High School Algebra I for 8th Grade

West Virginia math4life

Overview of the West Virginia College- and Career-Readiness Standards for Mathematics

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards. These standards address the skills, knowledge, and dispositions that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge – what students need to understand, know, and be able to do. The standards also require that the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards be connected. These connections are essential to support the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level or course-specific Mathematics Content Standards are. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Mathematical Habits of Mind

Overarching Habits of Mind of a Productive Mathematical Thinker

MHM1
Make senses of problems and persevere in solving them

MHM6
Attend to precision

Reasoning and Explaining	Modeling and Using Tools	Seeing Structure and Generalizing
MHM2 *Reason abstracting and quantitatively*	**MHM4** *Model with mathematics*	**MHM7** *Look for and make use of structure*
MHM3 *Construct viable arguments and critique the reasoning of others*	**MHM5** *Use appropriate tools strategically*	**MHM8** *Look for and express regularity in repeated reasoning*

The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important "processes and proficiencies " with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics' process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding it Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010).

Ideally, several Mathematical Habits of Mind will be evident in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MHM1), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck's research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender a "growth mindset." In Dweck's estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind are able to use these skills not only in mathematics, but across disciplines and into their lives beyond school, college, and career.

Policy 2520.2B

West Virginia College- and Career-Readiness Standards for Mathematics

Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a

flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$ and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics – High School Algebra I for 8th Grade

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Students in this course will focus on five critical units that deepen and extend understanding of linear and exponential relationships by contrasting them with each other and by applying linear models to data that exhibit a linear trend, and students engage in methods for analyzing, solving, and using quadratic functions. Students are introduced to methods for analyzing and using quadratic functions, including manipulating expressions for them, and solving quadratic equations. Students in 8th grade High School Algebra understand and apply the Pythagorean theorem, and use quadratic functions to model and solve problems. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from seventh grade, the following chart represents the mathematical understandings that will be developed:

Relationships between Quantities and Reasoning with Equations	Linear and Exponential Relationships
• Solve problems with a wide range of units and solve problems by thinking about units. (e.g., The Trans Alaska Pipeline System is 800 miles long and cost $8 billion to build. Divide one of these numbers by the other. What is the meaning of the answer? Greenland has a population of 56,700 and a land area of 2,175,600 square kilometers. By what factor is the population density of the United States, 80 persons per square mile, larger than the population density of Greenland?)	• Understand contextual relationships of variables and constants. (e.g., Annie is picking apples with her sister. The number of apples in her basket is described by n = 22t + 12, where t is the number of minutes Annie spends picking apples. What do the numbers 22 and 12 tell you about Annie's apple picking?)
Descriptive Statistics	**Expressions and Equations**
• Use linear regression techniques to describe the relationship between quantities and assess the fit of the model. (e.g., Use the high school and university grades for 250 students to create a model that can be used to predict a student's university GPA based on his high school GPA.)	• Interpret algebraic expressions and transforming them purposefully to solve problems. (e.g., In solving a problem about a loan with interest rate r and principal P, seeing the expression $P(1+r)^n$ as a product of P with a factor not depending on P.)
Quadratic Functions and Modeling	
• Solve real-world and mathematical problems by writing and solving nonlinear equations, such as quadratic equations ($ax^2 + bx + c = 0$).	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Relationships between Quantities and Reasoning with Equations	
Reason quantitatively and use units to solve problems.	Standards 1-3
Interpret the structure of expressions.	Standard 4
Create equations that describe numbers or relationships.	Standards 5-8
Understand solving equations as a process of reasoning and explain the reasoning.	Standard 9
Solve equations and inequalities in one variable.	Standard 10

Linear and Exponential Relationships	
Extend the properties of exponents to rational exponents.	Standards 11-12
Analyze and solve linear equations and pairs of simultaneous linear equations.	Standard 13
Solve systems of equations.	Standards 14-15
Represent and solve equations and inequalities graphically.	Standards 16-18
Define, evaluate and compare functions.	Standards 19-21
Understand the concept of a function and use function notation.	Standards 22-24
Use functions to model relationships between quantities.	Standards 25-26
Interpret functions that arise in applications in terms of a context.	Standards 27-29
Analyze functions using different representations.	Standards 30-31
Build a function that models a relationship between two quantities.	Standards 32-33
Build new functions from existing functions.	Standard 34
Construct and compare linear, quadratic, and exponential models and solve problems.	Standards 35-37
Interpret expressions for functions in terms of the situation they model.	Standard 38
Descriptive Statistics	
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 39-41
Investigate patterns of association in bivariate data.	Standards 42-45
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standards 46-47
Interpret linear models.	Standards 48-50
Expressions and Equations	
Interpret the structure of equations.	Standards 51-52
Write expressions in equivalent forms to solve problems.	Standard 53
Perform arithmetic operations on polynomials.	Standard54
Create equations that describe numbers or relationships.	Standards 55-57
Solve equations and inequalities in one variable.	Standard 58
Solve systems of equations.	Standard 59
Quadratic Functions and Modeling	
Use properties of rational and irrational numbers.	Standard 60
Understand and apply the Pythagorean theorem.	Standards 61-63
Interpret functions that arise in applications in terms of a context.	Standards 64-66
Analyze functions using different representations.	Standards 67-69
Build a function that models a relationship between two quantities.	Standard 70
Build new functions from existing functions.	Standards 71-72
Construct and compare linear, quadratic and exponential models and solve problems.	Standard 73

