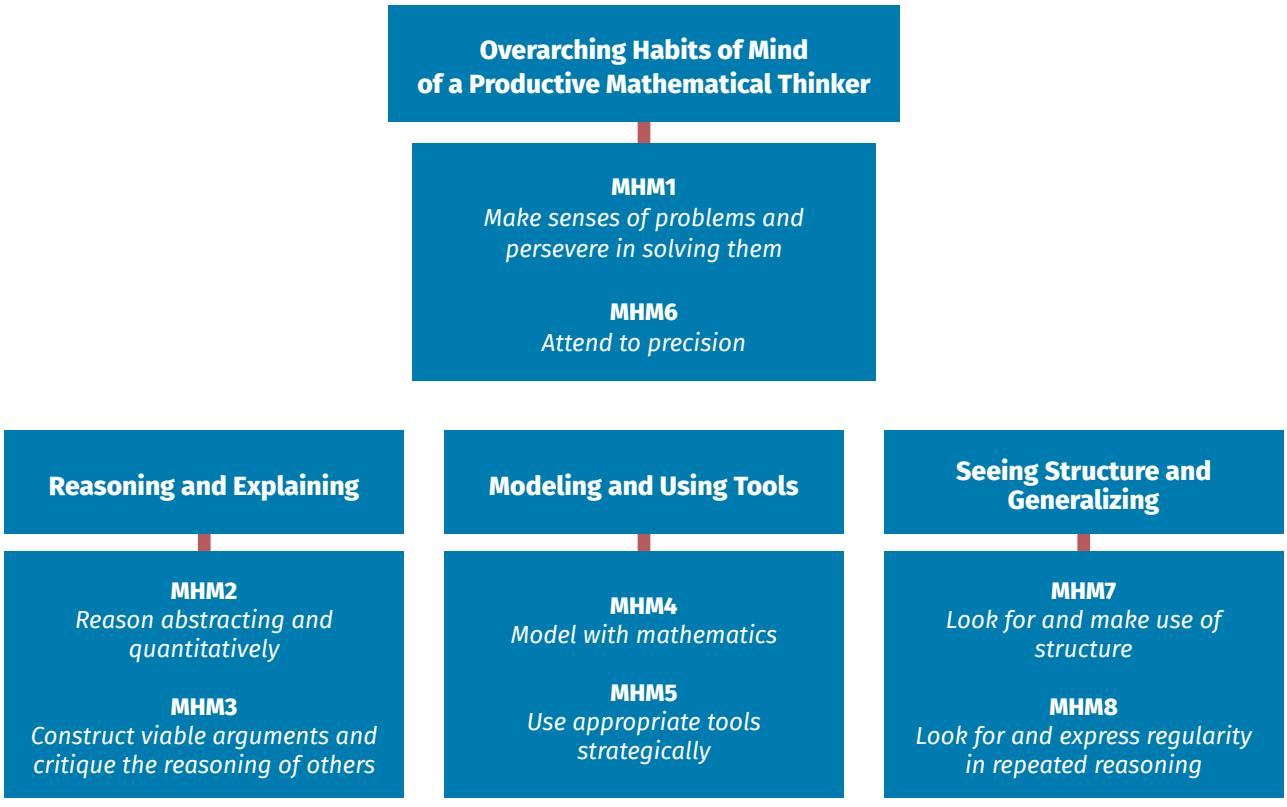
Overview of the West Virginia College- and Career-Readiness Standards for Mathematics

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards. These standards address the skills, knowledge, and dispositions that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge – what students need to understand, know, and be able to do. The standards also require that the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards be connected. These connections are essential to support the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level or course-specific Mathematics Content Standards are. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Mathematical Habits of Mind



The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important "processes and proficiencies " with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics' process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding it Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010).

Ideally, several Mathematical Habits of Mind will be evident in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MHM1), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck's research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender a "growth mindset." In Dweck's estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind are able to use these skills not only in mathematics, but across disciplines and into their lives beyond school, college, and career.

Policy 2520.2B

West Virginia College- and Career-Readiness Standards for Mathematics

Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, “Does this make sense?” They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a

flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7 × 8 equals the well-remembered 7 × 5 + 7 × 3, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2 × 7 and the 9 as 2 + 7. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$ and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics – Quantitative Reasoning

This course was created in conjunction with the West Virginia Higher Education Policy Commission (HEPC) and is intended to be a dual credit course.

