

**WEST VIRGINIA  
DEPARTMENT OF EDUCATION**



# MATHEMATICS

**GRADE 6**

***Fitting Boxes into Boxes***

# Table of Contents

<b>Rationale for Lesson and Associated Tasks</b>	<b>Page 1</b>
<b>Lesson and Associated Tasks Overview</b>	<b>Pages 1 - 2</b>
<b>West Virginia College-and Career-Readiness Standards</b>	<b>Page 2</b>
<b>Mathematical Habits of Mind (MHM)</b>	<b>Page 2</b>
<b>Mathematics Teaching Practices to Support Student Growth</b>	<b>Page 2</b>
<b>Essential Understandings</b>	<b>Page 2</b>
<b>Set-up Phase</b>	<b>Page 3</b>
<b>Establish Small Groups</b>	<b>Pages 3 - 4</b>
<b>Develop Open-Ended Questions</b>	<b>Page 4</b>
<b>Gather Materials</b>	<b>Page 4</b>
<b>Anticipated Common Student Misconceptions</b>	<b>Pages 4 - 5</b>
<b>Explore Phase</b>	<b>Page 5</b>
<b>Prior Instruction/Knowledge</b>	<b>Pages 5 - 6</b>
<b>Implementation Phase</b>	<b>Pages 6 - 10</b>
<b>Share, Discuss and Analyze Phase</b>	<b>Pages 10 - 12</b>
<b>Task in Action</b>	<b>Page 12</b>



**Task Title:** *Fitting Boxes into Boxes*

**Grade or Content Area:** 6th Grade

**Toolkit Author:** Mary Scott, Nada Waddell, and Rachel Moon

**Original Task Creator:** Illustrative Math

**Quarter:** 1

### **Rationale for Lesson and Associated Tasks**

A local artist who is in business for himself received a major order from a well-known brand department store. The order is for 270 necklaces. Since he is a businessman, the artist wants to ship the necklaces as cheaply as possible in order to make a profit. The artist packs each necklace in a jewelry box with the dimensions of  $1\frac{3}{4}$  inches by  $2\frac{1}{4}$  inches by  $\frac{3}{4}$  inch. He needs to find the cheapest way to ship the 270 jewelry boxes in flat-rate containers. The postal service has small, medium, and large mailing boxes with different costs of shipping. Yet, he wonders which size of mailing boxes would be the least expensive way to mail the 270 jewelry boxes to the department store. Which one of the flat rate boxes should the artist use to minimize or lower his cost of shipping the jewelry?

The associated tasks with this lesson provide the students the opportunity to multiply and divide fractions, convert mixed numbers to improper fractions, and apply the formula

$V = \text{Length} \times \text{Width} \times \text{Height}$  to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems. This task also requires students to seek understanding, analyze information, and look for measurements of rectangular prisms to choose by using deductive reasoning.

### **Lesson and Associated Tasks Overview**

*Fitting Boxes into Boxes*

Teacher Plan: ([click here](#))

Student Plan: ([click here](#))

For Grade 6 students, this lesson and associated tasks may best serve as an inquiry style introduction to finding and comparing the costs of boxes after students determine unit volumes for various options of boxes. The task can serve to connect students' prior knowledge with determining the volume of rectangular prisms with whole-number measurements by applying the formulas  $V = lwh$  and  $V = Bh$  to find the volume to rectangular prisms with fractional measurements.