Cluster	Reason quantitatively and use units to solve problems
M.A18.1	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.A18.2	Define appropriate quantities for the purpose of descriptive modeling. Instructional Note: Working with quantities and the relationships between them provides grounding for work with expressions, equations, and functions.
M.A18.3	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Cluster	Interpret the structure of expressions.
M.A18.4	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Limit to linear expressions and to exponential expressions with integer exponents.

Cluster	Create equations that describe numbers or relationships.
M.A18.5	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.
M.A18.6	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Limit to linear and exponential equations, and, in the case of exponential equations, limit to situations requiring evaluation of exponential functions at integer inputs.
M.A18.7	Represent constraints by equations or inequalities, and by systems of equations and/or inequalities, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: Limit to linear equations and inequalities.
M.A18.8	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Limit to formulas with a linear focus.

Cluster	Understand solving equations as a process of reasoning and explain the reasoning.
M.A18.9	Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method. Instructional Note: Students should focus on linear equations and be able to extend and apply their reasoning to other types of equations in future units and courses. Students will solve exponential equations in Algebra II.

Cluster	Solve equations and inequalities in one variable.
M.A18.10	Solve linear equations and inequalities in one variable, including equations with coefficients represented by letters. Instructional Note: Extend earlier work with solving linear equations to solving linear inequalities in one variable and to solving literal equations that are linear in the variable being solved for. Include simple exponential equations that rely only on application of the laws of exponents, such as $5^x = 125$ or $2^x = 1/16$.

Linear and Exponential Relationships

Cluster	Extend the properties of exponents to rational exponents.
M.A18.11	Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define $5^{1/3}$ to be the cube root of 5 because we want $(5^{1/3})^3 = 5^{(1/3)3}$ to hold, so $(5^{1/3})^3$ must equal 5. Instructional Note: Address this standard before discussing exponential functions with continuous domains.
M.A18.12	Rewrite expressions involving radicals and rational exponents using the properties of exponents. Instructional Note: Address this standard before discussing exponential functions with continuous domains.

Cluster	Analyze and solve linear equations and pairs of simultaneous linear equations.
M.A18.13	Analyze and solve pairs of simultaneous linear equations. a. Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously. b. Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection. For example, $3x + 2y = 5$ and $3x + 2y = 6$ have no solution because $3x + 2y$ cannot simultaneously be 5 and 6. c. Solve real-world and mathematical problems leading to two linear equations in two variables. (e.g., Given coordinates for two pairs of points, determine whether the line through the first pair of points intersects the line through the second pair.) Instructional Note: While this content is likely subsumed by M.A18.10, 14, and 15, it could be used for scaffolding instruction to the more sophisticated content found there.

Cluster	**Solve systems of equations.**
M.A18.14	Prove that, given a system of two equations in two variables, replacing one equation by the sum of that equation and a multiple of the other produces a system with the same solutions. Instructional Note: Include cases where two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution).
M.A18.15	Solve systems of linear equations exactly and approximately (e.g., with graphs), focusing on pairs of linear equations in two variables. Instructional Note: Include cases where two equations describe the same line (yielding infinitely many solutions) and cases where two equations describe parallel lines (yielding no solution).

Cluster	**Represent and solve equations and inequalities graphically.**
M.A18.16	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line). Instructional Note: Focus on linear and exponential equations and be able to adapt and apply that learning to other types of equations in future courses.
M.A18.17	Explain why the x-coordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately (e.g., using technology to graph the functions, make tables of values or find successive approximations). Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential and logarithmic functions. Instructional Note: Focus on cases where f(x) and g(x) are linear or exponential.
M.A18.18	Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality), and graph the solution set to a system of linear inequalities in two variables as the intersection of the corresponding half-planes.

