometric knowledge from prior grades and develop connections between the algebraic and geometric ideas studied. Mathematical habits of mind, which, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from seventh grade, the following chart represents the mathematical understandings that will be developed:

Relationships between Quantities

- Solve problems with a wide range of units and solve problems by thinking about units. (e.g., The Trans Alaska Pipeline System is 800 miles long and cost $\$ 8$ billion to build. Divide one of these numbers by the other. What is the meaning of the answer? Greenland has a population of 56,700 and a land area of $2,175,600$ square kilometers. By what factor is the population density of the United States, 80 persons per square mile, larger than the population density of Greenland?)

Linear and Exponential Relationships

- Understand contextual relationships of variables and constants. (e.g., Annie is picking apples with her sister. The number of apples in her basket is described by $\mathrm{n}=22 \mathrm{t}+12$, where t is the number of minutes Annie spends picking apples. What do the numbers 22 and 12 tell you about Annie's apple picking?)
- Translate between various forms of linear equations. (e.g., The perimeter of a rectangle is given by $\mathrm{P}=2 \mathrm{~W}+2 \mathrm{~L}$. Solve for W and restate in words the meaning of this new formula in terms of the meaning of the other variables.).
- Explore systems of equations, find and interpret their solutions. (e.g., The high school is putting on the musical Footloose. The auditorium has 300 seats. Student tickets are \$3 and adult tickets are $\$ 5$. The royalty for the musical is $\$ 1300$. What combination of student and adult tickets do you need to fill the house and pay the royalty? How could you change the price of tickets so more students can go?)
Congruence, Proof, and Constructions
- Given a transformation, work backwards to discover the sequence that led to the transformation.
- Given two quadrilaterals that are reflections of each other, find the line of that reflection.


## Descriptive Statistics

- Use linear regression techniques to describe the relationship between quantities and assess the fit of the model. (e.g., Use the high school and university grades for 250 students to create a model that can be used to predict a student's university GPA based on his high school GPA.)

Connecting Algebra and Geometry through Coordinates

- Use a rectangular coordinate system and build on understanding of the Pythagorean Theorem to find distances. (e.g., Find the area and perimeter of a real-world shape using a coordinate grid and Google Earth.)
- Analyze the triangles and quadrilaterals on the coordinate plane to determine their properties. (e.g., Determine whether a given quadrilateral is a rectangle.)


## Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

## Relationships between Quantities

| Reason quantitatively and use units to solve problems. | Standards 1-3 |
| :--- | :--- |
| Interpret the structure of expressions. | Standard 4 |
| Create equations that describe numbers or relationships. | Standards 5-8 |
| Linear and Exponential Relationships |  |
| Represent and solve equations and inequalities graphically. | Standards 9-11 |
| Define, evaluate, and compare functions. | Standards 12-14 |


| Understand the concept of a function and use function notation. |  | Standards 15-17 |
| :---: | :---: | :---: |
| Use functions to model relationships between quantities. |  | Standards 18-19 |
| Interpret functions that arise in applications in terms of a context. |  | Standards 20-22 |
| Analyze functions using different representations. |  | Standards 23-24 |
| Build a function that models a relationship between two quantities. |  | Standards 25-26 |
| Build new functions from existing functions. |  | Standard 27 |
| Construct and compare linear, quadratic, and exponential models and solve problems. |  | Standards 28-30 |
| Interpret expressions for functions in terms of the situation they model. |  | Standard 31 |
| Reasoning with Equations |  |  |
| Understand solving equations as a process of reasoning and explain the reasoning. |  | Standard 32 |
| Solve equations and inequalities in one variable. |  | Standard 33 |
| Analyze and solve linear equations and pairs of simultaneous linear equations. |  | Standard 34 |
| Solve systems of equations. |  | Standards 35-36 |
| Descriptive Statistics |  |  |
| Summarize, represent, and interpret data on a single count or measurement variable. |  | Standards 37-39 |
| Investigate patterns of association in bivariate data. |  | Standards 40-43 |
| Summarize, represent, and interpret data on two categorical and quantitative variables. |  | Standards 44-45 |
| Interpret linear models. |  | Standards 46-48 |
| Congruence, Proof, and Constructions |  |  |
| Experiment with transformations in the plane. |  | Standards 49-53 |
| Understand congruence in terms of rigid motions. |  | Standards 54-56 |
| Make geometric constructions. |  | Standards 57-58 |
| Understand and apply the Pythagorean theorem. |  | Standards 59-61 |
| Connecting Algebra and Geometry through Coordinates |  |  |
| Use coordinates to prove simple geometric theorems algebraically. |  | Standards 62-64 |
| Relationships between Quantities |  |  |
| Cluster | Reason quantitatively and use units to solve problems. |  |
| M.1HS8.1 | Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays. |  |
| M.1HS8.2 | Define appropriate quantities for the purpose of descriptive modeling. Instructional Note: Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions. |  |
| M.1HS8.3 | Choose a level of accuracy appropriate to limitations on measurement when reporting quantities. |  |


