Overview of the West Virginia College- and Career-Readiness Standards for Mathematics

Included in Policy 2520.2B, the West Virginia College- and Career-Readiness Standards for Mathematics are two types of standards: the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards. These standards address the skills, knowledge, and dispositions that students should develop to foster mathematical understanding and expertise, as well as concepts, skills, and knowledge – what students need to understand, know, and be able to do. The standards also require that the Mathematical Habits of Mind and the grade-level or course-specific Mathematics Content Standards be connected. These connections are essential to support the development of students' broader mathematical understanding, as students who lack understanding of a topic may rely too heavily on procedures. The Mathematical Habits of Mind must be taught as carefully and practiced as intentionally as the grade-level or course-specific Mathematics Content Standards are. Neither type should be isolated from the other; mathematics instruction is most effective when these two aspects of the West Virginia College- and Career-Readiness Standards for Mathematics come together as a powerful whole.

Mathematical Habits of Mind

Overarching Habits of Mind of a Productive Mathematical Thinker

MHM1
Make senses of problems and persevere in solving them

MHM6
Attend to precision

Reasoning and Explaining

MHM2
Reason abstracting and quantitatively

MHM3
Construct viable arguments and critique the reasoning of others

Modeling and Using Tools

MHM4
Model with mathematics

MHM5
Use appropriate tools strategically

Seeing Structure and Generalizing

MHM7
Look for and make use of structure

MHM8
Look for and express regularity in repeated reasoning

The eight Mathematical Habits of Mind (MHM) describe the attributes of mathematically proficient students and the expertise that mathematics educators at all levels should seek to develop in their students. The Mathematical Habits of Mind provide a vehicle through which students engage with and learn mathematics. As students move from elementary school through high school, the Mathematical Habits of Mind are integrated in the tasks as students engage in doing mathematics and master new and more advanced mathematical ideas and understandings.

The Mathematical Habits of Mind rest on important "processes and proficiencies " with longstanding importance in mathematics education. The first of these are the National Council of Teachers of Mathematics' process standards of problem solving, reasoning and proof, communication, representation, and connections. The second are the strands of mathematical proficiency specified in the National Research Council's report Adding it Up: adaptive reasoning, strategic competence, conceptual understanding, procedural fluency, and productive disposition (NGA/CCSSO 2010).

Ideally, several Mathematical Habits of Mind will be evident in each lesson as they interact and overlap with each other. The Mathematical Habits of Mind are not a checklist; they are the basis for mathematics instruction and learning. To help students persevere in solving problems (MHM1), teachers need to allow their students to struggle productively, and they must be attentive to the type of feedback they provide to students. Dr. Carol Dweck's research (Dweck 2006) revealed that feedback offering praise of effort and perseverance seems to engender a "growth mindset." In Dweck's estimation, growth-minded teachers tell students the truth about being able to close the learning gap between them and their peers and then give them the tools to close the gap (Dweck 2006).

Students who are proficient in the eight Mathematical Habits of Mind are able to use these skills not only in mathematics, but across disciplines and into their lives beyond school, college, and career.

Policy 2520.2B

West Virginia College- and Career-Readiness Standards for Mathematics

Mathematical Habits of Mind

The Mathematical Habits of Mind (hereinafter MHM) describe varieties of expertise that mathematics educators at all levels should develop in their students.

MHM1. Make sense of problems and persevere in solving them.
Mathematically proficient students start by explaining to themselves the meaning of a problem and looking for entry points to its solution. They analyze givens, constraints, relationships and goals. They make conjectures about the form and meaning of the solution and plan a solution pathway rather than simply jumping into a solution attempt. They consider analogous problems and try special cases and simpler forms of the original problem in order to gain insight into its solution. They monitor and evaluate their progress and change course if necessary. Older students might, depending on the context of the problem, transform algebraic expressions or change the viewing window on their graphing calculator to get the information they need. Mathematically proficient students can explain correspondences between equations, verbal descriptions, tables and graphs or draw diagrams of important features and relationships, graph data and search for regularity or trends. Younger students might rely on using concrete objects or pictures to help conceptualize and solve a problem. Mathematically proficient students check their answers to problems using a different method and they continually ask themselves, "Does this make sense?" They can understand the approaches of others to solving complex problems and identify correspondences between different approaches.