Cluster	**Define, evaluate and compare functions.** Instructional Note: While this content is likely subsumed by M.A18.22-24 and M.A18.30a it could be used for scaffolding instruction to the more sophisticated content found there.
M.A18.19	Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.
M.A18.20	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.)
M.A18.21	Interpret the equation y = mx + b as defining a linear function, whose graph is a straight line; give examples of functions that are not linear. (e.g., The function $A = s^2$ giving the area of a square as a function of its side length is not linear because its graph contains the points (1,1), (2,4) and (3,9), which are not on a straight line.)

Cluster	**Understand the concept of a function and use function notation.**
M.A18.22	Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x). Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Constrain examples to linear functions and exponential functions having integral domains.
M.A18.23	Use function notation, evaluate functions for inputs in their domains and interpret statements that use function notation in terms of a context. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Constrain examples to linear functions and exponential functions having integral domains.
M.A18.24	Recognize that sequences are functions, sometimes defined recursively, whose domain is a subset of the integers. For example, the Fibonacci sequence is defined recursively by f(0) = f(1) = 1, f(n+1) = f(n)+ f(n-1) for n ≥ 1. Instructional Note: Students should experience a variety of types of situations modeled by functions. Detailed analysis of any particular class of function at this stage is not advised. Students should apply these concepts throughout their future mathematics courses. Constrain examples to linear functions and exponential functions having integral domains. Draw connection to M.A18.33, which requires students to write arithmetic and geometric sequences.)

Cluster	**Use functions to model relationships between quantities.** **Instructional Note: While this content is likely subsumed by M.A18.27and M.A18.32a, it could be used for scaffolding instruction to the more sophisticated content found there.**
M.A18.25	Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.
M.A18.26	Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

Cluster	**Interpret functions that arise in applications in terms of a context.**
M.A18.27	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on linear and exponential functions.
M.A18.28	Relate the domain of a function to its graph and where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Instructional Note: Focus on linear and exponential functions.
M.A18.29	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on linear functions and intervals for exponential functions whose domain is a subset of the integers. The Quadratic Functions and Modeling unit of this course and Algebra II course will address other function types.

Cluster	**Analyze functions using different representations.**
M.A18.30	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph exponential and logarithmic functions, showing intercepts and end behavior and trigonometric functions, showing period, midline and amplitude. Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \cdot 2^n$.
M.A18.31	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Focus on linear and exponential functions. Include comparisons of two functions presented algebraically. For example, compare the growth of two linear functions, or two exponential functions such as $y = 3^n$ and $y = 100 \cdot 2^n$.

Cluster	**Build a function that models a relationship between two quantities.**
M.A18.32	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Instructional Note: Limit to linear and exponential functions.
M.A18.33	Write arithmetic and geometric sequences both recursively and with an explicit formula, use them to model situations, and translate between the two forms. Instructional Note: Connect arithmetic sequences to linear functions and geometric sequences to exponential functions.

Cluster	**Build new functions from existing functions.**
M.A18.34	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on vertical translations of graphs of linear and exponential functions. Relate the vertical translation of a linear function to its y-intercept. While applying other transformations to a linear graph is appropriate at this level, it may be difficult for students to identify or distinguish between the effects of the other transformations included in this standard.

Cluster	**Construct and compare linear, quadratic, and exponential models and solve problems.**
M.A18.35	Distinguish between situations that can be modeled with linear functions and with exponential functions. a. Prove that linear functions grow by equal differences over equal intervals; exponential functions grow by equal factors over equal intervals. b. Recognize situations in which one quantity changes at a constant rate per unit interval relative to another. c. Recognize situations in which a quantity grows or decays by a constant percent rate per unit interval relative to another.
M.A18.36	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship or two input-output pairs (include reading these from a table).
M.A18.37	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Limit to comparisons between linear and exponential models.

Cluster	**Interpret expressions for functions in terms of the situation they model.**
M.A18.38	Interpret the parameters in a linear or exponential function in terms of a context. Instructional Note: Limit exponential functions to those of the form $f(x) = b^x + k$.

Descriptive Statistics

Cluster	**Summarize, represent, and interpret data on a single count or measurement variable.**
M.A18.39	Represent data with plots on the real number line (dot plots, histograms, and box plots).
M.A18.40	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets. Instructional Note: In grades 6 – 7, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.

M.A18.41	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers). Instructional Note: In grades 6 – 7, students describe center and spread in a data distribution. Here they choose a summary statistic appropriate to the characteristics of the data distribution, such as the shape of the distribution or the existence of extreme data points.

