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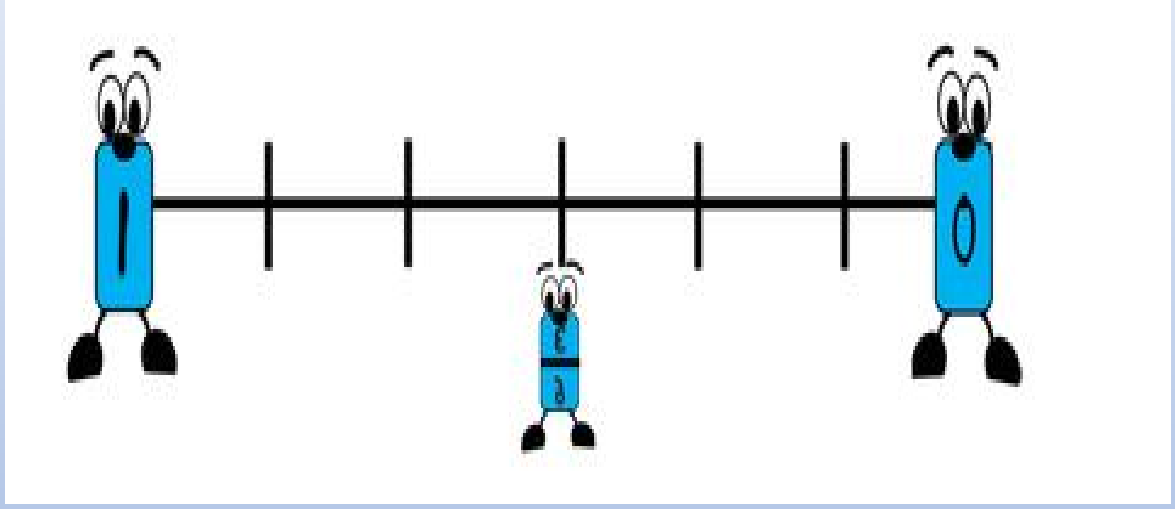

MATHEMATICS

GRADE 4

Comparing Fractions Using Benchmark Fractions

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Task Title: Comparing Fractions Using Benchmark Fractions
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Original Task Creator: Illustrative Math
Quarter: 3rd

Rationale for Lesson and Associated Tasks

Through a fraction card task, students deepen their understanding of fractions by using benchmark fractions of ½ and 1 to determine if a fraction is greater than, less than, or equal to another fraction. The goal of this task is to compare fractions and present justifications or explanations that demonstrate deep conceptual understanding. Proficiency with fractions is an important foundation for learning more advanced mathematics.
Note: The activity may be referred to as a game. However, since there is no competition, no winner or loser, it is a learning activity for students to complete together to apply their knowledge of fractions.

Students use an activity format to practice and review benchmark fractions. By using representations to compare fractions, this learning task helps students develop number sense about fraction size. This understanding helps students appreciate and internalize that strategies used to compare whole numbers may not necessarily extend in comparing fractions ($\frac{1}{2}$ is greater than $\frac{1}{6}$ while the whole number 6 is greater than 2). Students need opportunities to work with a variety of representations of fractions, including set and region models. They need to develop a concrete understanding of a fraction. Just as they use counters to help form a mental image of a whole number, students can use number line benchmarks to show how a fraction can be inserted between any two fractions. Experiences such as this activity assist in developing the necessary background students need as the beginning step into the abstract, advanced levels of mathematics such as algebra.

Lesson Activity and Associated Tasks Overview

Task: (Click here)
For Grade 4 students, this lesson and associated tasks may best serve as an activity to review and practice fractions ordering fractions to develop number sense.

Part 1: Introduction of the Fraction Activity

- Create student pairs. Introduce a review of fractional terms and explain the fractional activity.
- Provide the student handouts with card pieces, fraction mat, and optional items the students need to determine the fraction's location on the mat (benchmark page). Check for Understanding about the task by demonstrating a sample play in the activity.

Part 2: Comparing Fractions Using Benchmarks/Discussion of Observations

- Students begin the activity.
- Students reflect on understandings developed from activity.

West Virginia College- and Career-Readiness State Standard

M.4.12
Explain why a fraction a/b is equivalent to a fraction (n × a)/(n × b) by using visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size. Use this principle to recognize and generate equivalent fractions.