This lesson and associated tasks are scheduled to be completed over 2 or more class periods per the following suggested sequence:

#### **Day 1**

- Introduce the *Fitting Boxes Into Boxes* task to students (whole class)
- Provide students with the *Fitting Boxes Into Boxes* task ([click here](#))
- Provide students with the USPS flat rate box information
- Complete plans for determining the most economical shipping box combination

#### **Day 2**

- Present plans for determining the most economical shipping box combination
- Evaluate the volume of all shipping boxes

- Evaluate all shipping boxes for the number of jewelry boxes each will hold
- Determine the most economical shipping box combination

### Day 3

- Revisit mathematical calculations
- Develop a group display/drawing with the following: the selected shipping box (l, w, h labeled), the jewelry box (l, w, h labeled), all mathematical calculations, the final shipping cost.
- Gallery Walk using the *I Wonder...I Notice Protocol*

### West Virginia College-and-Career-Readiness State Standard

**Cluster: Apply and extend previous understandings of multiplication and division to divide fractions by fractions.**

#### M.6.4

Interpret and compute quotients of fractions and solve word problems involving division of fractions by fractions by using visual fraction models and equations to represent the problem. (e.g., Create a story context for  $(2/3) \div (3/4)$  and use a visual fraction model to show the quotient; use the relationship between multiplication and division to explain that  $(2/3) \div (3/4) = 8/9$  because  $3/4$  of  $8/9$  is  $2/3$ . (In general,  $(a/b) \div (c/d) = ad/bc$ .) How much chocolate will each person get if 3 people share  $1/2$  lb. of chocolate equally? How many  $3/4$ -cup servings are in  $2/3$  of a cup of yogurt? How wide is a rectangular strip of land with length  $3/4$  mi and area  $1/2$  square mi?)

**Cluster: Solve real-world and mathematical problems involving area, surface area, and volume.**

#### M.6.22

Find the volume of a right rectangular prism with fractional edge lengths by packing it with unit cubes of the appropriate unit fraction edge lengths and show that the volume is the same as would be found by multiplying the edge lengths of the prism. Apply the formulas  $V = lwh$  and  $V = Bh$  to find volumes of right rectangular prisms with fractional edge lengths in the context of solving real-world and mathematical problems.

### Mathematical Habits of Mind (MHM)

\*While several MHM are listed, the one MHM in bold font is the focus of the lesson and associated tasks.

MHM1. Make sense of problems and persevere in solving them.

**MHM3. Construct viable arguments and critique the reasoning of others.**

**MHM4. Model with mathematics.**

### Mathematics Teaching Practices to Support Student Growth

- Implement tasks that promote reasoning and problem solving.
- Facilitate meaningful mathematical discourse.
- Support productive struggle in learning mathematics.

### Essential Understandings

- The ability to find the volume of a shape with whole-number measurements can be extended to shapes with fractional measurements.
- The ability to compute with fractions and interpret the results is an effective tool in addressing real-world contexts.

## Set-up Phase

### 1. Become an Expert Regarding All Lesson and Associated Task Content

It is essential that the teacher become very familiar with all *Fitting Boxes into Boxes* materials (e.g. Grade 6 Lesson Plan, Lesson PowerPoint, student handout, and completion rubrics). This lesson comes with many associated tasks. The use of each task may vary class to class depending on the time frame available for the lesson and the student levels of engagement and understanding. Reviewing, analyzing, and completing all tasks prior to the implementation with students is imperative to the success of this lesson. Breakdown of items is as follows:

- The teacher should obtain flat-rate shipping boxes from the US Post Office (they are free!) for students to use as a reference. Free flat-rate shipping boxes can be ordered online at [https://store.usps.com/store/results/shipping-supplies/flat-rate/\\_/N-7d0v8vZ1wrlxch](https://store.usps.com/store/results/shipping-supplies/flat-rate/_/N-7d0v8vZ1wrlxch)
- The teacher reads through the task several times to identify questions students may have or areas in which they may struggle.
- The teacher works through the task to get a better appreciation of the activities the students are being asked to complete. This allows the teacher to gain insight into the different ways students may approach the task to develop their solution.