| Cluster | Interpret the structure of expressions. |
| :--- | :--- |
| M.1HS8.4 | Interpret expressions that represent a quantity in terms of its context. <br> a. Interpret parts of an expression, such as terms, factors, and <br> coefficients. |
|  | b. Interpret complicated expressions by viewing one or more of their <br> parts as a single entity. For example, interpret $\mathrm{P}(1+\mathrm{r})^{\mathrm{n}}$ as the product <br> of P and a factor not depending on P. <br> Instructional Note: Limit to linear expressions and to exponential <br> expressions with integer exponents. |


| Cluster | Create equations that describe numbers or relationships. |
| :--- | :--- |
| M.1HS8.5 | Create equations and inequalities in one variable and use them to solve <br> problems. Include equations arising from linear and quadratic functions and <br> simple rational and exponential functions. Instructional Note: Limit to linear <br> and exponential equations and in the case of exponential equations, limit to <br> situations requiring evaluation of exponential functions at integer inputs. |
| M.1HS8.6 | Create equations in two or more variables to represent relationships between <br> quantities; graph equations on coordinate axes with labels and scales. <br> Instructional Note: Limit to linear and exponential equations and in the case <br> of exponential equations, limit to situations requiring evaluation of <br> exponential functions at integer inputs. |
| M.1HS8.7 | Represent constraints by equations or inequalities, and by systems of <br> equations and/or inequalities, and interpret solutions as viable or non-viable <br> options in a modeling context. (e.g., Represent inequalities describing <br> nutritional and cost constraints on combinations of different foods.) <br> Instructional Note: Limit to linear equations and inequalities. |
| M.1HS8.8 | Rearrange formulas to highlight a quantity of interest, using the same <br> reasoning as in solving equations. (e.g., Rearrange Ohm's law $V=I R$ to <br> highlight resistance R.) Instructional Note: Limit to formulas with a linear <br> focus. |

## Linear and Exponential Relationships

| Cluster | Represent and solve equations and inequalities graphically. |
| :--- | :--- |
| M.1HS8.9 | Understand that the graph of an equation in two variables is the set of all its <br> solutions plotted in the coordinate plane, often forming a curve (which could <br> be a line). Instructional Note: Focus on linear and exponential equations and <br> be able to adapt and apply that learning to other types of equations in future <br> courses. |
| M.1HS8.10 | Explain why the $x$-coordinates of the points where the graphs of the <br> equations $y=f(x)$ and $y=g(x)$ intersect are the solutions of the equation $f(x)=$ <br> $g(x) ; ~ f i n d ~ t h e ~ s o l u t i o n s ~ a p p r o x i m a t e l y, ~(e . g ., ~ u s i n g ~ t e c h n o l o g y ~ t o ~ g r a p h ~ t h e ~$ <br> functions, make tables of values, or find successive approximations). Include |


|  | cases where $f(x)$ and/or $g(x)$ are linear, polynomial, rational, absolute value <br> exponential, and logarithmic functions. Instructional Note: Focus on cases <br> where $f(x)$ and $g(x)$ are linear or exponential. |
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| M.1HS8.11 | Graph the solutions to a linear inequality in two variables as a half-plane <br> (excluding the boundary in the case of a strict inequality) and graph the <br> solution set to a system of linear inequalities in two variables as the <br> intersection of the corresponding half-planes. |