M 4.13
Compare two fractions with different numerators and different denominators (e.g., by creating common

denominators or numerators, or by comparing to a benchmark fraction such as ½). Recognize that comparisons are valid only when the two fractions refer to the same whole. Record the results of comparisons with symbols >, = or <, and justify the conclusions by using a visual fraction model.

Mathematical Habits of Mind (MHM)

MHM.1 – Make sense of problems and persevere in solving them.
MHM.3 – Construct viable arguments and critique the reasoning of others.
MHM.4 – Model with mathematics.
MHM.5 – Use appropriate tools strategically

Mathematics Teaching Practices to Support Student Growth

- Implement tasks that promote reasoning and problem solving
- Facilitate meaningful mathematical discourse.
- Support productive struggle in learning mathematics.

Essential Understandings

- The ability to determine appropriate benchmarks for fractions which are categorized with the benchmarks between 0, 1/2, and 1 using such terms as greater than, less than or equal to another fraction can strengthen the understanding of fractions.
- The ability to explain the deep conceptual understanding of the represented fractions and reasons why the fractions are placed in a specific benchmark category form an important foundation for learning more advanced mathematics.
- The ability to compare fractions can be used as an effective tool in addressing real-world situations.

Set-up Phase

1. Become an Expert Regarding Comparative Fraction Activity Task Content

It is essential that the teacher complete the activity prior to classroom implementation to understand how the activity works and clearly appreciate the mathematics embedded in the activity. Taking this opportunity to analyze the task provides an opportunity to identify specific questions that may arise as students work through the activity. It provides the teacher with a better appreciation of the variety of opportunities that are embedded in the activity to deepen student understanding of comparing fractions. This learning task allows the teacher to differentiate the activity by having two types of cards available: fraction cards with pictures and fraction cards with no pictures, as well as, differentiated fraction mats. During the task, the teacher should circulate throughout the class to check for student understanding related to comparing fractions and to see if students need scaffolding support with the materials. Some students may still need to have visual supports.

Students' understanding of fractions begins in Grade 3 with an introduction to what is a fraction. Students use models such as number lines or bar models to compare two fractions with the same denominators or the same numerator. In Grade 4, students extend the use of these models to compare fractions with different numerators and denominators using several strategies: find equivalent fractions; if the denominator is the same – compare the numerator; if the numerator is the same – compare the denominators; and using a benchmark of ½ and 1.

The teacher should determine what documents students will create and hand in as they complete the task. The teacher should develop procedures and/or requirements for group work. As students

engage in classroom discourse, they may be provided with prompts such as:

- Would you explain your thinking more about this?
- Why did you place your fraction on that part of the mat?
- I respectfully disagree . . .

2. Establishing Groups of Two Students

Where to start?

This activity is best addressed by pairs of students. The teacher needs to take into consideration student skill levels, leadership skills, and personalities when creating small groups. Small group collaboration works best when students have been provided previous opportunities to work together on a regular basis.

Through weeks of teacher observation and making note of leadership skills, personalities, the ability to take criticism, to question, and think deeply about a task or problem will prove to be extremely helpful when creating small groups for this lesson and associated tasks. When creating the groups, all these factors help to eliminate the potential situation in which one student takes the lead and makes the decisions for the group. When you have this situation, one student is gaining all the benefits of the task, while others do not. Both students need to collaborate throughout the task and engage in a meaningful discussion about the mathematics involved.

3. Develop Open-Ended Questions

The teacher should create a list of open-ended questions designed to support and scaffold the introduction and the student reflection. These questions should purposefully direct students towards provided information, previously learned content, and similarities and differences in their explanations and conceptual understandings of fractions. Some questions might include:

- How could you compare your fraction into a benchmark?
- How can we compare or represent different quantities of fractions?
- What about the fractions can you compare?
- What about the fractions are you comparing?
- What did you do in this activity that was similar with the fractions? Different?
- Is there are strategy or method of comparison that is more efficient? Why or why not?
- Why is it important to compare and order fractions in this activity using benchmark numbers (zero, one-half, one)?