### 2. Establish Small Groups\*

The *Fitting Boxes into Boxes* activity is best completed in a small group arrangement of three or four students. In forming these groups, the teacher should take in consideration student skill levels, leadership skills, and personalities when creating small groups (e.g. three students). Small group collaboration works best when students have been provided previous opportunities to work together on a regular basis. Teacher observations of students' leadership skills, personalities, the ability to take criticism, to question, and to think deeply about a task or problem will prove to be extremely helpful when creating small groups for this lesson and associated tasks. When creating the groups, all these factors help to eliminate the potential situation in which one student takes the lead and makes the decisions for the group. In this situation, one student is gaining all the benefits of the task, while others do not. If a student is not engaged in conversation, this lesson and associated tasks will not be beneficial in helping all students to have meaningful discussions about the mathematics involved nor in analyzing the relationships inherent to the tasks. Students will be working both individually and together when placed in small groups (e.g. three students) to complete this lesson and associated tasks. If students have not previously developed standards for group work, provide students with an opportunity to collaborate to develop guidelines to help them stay focused during tasks and group work assignments. The teacher should post these standards in the classroom and reference them prior to group task activities.

- One way to arrange groups prior to implementing the problem task is provided below:
  - The Wheel Decide website <https://wheeldecide.com/> is a wonderful tool to create student groups.
  - One option would be to group students with similar abilities on the same wheel so that a diverse group of students and student strengths are represented within each group. For example, the teacher can place the names of students on each wheel so that each of the created groups has a student who is a leader within the class as well as a student who often struggles with content.
  - Students who should not work together should be placed on the same wheel.
  - During class, the teacher can spin the wheels to “randomly” create student groups.

If random selection is not appropriate, the teacher can form groups based upon his/her knowledge of the students. Whatever the method to decide the groups, remember, effective group work takes a lot of scaffolding. Do not expect students to know how to divide up the work on their own. Working together to break down and delegate responsibilities is one of the most challenging tasks for any group. Breaking down tasks ahead of time models for students how it can be done. Over time, consider transferring some of this responsibility to them.

- Possible group roles can include: Recorder, Material/Resource Person, and Presenter. In a group of four students, one of the students may be given the role of timekeeper. This role would include making sure everyone in the group is on task. In groups of three students, the group itself would decide who would be the timekeeper.

\* Optional: Rubric for Group Work

### 3. Develop Open-Ended Questions

The teacher should create a list of open-ended questions designed to support and scaffold the lesson and the associated tasks for the students. These questions should purposefully direct students towards provided information, previously learned content, and similarities and differences in their work among the group members. Some questions might include the following:

- What are we trying to determine?
- What information was provided?
- When using the information provided, what important details could help?
- How would you solve this problem? How could you find \_\_\_\_\_?
- What could you do to compare the boxes?
- What about the boxes could you compare?
- What options did you use to compare the boxes?
- In comparing the boxes, what did you do that was similar? Different?
- Is there a more efficient strategy? Why or why not?

### 4. Gather Materials

Preorganization of the supplies will eliminate delays, disruptions, and confusion as students prepare to work. The teacher might have the items in a plastic tub or any container in the center of the group for easy access. The materials in the math toolbox would include a variety to use while students work:

- Rulers and measuring tapes
- Tracing paper, graph paper, colored pencils, scissors, markers, and pens
- Copies of task statements (Student Plan Handout)
- Samples of USPS flat-rate boxes OR images and dimensions of these boxes
- Samples of jewelry boxes or a facsimile to represent the small box container where the artist would place one item of jewelry (Preferably a mock model of the dimensions  $1 \frac{3}{4}$  inches by  $2 \frac{1}{4}$  inches by  $\frac{3}{4}$  inch. If not, a small sample box will do to represent this part of the scenario.)
- Calculators (optional)
- Chart Paper (plain), interactive presentation board or whiteboard
- White poster paper or sticky chart paper (large) for students in groups to post their results
- Rubric for Group Work (optional)

### 5. Anticipated Common Student Misconceptions

- Students may solve for the surface area of the box containers instead of volume.
- Students may be unfamiliar with a jewelry box. The jewelry box in the problem is comparable to a small gift box. In this scenario, it is not referring to the storage case for keeping a person's

collection of necklaces, rings, or other items. The teacher may need to clarify the meaning of this term.