All West Virginia teachers are responsible for classroom instruction that integrates content standards and mathematical habits of mind. Quantitative Reasoning prepares students to reason, model, and draw conclusions or make decisions with mathematical, statistical, and quantitative information. Students will compare, analyze, and synthesize of multiple forms or sources of quantitative information. Students will use appropriate mathematical and statistical language in oral, written, and graphical forms; read and interpret authentic texts such as advertisements, consumer information, government forms, and newspaper articles containing quantitative information, including graphical displays of quantitative information. Students will develop an answer to an open-ended question requiring analysis and synthesis of multiple calculations, data summaries, and/or models. Students will draw conclusions or make decisions in quantitatively based situations that are dependent upon multiple factors and analyze how different situations would affect the decisions. Students will be able to critique and evaluate quantitative arguments that utilize mathematical, statistical, and quantitative information. Students will evaluate the validity and possible biases in arguments presented in authentic contexts based on multiple sources of quantitative information (e.g., advertising, internet postings, consumer information, political arguments). Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending

to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Logical Reasoning	**Algebraic Modeling and Number Sense**
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	• Create and analyze mathematical models to make decisions related to earning, investing, spending and borrowing money.
Descriptive Statistics	**Probability**
• Make decisions based on understanding, analysis and critique of reported statistical information and summaries. • Use basic rules of counting and probability to analyze and evaluate risk and return in the context of everyday situations. • Interpret categorical and quantitative date, make inferences and justify conclusions. • Create and analyze mathematical models to make decisions related to earning, investing, spending and borrowing money.	• Make conclusions based on understanding, analysis and critique of probabilities. (e.g., Compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.) • Develop a probability distribution. (e.g., Find the theoretical probability distribution for the number of correct answers obtained by guessing on all five questions of a multiple-choice test where each question has four choices, and find the expected grade under various grading schemes.)

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Quantitative Reasoning:

Logical Reasoning	
Math as a language.	Standards 1-7
Tools for problem solving.	
Algebraic Modeling and Number Sense	
Understand ratio concepts and use ratio reasoning to solve problems.	Standard 4
Work with integer exponents, scientific notation, and radicals.	Standards 5-7
Reason quantitatively and use units to solve problems.	Standard 8
Represent and solve equations and inequalities graphically.	Standard 9
Explain volume formulas and use them to solve problems.	Standard 10
Understand financial models.	Standards 11-13
Reason quantitatively and use units to solve problems.	Standards 14-15
Create equations that describe numbers or relationships.	Standards 16-17

Construct and compare linear, quadratic, and exponential models and solve problems.	Standard 18
Build a function that models a relationship between two quantities.	Standard 19
Interpret linear models.	Standard 20
Descriptive Statistics	
Summarize, represent, and interpret data on two categorical and quantitative variables.	Standard 21
Summarize, represent, and interpret data on a single count or measurement variable.	Standards 22-24
Conduct statistical analysis.	Standard 25
Communicate statistical information.	Standards 26-27
Probability	
Analyze information using probability and counting.	Standards 28-29
Use probability to evaluate outcomes of decisions.	Standard 30
Manage uncertainty.	Standard 31
Understand independence and conditional probability and use them to interpret data.	Standards 32-36
Use the rules of probability to compute probabilities of compound events in a uniform probability model.	Standards 37-40
Use probability to evaluate outcomes of decisions.	Standards 41-42

Logical Reasoning

Cluster	**Math as a language**
M.QR.1	Demonstrate reasoning skills in developing, explaining, and justifying sound mathematical arguments and analyze the soundness of mathematical arguments of others.
M.QR.2	Communicate with and about mathematics orally and in writing as part of independent and collaborative work, including making accurate and clear presentations of solutions to problems.
Cluster	**Tools for problem solving**
M.QR.3	Gather data, conduct investigations and apply mathematical concepts and models to solve problems in mathematics and other disciplines.

Cluster	**Understand ratio concepts and use ratio reasoning to solve problems.**
M.QR.4	Use ratio and rate reasoning to solve real-world and mathematical problems. a. Make tables of equivalent ratios relating quantities with whole number measurements, find missing values in the tables, and plot the pairs of values on the coordinate plane. Use tables to compare ratios. b. Solve unit rate problems including those involving unit pricing and constant speed. (e.g., If it took 7 hours to mow 4 lawns, then at that rate, how many lawns could be mowed in 35 hours? At what rate were lawns being mowed?) c. Find a percent of a quantity as a rate per 100 (e.g., 30% of a quantity means 30/100 times the quantity); solve problems involving finding the whole, given a part and the percent. d. Use ratio reasoning to convert measurement units; manipulate and transform units appropriately when multiplying or dividing quantities.
Cluster	**Work with integer exponents, scientific notation, and radicals.**
M.QR.5	Know and apply the properties of integer exponents to generate equivalent numerical expressions.
M.QR.6	Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities. (e.g., Use millimeters per year for seafloor spreading.) Interpret scientific notation that has been generated by technology.
M.QR.7	Rewrite expressions involving radicals and rational exponents using the properties of exponents.
Cluster	**Reason quantitatively and use units to solve problems.**
M.QR.8	Define appropriate quantities for the purpose of descriptive modeling.
Cluster	**Represent and solve equations and inequalities graphically.**
M.QR.9	Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
Cluster	**Explain volume formulas and use them to solve problems.**
M.QR.10	Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone.
Cluster	**Understand financial models**
M.QR.11	Determine, represent and analyze mathematical models for loan amortization and the effects of different payments and/or finance terms (e.g., Auto, Mortgage, and/or Credit Card).
M.QR.12	Determine, represent and analyze mathematical models for investments involving simple and compound interest with and without additional deposits. (e.g., Savings accounts, bonds, and/or certificates of deposit.)
M.QR.13	Research and analyze taxes including payroll, sales, personal property, real estate and income tax returns.