4. Gather Materials

- Copies of Task – Comparing Fractions Using Benchmarks student view – (click here)
- Copies of Fraction Cards No Pictures
- Copies of Fraction Cards with Pictures
- Copies of Benchmark Fraction Mat
- Paper/Pencil
- Cuisenaire Rods (optional)
- Number line (optional)
- Fraction bars (optional)
- Yarn (optional for group discussion)
- 3 Post-it notes with benchmark fractions – 0, ½, 1 written on the sticky notes (optional for the group discussion)
- Plastic baggies for the fractional activity pieces (optional)

Preorganization of the materials needed to complete this activity will eliminate delays, disruptions, and confusion as students prepare to work. The teacher might have the items in a plastic tub or any container in the center of the group for easy access and reachable. Items such as sticky notes, fractional bars, yarn, Cuisenaire Rods, number line strips, and any other optional mathematical tools can be placed in a math tub or toolbox for ready access by students during the activity if needed.

Have the following materials organized and ready to distribute:

- Fractional cards are cut and placed in plastic baggies
- Mats are printed and may be laminated for easy use (1 mat per student pair)

*The teacher may determine or allow the student pairs to choose the set of fraction pieces to be used. Two sets of fractional cards are available – one set with pictures and one set without pictures. Students may also be permitted to transition from the set with pictures to the set without pictures as they activity proceeds. If student choice is allowed, the teacher needs to have enough sets of fractional card sets (with pictures and without pictures) available. If the teacher is determining which student pairs will have the support of pictures available, it would be wise to have extra sets of each set available for student pairs who are able to transition from cards with pictures to cards without pictures and for the possible student pair(s) who the teacher finds need the additional support as the activity unfolds.

5. Anticipated Common Student Misconceptions

- Students may not relate the fractional amount to a benchmark whole number. An example: "$\frac{1}{3}, \frac{2}{3}, \frac{3}{3}, \frac{4}{3}, \frac{5}{3}, 1$"
- Students do not look at the relationship between the size of the numerator and denominator when comparing fractions. Students misidentify a fraction as being larger than it is because the "bottom number is bigger." Example: Students place the fraction $\frac{1}{4}$ between the benchmark $\frac{1}{2}$ and 1 because the denominator 4 is bigger than 2 in their reasoning.
- Students who believe that the fraction $\frac{3}{4}$ would be placed on the number line between 3 and 4.
- Students who see numerators and denominators as separate numbers rather than thinking of the fraction as a single number.
- Video with information about misconceptions with fractions: https://youtu.be/0ZGNBME8v18

Explore Phase

Prior to using this task activity and the associated activities the following content might be reviewed (e.g. as warm-up activities):

- Opportunities to explain a comparison of fractions.
  Sample:
  Johnny tells his classmates the following explanation for why $\frac{1}{5} < \frac{2}{7}$:
  I can compare both $\frac{1}{5}$ and $\frac{2}{7}$ to $\frac{1}{4}$.
  Since $\frac{1}{5}$ and $\frac{1}{4}$ are unit fractions and fifths are smaller than fourths, I know that $\frac{1}{5} < \frac{1}{4}$. I also know that $\frac{1}{4}$ is the same as $\frac{2}{8}$, so $\frac{2}{7}$ is bigger than $\frac{1}{4}$.
  Therefore $\frac{1}{5} < \frac{2}{7}$
- Use the vocabulary as terms in a sentence for clarification:
  *Numerator, denominator: The math teacher drew a line and instructed the students to put the numerator first and then write the denominator underneath.

* Equivalent, $\frac{1}{2}$ is equivalent to $\frac{2}{4}$
* And the symbols: >, <. =.

Prior Instruction/Knowledge:

Basic Concepts:

- Grade 4 students learn a fundamental property of equivalent fractions: multiplying the numerator and denominator of a fraction by the same non-zero whole number results in a fraction that represents the same number as the original fraction. Students use visual fraction models, with attention to how the number and size of the parts differ even though the two fractions themselves are the same size (M.4.12). This property forms the basis for much of the work with fractions in grade four, including comparing, adding, and subtracting fractions and the introduction of finite decimals.
- Students apply their new understanding of equivalent fractions to compare two fractions with different numerators and different denominators (M.4.13). They compare fractions by using benchmark fractions and finding common denominators or common numerators. Students explain their reasoning and record their results using the symbols >, =, and <.
- Examples: Source: Comparing Fractions M.4.13 from WVDE's Educator's Guide for Mathematics)