Cluster	**Investigate patterns of association in bivariate data.** Instructional Note: While this content is likely subsumed by M.A18.47-50, it could be used for scaffolding instruction to the more sophisticated content found there.
M.A18.42	Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.
M.A18.43	Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.
M.A18.44	Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept. (e.g., In a linear model for a biology experiment, interpret a slope of 1.5 cm/hr as meaning that an additional hour of sunlight each day is associated with an additional 1.5 cm in mature plant height.)
M.A18.45	Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables. (e.g., Collect data from students in your class on whether or not they have a curfew on school nights and whether or not they have assigned chores at home. Is there evidence that those who have a curfew also tend to have chores?)

Cluster	**Summarize, represent, and interpret data on two categorical and quantitative variables.**
M.A18.46	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal and conditional relative frequencies). Recognize possible associations and trends in the data.

M.A18.47	Represent data on two quantitative variables on a scatter plot, and describe how the variables are related. a. Fit a function to the data; use functions fitted to data to solve problems in the context of the data. Use given functions or choose a function suggested by the context. Instructional Note: Emphasize linear and exponential models. b. Informally assess the fit of a function by plotting and analyzing residuals. Instructional Note: Focus should be on situations for which linear models are appropriate, but may be used to preview quadratic functions in the Quadratic Functions and Modeling Unit. c. Fit a linear function for scatter plots that suggest a linear association. Instructional Note: Students take a more sophisticated look at using a linear function to model the relationship between two numerical variables. In addition to fitting a line to data, students assess how well the model fits by analyzing residuals.

Cluster	**Interpret linear models.**
M.A18.48	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data. Instructional Note: Build on students' work with linear relationships and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.A18.49	Compute (using technology) and interpret the correlation coefficient of a linear fit. Instructional Note: Build on students' work with linear relationships and introduce the correlation coefficient. The focus here is on the computation and interpretation of the correlation coefficient as a measure of how well the data fit the relationship.
M.A18.50	Distinguish between correlation and causation. Instructional Note: The important distinction between a statistical relationship and a cause-and-effect relationship arises here.

Expressions and Equations

Cluster	**Interpret the structure of equations.**
M.A18.51	Interpret expressions that represent a quantity in terms of its context. a. Interpret parts of an expression, such as terms, factors, and coefficients. b. Interpret complicated expressions by viewing one or more of their parts as a single entity. For example, interpret $P(1 + r)^n$ as the product of P and a factor not depending on P. Instructional Note: Focus on quadratic and exponential expressions. For M.A18.51b, exponents are extended from integer found in the unit on Relationships between Quantities to rational exponents focusing on those that represent square roots and cube roots.
M.A18.52	Use the structure of an expression to identify ways to rewrite it. For example, see $x^4 - y^4$ as $(x^2)^2 - (y^2)^2$, thus recognizing it as a difference of squares that can be factored as $(x^2 - y^2)(x^2 + y^2)$.

Cluster	**Write expressions in equivalent forms to solve problems.**
M.A18.53	Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression. a. Factor a quadratic expression to reveal the zeros of the function it defines. b. Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines. c. Use the properties of exponents to transform expressions for exponential functions. For example the expression 1.15^t can be rewritten as $(1.15^{1/12})^{12t} \approx 1.012^{12t}$ to reveal the approximate equivalent monthly interest rate if the annual rate is 15%. Instructional Note: It is important to balance conceptual understanding and procedural fluency in work with equivalent expressions. For example, development of skill in factoring and completing the square goes hand-in-hand with understanding what different forms of a quadratic expression reveal.

Cluster	**Perform arithmetic operations on polynomials.**
M.A18.54	Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials. Instructional Note: Focus on polynomial expressions that simplify to forms that are linear or quadratic in a positive integer power of x.

Cluster	**Create equations that describe numbers or relationships.**
M.A18.55	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions. Instructional Note: Extend work on linear and exponential equations in the unit on Relationships between Quantities to include quadratic equations.
M.A18.56	Create equations in two or more variables to represent relationships between quantities; graph equations on coordinate axes with labels and scales. Instructional Note: Extend work on linear and exponential equations in the unit on Relationships between Quantities to include quadratic equations.
M.A18.57	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations. (e.g., Rearrange Ohm's law V = IR to highlight resistance R.) Instructional Note: Extend work on linear and exponential equations in the unit on Relationships between Quantities to include quadratic equations. Extend M.A18.57 to formulas involving squared variables.

Cluster	**Solve equations and inequalities in one variable.**
M.A18.58	Solve quadratic equations in one variable. a. Use the method of completing the square to transform any quadratic equation in x into an equation of the form $(x - p)^2 = q$ that has the same solutions. Derive the quadratic formula from this form. b. Solve quadratic equations by inspection (e.g., for $x^2 = 49$), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as $a \pm bi$ for real numbers a and b. Instructional Note: Students should learn of the existence of the complex number system, but will not solve quadratics with complex solutions until Algebra II.