MHM2. Reason abstractly and quantitatively.
Mathematically proficient students make sense of quantities and their relationships in problem situations. They bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize—to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize - to pause as needed during the manipulation process in order to probe into the referents for the symbols involved. Quantitative reasoning entails habits of creating a coherent representation of the problem at hand, considering the units involved, attending to the meaning of quantities, not just how to compute them, and knowing and flexibly using different properties of operations and objects.

MHM3. Construct viable arguments and critique the reasoning of others.
Mathematically proficient students understand and use stated assumptions, definitions, and previously established results in constructing arguments. They make conjectures and build a logical progression of statements to explore the truth of their conjectures. They are able to analyze situations by breaking them into cases and can recognize and use counterexamples. They justify their conclusions, communicate them to others, and respond to the arguments of others. They reason inductively about data, making plausible arguments that take into account the context from which the data arose. Mathematically proficient students are also able to compare the effectiveness of two plausible arguments, distinguish correct logic or reasoning from that which is flawed, and—if there is a

flaw in an argument—explain what it is. Elementary students can construct arguments using concrete referents such as objects, drawings, diagrams and actions. Such arguments can make sense and be correct, even though they are not generalized or made formal until later grades. Later, students learn to determine domains to which an argument applies. Students at all grades can listen or read the arguments of others, decide whether they make sense and ask useful questions to clarify or improve the arguments.

MHM4. Model with mathematics.
Mathematically proficient students can apply the mathematics they know to solve problems arising in everyday life, society and the workplace. In early grades, this might be as simple as writing an addition equation to describe a situation. In middle grades, a student might apply proportional reasoning to plan a school event or analyze a problem in the community. By high school, a student might use geometry to solve a design problem or use a function to describe how one quantity of interest depends on another. Mathematically proficient students who can apply what they know are comfortable making assumptions and approximations to simplify a complicated situation, realizing that these may need revision later. They are able to identify important quantities in a practical situation and map their relationships using such tools as diagrams, two-way tables, graphs, flowcharts and formulas. They can analyze those relationships mathematically to draw conclusions. They routinely interpret their mathematical results in the context of the situation and reflect on whether the results make sense, possibly improving the model if it has not served its purpose.

MHM5. Use appropriate tools strategically.
Mathematically proficient students consider the available tools when solving a mathematical problem. These tools might include pencil and paper, concrete models, a ruler, a protractor, a calculator, a spreadsheet, a computer algebra system, a statistical package or dynamic geometry software. Proficient students are sufficiently familiar with tools appropriate for their grade or course to make sound decisions about when each of these tools might be helpful, recognizing both the insight to be gained and their limitations. For example, mathematically proficient high school students analyze graphs of functions and solutions generated using a graphing calculator. They detect possible errors by strategically using estimation and other mathematical knowledge. When making mathematical models, they know that technology can enable them to visualize the results of varying assumptions, explore consequences and compare predictions with data. Mathematically proficient students at various grade levels are able to identify relevant external mathematical resources, such as digital content located on a website and use them to pose or solve problems. They are able to use technological tools to explore and deepen their understanding of concepts.

MHM6. Attend to precision.
Mathematically proficient students try to communicate precisely to others. They try to use clear definitions in discussion with others and in their own reasoning. They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. They are careful about specifying units of measure, and labeling axes to clarify the correspondence with quantities in a problem. They calculate accurately and efficiently, express numerical answers with a degree of precision appropriate for the problem context. In the elementary grades, students give carefully formulated explanations to each other. By the time they reach high school they have learned to examine claims and make explicit use of definitions.