Examples: Comparing Fractions	**M.4.13**
1. Students compare fractions to benchmark fractions—for example, comparing to $\frac{1}{2}$ when comparing $\frac{3}{8}$ and $\frac{2}{3}$. Students see that $\frac{3}{8} < \frac{4}{8}$ $(\frac{4}{8} = \frac{1}{2})$, and that since $\frac{2}{3} = \frac{4}{6}$ and $\frac{4}{6} > \frac{3}{6}$ $(\frac{3}{6} = \frac{1}{2})$, it must be true that $\frac{3}{8} < \frac{2}{3}$.	
2. Students compare $\frac{5}{8}$ and $\frac{7}{12}$ by writing them with a common denominator. They find that $\frac{5}{8} = \frac{5\times12}{8\times12} = \frac{60}{96}$ and $\frac{7}{12} = \frac{7\times8}{12\times8} = \frac{56}{96}$ and reason therefore that $\frac{5}{8} > \frac{7}{12}$. **Notice that students do not need to find the smallest common denominator for two fractions; any common denominator will work.**	

Please review the following:

Educators Guide for Mathematics: Grade 4 (pages 22- 31, pdf pages 24-40) (Click here)

Prerequisite Skills

- Represent fractions concretely and symbolically, including representing whole numbers as fractions.
- Compare fractions with the same numerator or denominator concretely and symbolically.

Supporting Skills

- Compare fractions with the same numerator or denominator concretely and symbolically**.**
- Locate points on a number line.
- Measure lengths in inches/centimeters using appropriate tools and units.

Impending Skills

- Model and identify mixed numbers and their equivalent fractions.
- Write equivalent fractions with smaller or larger denominators.
- Represent division of a unit fraction by a whole number or a whole number by a unit fraction using models to explain the process in number and word problems.

- Understand that the fractional relationships that occur between zero and one also occur between every two consecutive numbers.
- Organize, display, and interpret information in circle graphs.
- Round decimals to a given place value; round fractions and mixed numbers to a whole number or a given fractional place value.
- Indicate the probability of a chance event with or without models as certain, impossible, more likely, less likely, or neither likely nor unlikely using benchmark probabilities of 0, 1/2, and 1.

Source: *The Quantile Framework for Mathematics*
https://metametricsinc.com/educators/quantile-for-educators/

Implementation Phase

Part 1: Introduction of the Fraction Activity

- Create student pairs. Introduce a review of fractional terms and explain the fractional activity.
- Provide the student handouts with card pieces, fraction mat, and optional items the students need to determine the fraction's location on the mat (benchmark page). Check for Understanding about the task by demonstrating a sample play in the activity.

Part 1 Teacher Notes:

Preset Activities- Prepare materials for students: student handout with activity outlined, fractional cards (with or without images) may be cut and divided into plastic baggies for easy distribution, 1 benchmark mat for each group (may laminate or placed on card stock for durability).

- Fraction Cards No Picture
- Fraction Cards With Picture
- Benchmark Fraction Mat
- Differentiated Fraction Mats

Pair students up based on their needs. Provide any student or student pair needing scaffolding supports with the set of fraction cards with pictures. Provide all other student pairs with the fraction cards without pictures (see Note in Gather Materials section.)

Create a list of questions to support and scaffold the task for any struggling students. These may include:

- Is the numerator larger than the denominator?
- Is the numerator smaller than the denominator?
- What is half of the denominator? Is half of the denominator greater than, less than, or equal to the numerator?

Thinking starter for students: Today, we are going to apply what we know about fractions in the Comparing Fractions to Benchmarks Activity. The teacher will introduce the task and learning target to students. The target for the lesson is, "I can compare fractions using benchmark fractions."

Part 2: Comparing Fractions Using Benchmarks/Discussion of Observations

- Students begin the activity.
- Students reflect on understandings developed from activity.

Part 2 Teacher Notes:
Provide each student pair with a copy of the directions, a set of fraction cards, and a fraction benchmark. As a Check for Understanding after explaining the directions for the activity, in a whole-class format, play a round demonstrating the role of Student 1 and Student 2 in a round.

Directions:

- Both Student 1 and Student 2 select a card.
- Student 1 places his/her card in the appropriate category on the mat and explains why the card should be placed there.
- Student 2 agrees or disagrees with the placement.
- If the partners disagree, they discuss until an agreement is reached. If the partners agree, then Student 2 places his/her card in the appropriate category on the mat and explains why the card should be placed there.
- Student 1 agrees or disagrees with the placement.
- This process continued for 5 rounds with students recording their agreed-upon decisions for card placement.