| Cluster | Define, evaluate, and compare functions. <br> Instructional Note: While this content is likely subsumed by M.1HS8.12-14 and <br> M.1HS8.26a, it could be used for scaffolding instruction to the more <br> sophisticated content found there. |
| :--- | :--- |
| M.1HS8.12 | Understand that a function is a rule that assigns to each input exactly one <br> output. The graph of a function is the set of ordered pairs consisting of an <br> input and the corresponding output. |
| M.1HS8.13 | Compare properties of two functions each represented in a different way <br> (algebraically, graphically, numerically in tables, or by verbal descriptions). <br> (e.g., Given <br> a linear function represented by a table of values and a linear function <br> represented by an algebraic expression, determine which function has the <br> greater rate of change.) |
| M.1HS8.14 | Interpret the equation $y=m x+b$ as defining a linear function, whose graph is <br> a straight line; give examples of functions that are not linear. (e.g., The <br> function A = s² giving the area of a square as a function of its side length is <br> not linear because its graph contains the points (1,1), (2,4) and (3,9), which are <br> not on a straight line.) |


| Cluster | Understand the concept of a function and use function notation. |
| :--- | :--- |
| M.1HS8.15 | Understand that a function from one set (called the domain) to another set <br> (called the range) assigns to each element of the domain exactly one element <br> of the range. If $f$ is a function and $x$ is an element of its domain, then $f(x)$ <br> denotes the output of $f$ corresponding to the input $x$. The graph of $f$ is the <br> graph of the equation $y=f(x)$ Instructional Note: Students should experience <br> a variety of types of situations modeled by functions. Detailed analysis of <br> any particular class of function at this stage is not advised. Students should <br> apply these concepts throughout their future mathematics courses. <br> Constrain to linear functions and exponential functions having integral <br> domains. |
| M.1HS8.16 | Use function notation, evaluate functions for inputs in their domains and <br> interpret statements that use function notation in terms of a context. |
| Instructional Note: Students should experience a variety of types of <br> situations modeled by functions. Detailed analysis of any particular class of <br> function at this stage is not advised. Students should apply these concepts |  |


|  | throughout their future mathematics courses. Constrain to linear functions <br> and exponential functions having integral domains. |
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| M.1HS8.17 | Recognize that sequences are functions, sometimes defined recursively, <br> whose domain is a subset of the integers. (e.g., The Fibonacci sequence is <br> defined recursively by $f(0)=f(1)=1, f(n+1)=f(n)+f(n-1)$ for $n \geq 1$.$) Instructional$ <br> Note: Students should experience a variety of types of situations modeled by <br> functions. Detailed analysis of any particular class of function at this stage is <br> not advised. Students should apply these concepts throughout their future <br> mathematics courses. Constrain to linear functions and exponential <br> functions having integral domains. Draw connection to M.1HS8.26, which <br> requires students to write arithmetic and geometric sequences. |


| Cluster | Use functions to model relationships between quantities. <br> Instructional Note: While this content is likely subsumed by M.1HS8.20 and <br> M.1HS8.25a, it could be used for scaffolding instruction to the more <br> sophisticated content found there. |
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| M.1HS8.18 | Construct a function to model a linear relationship between two quantities. <br> Determine the rate of change and initial value of the function from a <br> description of a relationship or from two ( $x, y$ y values, including reading these <br> from a table or from a graph. Interpret the rate of change and initial value of <br> a linear function in terms of the situation it models, and in terms of its graph <br> or a table of values. |
| M.1HS8.19 | Describe qualitatively the functional relationship between two quantities by <br> analyzing a graph (e.g., where the function is increasing or decreasing, linear <br> or nonlinear). Sketch a graph that exhibits the qualitative features of a <br> function that has been described verbally. |


| Cluster | Interpret functions that arise in applications in terms of a context. |
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| M.1HS8.20 | For a function that models a relationship between two quantities, interpret <br> key features of graphs and tables in terms of the quantities and sketch <br> graphs showing key features given a verbal description of the relationship. <br> Key features include: intercepts; intervals where the function is increasing, <br> decreasing, positive or negative; relative maximums and minimums; <br> symmetries; end behavior; and periodicity. Instructional Note: Focus on <br> linear and exponential functions. |
| M.1HS8.21 | Relate the domain of a function to its graph and, where applicable, to the <br> quantitative relationship it describes. (e.g., If the function h(n) gives the <br> number of person-hours it takes to assemble n engines in a factory, then the <br> positive integers would be an appropriate domain for the function.) <br> Instructional Note: Focus on linear and exponential functions. |
| M.1HS8.22 | Calculate and interpret the average rate of change of a function (presented <br> symbolically or as a table) over a specified interval. Estimate the rate of <br> change from a graph. Instructional Note: Focus on linear functions and |