MHM7. Look for and make use of structure.
Mathematically proficient students look closely to discern a pattern or structure. Young students, for example, might notice that three and seven more is the same amount as seven and three more or they may sort a collection of shapes according to how many sides the shapes have. Later, students will see 7×8 equals the well-remembered $7 \times 5 + 7 \times 3$, in preparation for learning about the distributive property. In the expression $x^2 + 9x + 14$, older students can see the 14 as 2×7 and the 9 as $2 + 7$. They recognize the significance of an existing line in a geometric figure and can use the strategy of drawing an auxiliary line for solving problems. They also can step back for an overview and shift perspective. They can see complicated things, such as some algebraic expressions, as single objects or as being composed of several objects. For example, they can see $5 – 3(x – y)^2$ as 5 minus a positive number times a square and use that to realize that its value cannot be more than 5 for any real numbers x and y.

MHM8. Look for and express regularity in repeated reasoning.
Mathematically proficient students notice if calculations are repeated, and look both for general methods and for shortcuts. Upper elementary students might notice when dividing 25 by 11 that they are repeating the same calculations over and over again, and conclude they have a repeating decimal. By paying attention to the calculation of slope as they repeatedly check whether points are on the line through (1, 2) with slope 3, middle school students might abstract the equation $(y – 2)/(x – 1) = 3$. Noticing the regularity in the way terms cancel when expanding $(x – 1)(x + 1)$, $(x – 1)(x^2 + x + 1)$ and $(x – 1)(x^3 + x^2 + x + 1)$ might lead them to the general formula for the sum of a geometric series. As they work to solve a problem, mathematically proficient students maintain oversight of the process, while attending to the details. They continually evaluate the reasonableness of their intermediate results.

Mathematics – STEM Readiness

All West Virginia teachers are responsible for classroom instruction that integrates content standards and objectives and mathematical habits of mind. This course is designed for students who have completed the Math III (LA) course and subsequently decided they are interested in pursuing a STEM career. It includes standards that would have been covered in Mathematics III (STEM) but not in Mathematics III (LA) (i.e. standards that are marked with a “+”), selected topics from the Mathematics IV course, and topics drawing from standards covered in Mathematics I and Mathematics II as needed for coherence. Mathematical habits of mind, which should be integrated in these content areas, include: making sense of problems and persevering in solving them, reasoning abstractly and quantitatively; constructing viable arguments and critiquing the reasoning of others; modeling with mathematics; using appropriate tools strategically; attending to precision, looking for and making use of structure; and looking for and expressing regularity in repeated reasoning. Students will continue developing mathematical proficiency in a developmentally-appropriate progressions of standards. Continuing the skill progressions from previous courses, the following chart represents the mathematical understandings that will be developed:

Arithmetic and Algebra of Complex Numbers	**Polynomial, Rational, and Radical Relationships**
• Understand that the arithmetic and algebra of expressions involving rational numbers is governed by the same rules as the arithmetic and algebra of real numbers.	• Derive the formula for the sum of a geometric series, and use the formula to solve problems. (e.g., Calculate mortgage payments.)
Probability for Decisions	**Trigonometry of General Triangles**
• Make inferences and justify conclusions from sample surveys, experiments, and observational studies.	• Apply knowledge of the Law of Sines and the Law of Cosines to determine distances in realistic situations. (e.g., Determine heights of inaccessible objects.)
Functions and Modeling	
• Analyze real-world situations using mathematics to understand the situation better and optimize, troubleshoot, or make an informed decision. (e.g., Estimate water and food needs in a disaster area, or use volume formulas and graphs to find an optimal size for an industrial package.)	

Numbering of Standards

The following Mathematics Standards will be numbered continuously. The following ranges relate to the clusters found within Mathematics:

Arithmetic and Algebra of Complex Numbers	
Perform arithmetic operations with complex numbers.	Standards 1-3
Represent complex numbers and their operations on the complex plane.	Standards 4-6
Use complex numbers in polynomial identities and equations.	Standards 7-9
Polynomial, Rational, and Radical Relationships	
Use polynomial identities to solve problems.	Standard 10
Rewrite rational expressions.	Standard 11
Probability for Decisions	
Use probability to evaluate outcomes of decisions.	Standards 12-13
Trigonometry of General Triangles	
Apply trigonometry to general triangles.	Standards 14-16