Cluster	**Solve systems of equations.**
M.A18.59	Solve a simple system consisting of a linear equation and a quadratic equation in two variables algebraically and graphically. (e.g., Find the points of intersection between the line $y = -3x$ and the circle $x^2 + y^2 = 3$.) Instructional Note: Include systems consisting of one linear and one quadratic equation. Include systems that lead to work with fractions. For example, finding the intersections between $x^2 + y^2 = 1$ and $y = (x+1)/2$ leads to the point (3/5, 4/5) on the unit circle, corresponding to the Pythagorean triple $3^2 + 4^2 = 5^2$.

Quadratic Functions and Modeling

Cluster	**Use properties of rational and irrational numbers.**
M.A18.60	Explain why the sum or product of two rational numbers is rational; that the sum of a rational number and an irrational number is irrational; and that the product of a nonzero rational number and an irrational number is irrational. Instructional Note: Connect to physical situations (e.g., finding the perimeter of a square of area 2).

Cluster	**Understand and apply the Pythagorean theorem.**
M.A18.61	Explain a proof of the Pythagorean Theorem and its converse
M.A18.62	Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical problems in two and three dimensions. Instructional Note: Discuss applications of the Pythagorean theorem and its connections to radicals, rational exponents, and irrational numbers.
M.A18.63	Apply the Pythagorean Theorem to find the distance between two points in a coordinate system. Instructional Note: Discuss applications of the Pythagorean theorem and its connections to radicals, rational exponents, and irrational numbers.

Cluster	**Interpret functions that arise in applications in terms of a context.**
M.A18.64	For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship. Key features include: intercepts; intervals where the function is increasing, decreasing, positive, or negative; relative maximums and minimums; symmetries; end behavior; and periodicity. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studies in the unit on Linear and Exponential Functions.
M.A18.65	Relate the domain of a function to its graph and, where applicable, to the quantitative relationship it describes. For example, if the function h(n) gives the number of person-hours it takes to assemble n engines in a factory, then the positive integers would be an appropriate domain for the function. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studies in the unit on Linear and Exponential Functions.

M.A18.66	Calculate and interpret the average rate of change of a function (presented symbolically or as a table) over a specified interval. Estimate the rate of change from a graph. Instructional Note: Focus on quadratic functions; compare with linear and exponential functions studies in the unit on Linear and Exponential Functions.

Cluster	**Analyze functions using different representations.**
M.A18.67	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases. a. Graph linear and quadratic functions and show intercepts, maxima, and minima. b. Graph square root, cube root, and piecewise-defined functions, including step functions and absolute value functions. Instructional Note: Compare and contrast absolute value, step and piecewise-defined functions with linear, quadratic, and exponential functions. Highlight issues of domain, range, and usefulness when examining piecewise-defined functions. Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic function can be factored.
M.A18.68	Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function. a. Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context. b. Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)^{12t}$, $y = (1.2)^{t/10}$, and classify them as representing exponential growth or decay. Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic function can be factored. This unit, and in particular in M.A18.68b, extends the work begun in Unit 2 on exponential functions with integral exponents.
M.A18.69	Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). (e.g., Given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.) Instructional Note: Extend work with quadratics to include the relationship between coefficients and roots, and that once roots are known, a quadratic function can be factored.

Cluster	**Build a function that models a relationship between two quantities.**
M.A18.70	Write a function that describes a relationship between two quantities. a. Determine an explicit expression, a recursive process, or steps for calculation from a context. b. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. Instructional Note: Focus on situations that exhibit a quadratic relationship.

Cluster	Build new functions from existing functions.
M.A18.71	Identify the effect on the graph of replacing f(x) by f(x) + k, k f(x), f(kx), and f(x + k) for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them. Instructional Note: Focus on quadratic functions, and consider including absolute value functions.
M.A18.72	Find inverse functions. a. Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, $f(x) = 2x^3$ or f(x) = (x+1)/(x-1) for x ≠ 1. Instructional Note: Focus on linear functions but consider simple situations where the domain of the function must be restricted in order for the inverse to exist, such as $f(x) = x^2$, x > 0.

Cluster	Construct and compare linear, quadratic and exponential models and solve problems.
M.A18.73	Observe using graphs and tables that a quantity increasing exponentially eventually exceeds a quantity increasing linearly, quadratically, or (more generally) as a polynomial function. Instructional Note: Compare linear and exponential growth to growth of quadratic growth.