|  | intervals for exponential functions whose domain is a subset of the integers. <br> Mathematics II and III will address other function types. |
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| Cluster | Analyze functions using different representations. |
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| M.1HS8.23 | Graph functions expressed symbolically and show key features of the graph, <br> by hand in simple cases and using technology for more complicated cases. <br> a. Graph linear and quadratic functions and show intercepts, maxima, <br> and minima. |
| b. Graph exponential and logarithmic functions, showing intercepts and |  |
| end behavior, and trigonometric functions, showing period, midline, |  |
| and amplitude. |  |


| Cluster | Build a function that models a relationship between two quantities. |
| :--- | :--- |
| M.1HS8.25 | Write a function that describes a relationship between two quantities. <br> a. Determine an explicit expression, a recursive process or steps for <br> calculation from a context. <br> b. Combine standard function types using arithmetic operations. (e.g., <br> Build a function that models the temperature of a cooling body by <br> adding a constant function to a decaying exponential, and relate these <br> functions to the model.) |
| M.1HS8.26 | Instructional Note: Limit to linear and exponential functions. |
| Write arithmetic and geometric sequences both recursively and with an <br> explicit formula, use them to model situations, and translate between the two <br> forms. Instructional Note: Limit to linear and exponential functions. <br> Connect arithmetic sequences to linear functions and geometric sequences <br> to exponential functions. |  |


| Cluster | Build new functions from existing functions. |
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| M.1HS8.27 | Identify the effect on the graph of replacing $f(x)$ by $f(x)+k, k f(x), f(k x)$, and $f(x$ <br> $+k)$ for specific values of $k$ (both positive and negative); find the value of $k$ <br> given the graphs. Experiment with cases and illustrate an explanation of the <br> effects on the graph using technology. Include recognizing even and odd |


|  | functions from their graphs and algebraic expressions for them. Instructional <br> Note: Focus on vertical translations of graphs of linear and exponential <br> functions. Relate the vertical translation of a linear function to its y-intercept. <br> While applying other transformations to a linear graph is appropriate at this <br> level, it may be difficult for students to identify or distinguish between the <br> effects of the other transformations included in this standard. |
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| Cluster | Construct and compare linear, quadratic, and exponential models and solve <br> problems. |
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| M.1HS8.28 | Distinguish between situations that can be modeled with linear functions and <br> with exponential functions. <br> a. Prove that linear functions grow by equal differences over equal <br> intervals; exponential functions grow by equal factors over equal <br> intervals. |
| b. Recognize situations in which one quantity changes at a constant rate |  |
| per unit interval relative to another. |  |
| c. Recognize situations in which a quantity grows or decays by a |  |
| constant percent rate per unit interval relative to another. |  |


| Cluster | Interpret expressions for functions in terms of the situation they model. |
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| M.1HS8.31 | Interpret the parameters in a linear or exponential function in terms of a <br> context. Instructional Note: Limit exponential functions to those of the form <br> $f(x)=b^{x}+k$. |

## Reasoning with Equations

| Cluster | Understand solving equations as a process of reasoning and explain the <br> reasoning. |
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| M.1HS8.32 | Explain each step in solving a simple equation as following from the equality <br> of numbers asserted at the previous step, starting from the assumption that <br> the original equation has a solution. Construct a viable argument to justify a <br> solution method. <br> Instructional Note: Students should focus on linear equations and be able to <br> extend and apply their reasoning to other types of equations in future <br> courses. Students will solve exponential equations with logarithms in <br> Mathematics III. |


| Cluster | Solve equations and inequalities in one variable. |
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| M.1HS8.33 | Solve linear equations and inequalities in one variable, including equations <br> with coefficients represented by letters. Instructional Note: Extend earlier <br> work with solving linear equations to solving linear inequalities in one <br> variable and to solving literal equations that are linear in the variable being <br> solved for. Include simple exponential equations that rely only on application <br> of the laws of exponents, such as $5^{\times}=125$ or $2^{x}=1 / 16$. |