Functions and Modeling	
Analyze functions using different representations.	Standards 17-19
Building a function that models a relationship between two quantities.	Standards 20-21
Build new functions from existing functions.	Standards 22-26
Extend the domain of trigonometric functions using the unit circle.	Standards 27-28
Model periodic phenomena using trigonometric functions.	Standards 29-30
Prove and apply trigonometric identities.	Standard 31

Arithmetic and Algebra of Complex Numbers

Cluster	Perform arithmetic operations with complex numbers
M.SRM.1	Know there is a complex number i such that $i^2 = -1$, and every complex number has the form a + bi with a and b real.
M.SRM.2	Use the relation $i^2 = -1$ and the commutative, associative and distributive properties to add, subtract and multiply complex numbers.
M.SRM.3	Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

Cluster	Represent complex numbers and their operations on the complex plane
M.SRM.4	Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers) and explain why the rectangular and polar forms of a given complex number represent the same number.
M.SRM.5	Represent addition, subtraction, multiplication and conjugation of complex numbers geometrically on the complex plane; use properties of this representation for computation. (e.g., $(-1 + \sqrt{3}\,i)^3 = 8$ because $(-1 + \sqrt{3}\,i)$ has modulus 2 and argument 120°.)
M.SRM.6	Calculate the distance between numbers in the complex plane as the modulus of the difference and the midpoint of a segment as the average of the numbers at its endpoints.

Cluster	Use complex numbers in polynomial identities and equations
M.SRM.7	Solve quadratic equations with real coefficients that have complex solutions.
M.SRM.8	Extend polynomial identities to the complex numbers. For example, rewrite $x^2 + 4$ as $(x + 2i)(x – 2i)$.
M.SRM.9	Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.

Polynomials, Rational, and Radical Relationships

Cluster	**Use polynomial identities to solve problems.**
M.SRM.10	Know and apply the Binomial Theorem for the expansion of $(x + y)^n$ in powers of x and y for a positive integer n, where x and y are any numbers, with coefficients determined for example by Pascal's Triangle.

Cluster	**Rewrite rational expressions**
M.SRM.11	Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication and division by a nonzero rational expression; add, subtract, multiply and divide rational expressions.

Probability for Decisions

Cluster	**Use probability to evaluate outcomes of decisions.**
M.SRM.12	Use probabilities to make fair decisions (e.g. drawing by lot or using a random number generator).
M.SRM.13	Analyze decisions and strategies using probability concepts (e.g., product testing, medical testing, and/or pulling a hockey goalie at the end of a game).

Trigonometry of General Triangles

Cluster	**Apply trigonometry to general triangles.**
M.SRM.14	Derive the formula A = 1/2 ab sin(C) for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
M.SRM.15	Prove the Laws of Sines and Cosines and use them to solve problems.
M.SRM.16	Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and non-right triangles (e.g., surveying problems or resultant forces).

Functions and Modeling

Cluster	**Analyze functions using different representations.**
M.SRM.17	Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
M.SRM.18	Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available and showing end behavior.
M.SRM.19	Graph exponential and logarithmic functions, showing intercepts and end behavior and trigonometric functions, showing period, midline, and amplitude.

Cluster	Building a function that models a relationship between two quantities.
M.SRM.20	Write a function that describes a relationship between two quantities.
M.SRM.21	Compose functions. (e.g., If T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.)

Cluster	Build new functions from existing functions.
M.SRM.22	Find inverse functions.
M.SRM.23	Verify by composition that one function is the inverse of another.
M.SRM.24	Read values of an inverse function from a graph or a table, given that the function has an inverse.
M.SRM.25	Produce an invertible function from a non-invertible function by restricting the domain.
M.SRM.26	Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

Cluster	Extend the domain of trigonometric functions using the unit circle.
M.SRM.27	Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for $\pi-x$, $\pi+x$, and $2\pi-x$ in terms of their values for x, where x is any real number.
M.SRM.28	Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

Cluster	Model periodic phenomena using trigonometric functions.
M.SRM.29	Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
M.SRM.30	Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.

Cluster	Prove and apply trigonometric identities.
M.SRM.31	Prove the addition and subtraction formulas for sine, cosine and tangent and use them to solve problems.