As students complete the activity, the teacher should circulate and monitor conversations among the student pairs. If there are questions or disputes that the students cannot reconcile, the teacher should encourage students to draw number lines or models on their paper or prompt and redirect student thinking. If the teacher notices student pairs struggling with the fraction cards without the pictures, provide the student pair a set of cards with pictures. If student pairs using the fraction cards with pictures show understanding, challenge them with the set without pictures. The teacher should also record the variety of ways students approach the comparison of fractions and any students who may need reteaching.

For student groups finishing quickly, ask the student pair to select a fraction that they placed in each of the columns on their completed fraction mat and to record this on a class fraction mat. This class mat can be used to facilitate mathematical discourse and justifications at the conclusion of the activity.

After most students have completed five rounds, in a whole-class conversation, discuss the variety of ways students used to compare fractions. Encourage students to share the different methods they used to compare a pair of fractions.

The following Group Directions for Student Reflection may be displayed. :

- Select one fraction from your fraction cards and your activity sheet where you did your recordings of comparisons.
- You will tell us how you compared the fraction on the benchmark mat on the fraction benchmark mat on the board in front of the classroom.

As an extension, the class can decide upon the placement of the fraction on a **yarn number line***. Discuss a variety of methods for determining the placement of fractions on the mat. Methods may include options such as “changed fraction to an equivalent fraction,” “used a picture/visual model,” etc.

Effective classroom discourse and group reporting “can make the difference between students’ feeling that they are just going through their paces and the sense that they are engaged in a powerful exchange of ideas.” As a Check for Understanding, the teacher asks students for examples from their individual records to add to the class mat. As each student shares the placement of a fraction, he/she explains to the class the reasoning for the placement of the fraction on the mat. The teacher can use prepared

questions to extend student reasoning. These questions can help students continue building their background knowledge of how to compare fractions. Examples include:

- Is the numerator larger than the denominator?
- Is the numerator smaller than the denominator?
- What is half of the denominator? Is half the denominator greater than, less than, or equal to the numerator?

The questions should prompt students into revealing their understanding about relationships among fractions, their numerators, and their denominators, and their relationship to benchmarks as they made comparisons. Student reasoning may include:

- If the denominators are the same, then the fraction with the larger numerator is the greater fraction.
- If the denominators are the same, the fraction with the smaller numerator is the lesser fraction.
- If the denominators are the same and the numerators are equal, the fractions are equivalent.
- Use < or > or = to compare the two fractions.
- Since equivalent fractions do not always have the same numerator and denominator, one way to determine if two fractions are equivalent is to find a common denominator and rewrite each fraction with that denominator. Once the two fractions have the same denominator, it is easy to check to see if the numerators are equal and that the two fractions are equal as well.

***Teacher Preparation for Yarn Number Line:**
Using yarn, 3 sticky notes with a benchmark fraction of 0, ½, and 1 written on individual notes, make a number line. Students will use their fraction cards and lay their fractions out along the number line.

Share, Discuss, and Analyze Phase

Essential Understanding #1

The ability to determine appropriate benchmarks for fractions which are categorized with the benchmarks between 0, 1/2, and 1 using such terms as greater than, less than or equal to another fraction can strengthen the understanding of fractions.

Share- Students begin the lesson with an activity designed to provide practice and to reinforce their understanding of fractions. It allows the students to verbalize how they will determine the placement of a fraction card on the benchmark fraction mat.

Discuss- Students should use and understand vocabulary that includes terms such as numerator, denominator, benchmark fractions, equivalent fractions, greater than, less than, and equal to. The teacher prompts students to continue to use the appropriate terms as they explain their reasoning about the placement of the fraction on the fraction mat. Students demonstrate their understanding of fractions as numbers in this activity. Students expand their understanding of the number system beyond whole numbers.

Analyze- The teacher analyzes the discussion among student pairs and in whole-class discussion and appropriately provides some student pairs the opportunity to work with the differentiated fraction mats. These mats challenge students to work with broader benchmark ranges. Based on the students' understanding and knowledge, these mats can be used in progression. The range varies for the mats:

- Fraction Mat 1: Students identify fractions as greater than or less than 1.
- Fraction Mat 2: Students identify fractions as equivalent to 1/2 and 1.
- Fraction Mat 3: Students use the prior understandings of Fraction Mat 1 and 2 to identify fractions that fall within the benchmarks of 1/2 and 1.