| Cluster | Analyze and solve linear equations and pairs of simultaneous linear <br> equations. |
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| M.1HS8.34 | Analyze and solve pairs of simultaneous linear equations. <br> a. Understand that solutions to a system of two linear equations in two <br> variables correspond to points of intersection of their graphs, because <br> points of intersection satisfy both equations simultaneously. <br> b. Solve systems of two linear equations in two variables algebraically, <br> and estimate solutions by graphing the equations. Solve simple cases <br> by inspection. For example, $3 x+2 y=5$ and $3 x+2 y=6$ have no solution <br> because $3 x+2 y$ cannot simultaneously be 5 and 6. |
| c. Solve real-world and mathematical problems leading to two linear |  |
| equations in two variables. For example, given coordinates for two |  |
| pairs of points, determine whether the line through the first pair of |  |
| points intersects the line through the second pair. |  |


| Cluster | Solve systems of equations. |
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| M.1HS8.35 | Prove that, given a system of two equations in two variables, replacing one <br> equation by the sum of that equation and a multiple of the other produces a <br> system with the same solutions. Instructional Note: Include cases where two <br> equations describe the same line (yielding infinitely many solutions) and <br> cases where two equations describe parallel lines (yielding no solution). |
| M.1HS8.36 | Solve systems of linear equations exactly and approximately (e.g., with <br> graphs), focusing on pairs of linear equations in two variables. Instructional <br> Note: Include cases where two equations describe the same line (yielding <br> infinitely many solutions) and cases where two equations describe parallel <br> lines (yielding no solution). |

## Descriptive Statistics

| Cluster | Summarize, represent, and interpret data on a single count or measurement <br> variable. |
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| M.1HS8.37 | Represent data with plots on the real number line (dot plots, histograms, and <br> box plots). |


| M.1HS8.38 | Use statistics appropriate to the shape of the data distribution to compare <br> center (median, mean) and spread (interquartile range, standard deviation) <br> of two or more different data sets. Instructional Note: In grades 6-7, <br> students describe center and spread in a data distribution. Here they choose <br> a summary statistic appropriate to the characteristics of the data <br> distribution, such as the shape of the distribution or the existence of extreme <br> data points. |
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| M.1HS8.39 | Interpret differences in shape, center and spread in the context of the data <br> sets, accounting for possible effects of extreme data points (outliers). |
| Instructional Note: In grades 6 - 7, students describe center and spread in a <br> data distribution. Here they choose a summary statistic appropriate to the <br> characteristics of the data distribution, such as the shape of the distribution <br> or the existence of extreme data points. |  |


| Cluster | Investigate patterns of association in bivariate data. <br> Instructional Note: While this content is likely subsumed by M.1HS8.45-48, it <br> could be used for scaffolding instruction to the more sophisticated content <br> found there. |
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| M.1HS8.40 | Construct and interpret scatter plots for bivariate measurement data to <br> investigate patterns of association between two quantities. Describe patterns <br> such as clustering, outliers, positive or negative association, linear <br> association and nonlinear association. |
| M.1HS8.41 | Know that straight lines are widely used to model relationships between two <br> quantitative variables. For scatter plots that suggest a linear association, <br> informally fit a straight line and informally assess the model fit by judging <br> the closeness of the data points to the line. |
| M.1HS8.42 | Use the equation of a linear model to solve problems in the context of <br> bivariate measurement data, interpreting the slope and intercept. (e.g., In a <br> linear model for a biology experiment, interpret a slope of 1.5 cm/hr as <br> meaning that an additional hour of sunlight each day is associated with an <br> additional 1.5 cm in mature plant height.) |
| M.1HS8.43 | Understand that patterns of association can also be seen in bivariate <br> categorical data by displaying frequencies and relative frequencies in a two- <br> way table. Construct and interpret a two-way table summarizing data on two <br> categorical variables collected from the same subjects. Use relative <br> frequencies calculated for rows or columns to describe possible association <br> between the two variables. (e.g., Collect data from students in your class on <br> whether or not they have a curfew on school nights and whether or not they <br> have assigned chores at home. Is there evidence that those who have a <br> curfew also tend to have chores?) |