Cluster	**Reason quantitatively and use units to solve problems.**
M.QR.14	Use units as a way to understand problems and to guide the solution of multi-step problems; choose and interpret units consistently in formulas; choose and interpret the scale and the origin in graphs and data displays.
M.QR.15	Choose a level of accuracy appropriate to limitations on measurement when reporting quantities.

Cluster	**Create equations that describe numbers or relationships.**
M.QR.16	Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions and simple rational and exponential functions.
M.QR.17	Rearrange formulas to highlight a quantity of interest, using the same reasoning as in solving equations.

Cluster	**Construct and compare linear, quadratic, and exponential models and solve problems.**
M.QR.18	Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).

Cluster	**Build a function that models a relationship between two quantities.**
M.QR.19	Write a function that describes a relationship between two quantities. a. Combine standard function types using arithmetic operations. For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model. b. Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.

Cluster	**Interpret linear models.**
M.QR.20	Interpret the slope (rate of change) and the intercept (constant term) of a linear model in the context of the data.

Descriptive Statistics

Cluster	**Summarize, represent, and interpret data on two categorical and quantitative variables.**
M.QR.21	Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.

Cluster	**Summarize, represent, and interpret data on a single count or measurement variable.**
M.QR.22	Represent data with plots on the real number line (dot plots, histograms, and box plots).

M.QR.23	Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.
M.QR.24	Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
Cluster	**Conduct statistical analysis**
M.QR.25	Create data displays for given data sets to investigate, compare, and estimate center, shape, spread and unusual features.
Cluster	**Communicate statistical information**
M.QR.26	Report results of statistical studies to a particular audience, including selecting an appropriate presentation format, creating graphical data displays and interpreting results in terms of the question studied.
M.QR.27	Communicate statistical results in both oral and written formats using appropriate statistical and nontechnical language.

Probability

Cluster	**Analyze information using probability and counting**
M.QR.28	Use the Fundamental Counting Principle, Permutations and Combinations to determine all possible outcomes for an event; determine probability and odds of a simple event; explain the significance of the Law of Large Numbers.
M.QR.29	Determine and interpret conditional probabilities and probabilities of compound events by constructing and analyzing representations, including tree diagrams, Venn diagrams, two-way frequency tables and area models, to make decisions in problem situations.
Cluster	**Use probability to evaluate outcomes of decisions.**
M.QR.30	Use probabilities to make and justify decisions about risks in everyday life.
Cluster	**Manage uncertainty**
M.QR.31	Calculate expected value to analyze mathematical fairness, payoff and risk.
Cluster	**Understand independence and conditional probability and use them to interpret data.**
M.QR.32	Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events (“or,” “and,” “not”).
M.QR.33	Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.

M.QR.34	Recognize the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
M.QR.35	Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities. For example, collect data from a random sample of students in your school on their favorite subject among math, science, and English. Estimate the probability that a randomly selected student from your school will favor science given that the student is in tenth grade. Do the same for other subjects and compare the results. Instructional Note: Build on work with two-way tables from Algebra I to develop understanding of conditional probability and independence.
M.QR.36	Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.
Cluster	**Use the rules of probability to compute probabilities of compound events in a uniform probability model.**
M.QR.37	Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
M.QR.38	Apply the Addition Rule, P(A or B) = P(A) + P(B) – P(A and B), and interpret the answer in terms of the model.
M.QR.39	Apply the general Multiplication Rule in a uniform probability model, P(A and B) = P(A) P(B|A) = P(B)P(A|B), and interpret the answer in terms of the model.
M.QR.40	Use permutations and combinations to compute probabilities of compound events and solve problems.
Cluster	**Use probability to evaluate outcomes of decisions.**
M.QR.41	Use probabilities to make fair decisions (e.g., drawing by lots and/or using a random number generator).
M.QR.42	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game).