Essential Understanding #2

The ability to explain the deep conceptual understanding of the represented fractions and reasons why the fractions are placed in a specific benchmark category form an important foundation for learning more advanced mathematics.

Share- The teacher stresses to students that their ability to justify or explain their comparison of fractions demonstrates a deeper level of knowledge and understanding. Using representations to compare fractions helps students develop number sense.

Discuss- After students place their fraction cards along the number line, they share and discuss their methods for comparing the fractions. Encourage students to share multiple methods for the comparing the fractions. Some students may compare fractions with the same numerator and other students may choose to compare fractions with the same denominator. In explaining their reasoning, students are internalizing their understanding of fractions and strengthening their number sense. Some students may need to have visuals to support their thinking.

Analyze- The teacher should look for/listen for important ideas as the students work. Students should understand that a fraction is greater than 1 if the numerator is larger than the denominator (i.e., $\frac{5}{4}$) and a fraction is less than 1 if the numerator is smaller than the denominator (i.e., $\frac{2}{3}$). A fraction is equal to one if the numerator is equal to denominator ($\frac{6}{6}$). Students might draw pictures or models of fractions to compare them with ½ and 1. The students may plot fractions the fractions on the number line to compare them with ½ and 1.

Essential Understanding #3

The ability to compare fractions can be used as an effective tool in addressing real-world situations.

Share- During the wrap-up, the teacher may discuss when students are doing mathematics, patterns emerge. These patterns support the students in making conjectures, supporting their reasoning, and proving mathematical claims and knowledge. The language of math might be confusing to the students and that the students must understand the concepts behind the language. Fractions are necessary because we use them in the real world every day. Fractions are used in many ways every day. By using real world examples of sports and food, students learn the importance of using fractions.

Discuss- Students see a variety of equivalent fractions, number patterns, unit fractions, and fraction cards greater than 1, less than $\frac{1}{2}$, greater than $\frac{1}{2}$. Questions to support student reasoning can include:

- What did you notice?
- What do you wonder?
- How did/could you look at this fraction differently?
- What strategy did you use to compare your fraction to the benchmark fractions?

The discussion should extend to real-world contexts with plausible numbers for situations that involve using or comparing fractions in recipes, sports, or even sharing cookies among friends.

Analyze: By the end of the lesson, students should understand and use vocabulary related to fractions to demonstrate their understanding of fractions. Student analysis of fractions allows them to appropriately place the fractions on the fraction mats. The teacher should carefully analyze students' use appropriate use of vocabulary as they explain their reasoning for placement of fractions on the fraction mat. Based on students' responses, the teacher may choose to extend the activity to provide students the opportunity to continue toward mastery of comparing fractions and of using benchmarks to compare fractions. Because rote memorization does not lead to long-term understanding of fractions, students need to internalize their understanding of concepts related to fractions.

Task In Action

The video clips below provide a demonstration of the task being implemented in a classroom as it aligns to the Effective Mathematics Teaching Practice indicated. These clips should be used by the teacher to model the implementation of the task in his or her own classroom.

- Establish Mathematical Goals to Focus Learning
  - Video Clip 1
  - Video Clip 2
  - Video Clip 3
  - Video Clip 4
- Implement Tasks that Promote Reasoning and Problem Solving
  - Video Clip 1
  - Video Clip 2
  - Video Clip 3
  - Video Clip 4
  - Video Clip 5
- Use and Connect Mathematical Representations
  - Video Clip 1
  - Video Clip 2
- Facilitate Meaningful Mathematical Discourse
  - Video Clip
- Pose Purposeful Questions
  - Video Clip 1
  - Video Clip 2
- Build Procedural Fluency from Conceptual Understanding
  - Video Clip 1
  - Video Clip 2
  - Video Clip 3
  - Video Clip 4
  - Video Clip 5
- Support Productive Struggle in Learning Mathematics
  - Video Clip 1
  - Video Clip 2
  - Video Clip 3
  - Video Clip 4
  - Video Clip 5
- Elicit and Use Evidence of Student Learning
  - Video Clip

Resources

1. Introduction Clip or Reteaching about Benchmark Fractions: https://youtu.be/vJXkxu7JIOo
2. Equivalent Fraction Song: https://youtu.be/vKXqzpz-G0s
3. Reteaching Clip: https://youtu.be/LCggbGgvQdM United States Dept. of Education
4. Online Fluency with Comparing Fractions: https://www.brainpop.com/games/battleshipnumberline/