| Cluster | Summarize, represent, and interpret data on two categorical and <br> quantitative variables. |
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| M.1HS8.44 | Summarize categorical data for two categories in two-way frequency tables. <br> Interpret relative frequencies in the context of the data (including joint, <br> marginal and conditional relative frequencies). Recognize possible <br> associations and trends in the data. |
| M.1HS8.45 | Represent data on two quantitative variables on a scatter plot, and describe <br> how the variables are related. <br> a. Fit a function to the data; use functions fitted to data to solve <br> problems in the context of the data. Use given functions or choose a <br> function suggested by the context. Emphasize linear and exponential <br> models. |
| b. Informally assess the fit of a function by plotting and analyzing |  |
| residuals. (Focus should be on situations for which linear models are |  |
| appropriate.) |  |
| c. Fit a linear function for scatter plots that suggest a linear association. |  |
| Instructional Note: Students take a more sophisticated look at using a linear |  |
| function to model the relationship between two numerical variables. In |  |
| addition to fitting a line to data, students assess how well the model fits by |  |
| analyzing residuals. |  |


| Cluster | Interpret linear models. |
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| M.1HS8.46 | Interpret the slope (rate of change) and the intercept (constant term) of a <br> linear model in the context of the data. Instructional Note: Build on <br> students' work with linear relationships and introduce the correlation <br> coefficient. The focus here is on the computation and interpretation of the <br> correlation coefficient as a measure of how well the data fit the relationship. |
| M.1HS8.47 | Compute (using technology) and interpret the correlation coefficient of a <br> linear fit. Instructional Note: Build on students' work with linear <br> relationships and introduce the correlation coefficient. The focus here is on <br> the computation and interpretation of the correlation coefficient as a <br> measure of how well the data fit the relationship. |
| M.1HS8.48 | Distinguish between correlation and causation. Instructional Note: The <br> important distinction between a statistical relationship and a cause-and- <br> effect relationship arises here. |

## Congruence, Proof, and Constructions

| Cluster | Experiment with transformations in the plane. |
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| M.1HS8.49 | Know precise definitions of angle, circle, perpendicular line, parallel line and <br> line segment, based on the undefined notions of point, line, distance along a <br> line, and distance around a circular arc. |
| M.1HS8.50 | Represent transformations in the plane using, example, transparencies and <br> geometry software; describe transformations as functions that take points in |


|  | the plane as inputs and give other points as outputs. Compare <br> transformations that preserve distance and angle to those that do not (e.g., <br> translation versus horizontal stretch). Instructional Note: Build on student <br> experience with rigid motions from earlier grades. Point out the basis of rigid <br> motions in geometric concepts, (e.g., translations move points a specified <br> distance along a line parallel to a specified line; rotations move objects along <br> a circular arc with a specified center through a specified angle). |
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| M.1HS8.51 | Given a rectangle, parallelogram, trapezoid or regular polygon, describe the <br> rotations and reflections that carry it onto itself. Instructional Note: Build on <br> student experience with rigid motions from earlier grades. Point out the basis <br> of rigid motions in geometric concepts, (e.g., translations move points a <br> specified distance along a line parallel to a specified line; rotations move <br> objects along a circular arc with a specified center through a specified angle). |
| M.1HS8.52 | Develop definitions of rotations, reflections and translations in terms of <br> angles, circles, perpendicular lines, parallel lines and line segments. <br> Instructional Note: Build on student experience with rigid motions from <br> earlier grades. Point out the basis of rigid motions in geometric concepts, <br> (e.g., translations move points a specified distance along a line parallel to a <br> specified line; rotations move objects along a circular arc with a specified <br> center through a specified angle). |
| M.1HS8.53 | Given a geometric figure and a rotation, reflection or translation draw the <br> transformed figure using, e.g., graph paper, tracing paper or geometry <br> software. Specify a sequence of transformations that will carry a given figure <br> onto another. Instructional Note: Build on student experience with rigid <br> motions from earlier grades. Point out the basis of rigid motions in geometric <br> concepts, (e.g., translations move points a specified distance along a line <br> parallel to a specified line; rotations move objects along a circular arc with a <br> specified center through a specified angle). |


| Cluster | Understand congruence in terms of rigid motions. |
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| M.1HS8.54 | Use geometric descriptions of rigid motions to transform figures and to <br> predict the effect of a given rigid motion on a given figure; given two figures, <br> use the definition of congruence in terms of rigid motions to decide if they <br> are congruent. Instructional Note: Rigid motions are at the foundation of the <br> definition of congruence. Students reason from the basic properties of rigid <br> motions (that they preserve distance and angle), which are assumed without <br> proof. Rigid motions and their assumed properties can be used to establish <br> the usual triangle congruence criteria, which can then be used to prove other <br> theorems. |
| M.1HS8.55 | Use the definition of congruence in terms of rigid motions to show that two <br> triangles are congruent if and only if corresponding pairs of sides and <br> corresponding pairs of angles are congruent. Instructional Note: Rigid <br> motions are at the foundation of the definition of congruence. Students |


|  | reason from the basic properties of rigid motions (that they preserve <br> distance and angle), which are assumed without proof. Rigid motions and <br> their assumed properties can be used to establish the usual triangle <br> congruence criteria, which can then be used to prove other theorems. |
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| M.1HS8.56 | Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow <br> from the definition of congruence in terms of rigid motions. Instructional <br> Note: Rigid motions are at the foundation of the definition of congruence. <br> Students reason from the basic properties of rigid motions (that they <br> preserve distance and angle), which are assumed without proof. Rigid <br> motions and their assumed properties can be used to establish the usual <br> triangle congruence criteria, which can then be used to prove other <br> theorems. |


| Cluster | Make geometric constructions. |
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| M.1HS8.57 | Make formal geometric constructions with a variety of tools and methods <br> (compass and straightedge, string, reflective devices, paper folding, dynamic <br> geometric software, etc.). Copying a segment; copying an angle; bisecting a <br> segment; bisecting an angle; constructing perpendicular lines, including the <br> perpendicular bisector of a line segment; and constructing a line parallel to a <br> given line through a point not on the line. Instructional Note: Build on prior <br> student experience with simple constructions. Emphasize the ability to <br> formalize and defend how these constructions result in the desired objects. <br> Some of these constructions are closely related to previous standards and <br> can be introduced in conjunction with them. |
| M.1HS8.58 | Construct an equilateral triangle, a square and a regular hexagon inscribed in <br> a circle. Instructional Note: Build on prior student experience with simple <br> constructions. Emphasize the ability to formalize and defend how these <br> constructions result in the desired objects. Some of these constructions are <br> closely related to previous standards and can be introduced in conjunction <br> with them. |


| Cluster | Understand and apply the Pythagorean theorem. |
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| M.1HS8.59 | Explain a proof of the Pythagorean theorem and its converse. |
| M.1HS8.60 | Apply the Pythagorean theorem to determine unknown side lengths in right <br> triangles in real-world and mathematical problems in two and three <br> dimensions. Instructional Note: Discuss applications of the Pythagorean <br> theorem and its connections to radicals, rational exponents, and irrational <br> numbers. |
| M.1HS8.61 | Apply the Pythagorean theorem to find the distance between two points in a <br> coordinate system. Instructional Note: Discuss applications of the <br> Pythagorean theorem and its connections to radicals, rational exponents, and <br> irrational numbers. |

## Connecting Algebra and Geometry through Coordinates

| Cluster | Use coordinates to prove simple geometric theorems algebraically. |
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| M.1HS8.62 | Use coordinates to prove simple geometric theorems algebraically. (e.g., <br> Prove or disprove that a figure defined by four given points in the coordinate <br> plane is a rectangle; prove or disprove that the point (1, $\sqrt{3}$ ) lies on the circle <br> centered at the origin and containing the point (0, 2).) Instructional Note: <br> Reasoning with triangles in this unit is limited to right triangles (e.g., derive <br> the equation for a line through two points using similar right triangles). |
| M.1HS8.63 | Prove the slope criteria for parallel and perpendicular lines; use them to <br> solve geometric problems. (e.g., Find the equation of a line parallel or <br> perpendicular to a given line that passes through a given point.) <br> Instructional Note: Reasoning with triangles in this unit is limited to right <br> triangles (e.g., derive the equation for a line through two points using similar <br> right triangles). Relate work on parallel lines to work on M.1HS8.35 involving <br> systems of equations having no solution or infinitely many solutions. |
| M.1HS8.64 | Use coordinates to compute perimeters of polygons and areas of triangles <br> and rectangles, (e.g., using the distance formula). Instructional Note: <br> Reasoning with triangles in this unit is limited to right triangles (e.g., derive <br> the equation for a line through two points using similar right triangles). This <br> standard provides practice with the distance formula and its connection with <br> the Pythagorean theorem. |