- When dividing mixed numbers, students may fail to first write them as improper fractions. Prompt students to look in notes (if available) or ask group members for ideas on how to correctly divide mixed numbers.
- Students may use the strategy of finding the volume of the shipping boxes and dividing it by the volume of the jewelry box to determine the number of jewelry boxes that it will hold. This strategy will result in large numerators which can lead to simple arithmetic errors. Encourage the students to round and estimate in order to determine if their answers seem logical.

### Explore Phase

Prior to introducing this the lesson, the following content should be reviewed (e.g. as “bell ringers”):

- Skills and understandings about a variety of methods to compute the quotient of fractions, including tape diagrams and the standard algorithm.
- Skills and understandings in computing the volume of right rectangular prisms, including those with fractional edge lengths. Students should be familiar with using the formulas length x width x height and area of base x height to determine the volume.
- Familiarity with the commutative property of multiplication ( $B \times h$  or  $h \times B$ ) allows students to appreciate that, regardless of how the box lies, its volume remains the same.

### Prior Instruction/Knowledge

During Grade 6, students demonstrate fluency in addition, subtraction, multiplication, and division with positive rational numbers, including fractions. Students also extend their understanding of length, area, and volume as they solve problems by applying formulas for the volume of rectangular prisms.

**Please review the following:**

Educators Guide for Mathematics: Grade 6 (pages 18-22; pdf 42-46) ([click here](#))

### Prerequisite Skills

- Calculate the areas of triangles, parallelograms, trapezoids, circles, and composite figures in number and word problems.
- Evaluate algebraic expressions in number and word problems.
- Model the concept of the volume of a solid figure using cubic units.
- Model and identify mixed numbers and their equivalent fractions.
- Identify additive inverses (opposites) and multiplicative inverses (reciprocals) and use them to solve number and word problems.
- Write equivalent fractions with smaller or larger denominators.
- Represent division of a unit fraction by a whole number or a whole number by a unit fraction using models to explain the process in number and word problems.

### Supporting Skills

- Use exponential notation and repeated multiplication to describe and simplify exponential expressions.
- Describe the effect of operations on size and order of numbers.
- Find multiples, common multiples, and the least common multiple of numbers; explain.
- Multiply two fractions or a fraction and a whole number in number and word problems.

## Impending Skills

- Calculate the volume of cylinders, pyramids, and cones in number and word problems.
- Use models to investigate the relationship of the volume of a cone to a cylinder and a pyramid to a prism with the same base and height.
- Determine the volume of composite figures in number and word problems.
- Divide two fractions or a fraction and a whole number in number or word problems.

**Source:** *The Quantile Framework for Mathematics*

<https://metametricsinc.com/educators/quantile-for-educators/>

2020 MetaMetrics Inc.

## Implementation Phase

### Day 1

- Introduce the scenario to the student groups. Pose the question: Which one of the flat rate boxes should the artist use to minimize or lower his cost of shipping the jewelry?
- Provide the Student Plan handout and in a whole group format ask students to develop a list of four given facts and one piece of additional information that is needed to solve this scenario. (Student Handout, Part 1, Question 1)
- Walk around the room to get an overview of the ideas or questions the students are generating.
- Check for understanding asking each group to share from its group's lists. As each group provides a fact or an additional needed piece of information, record responses until all groups agree that the question has been thoroughly addressed.
- Review the compiled list of facts and needed additional information. Discuss these lists to help students make sense of the task and determine what they need to know and do to find the most economical shipping box combination.
- Provide student groups an opportunity to develop a plan to find the most economical way to ship the necklaces. As student groups collaborate to address the task, take note of the plans developed by each group.
- Provide an opportunity for student groups to share, in a whole-class format, different plans for determining the most economical way to ship the necklaces.

### Day 1 Teacher Notes:

#### Preset Activities

During Day 1, students make sense of the task, outline what they need to know and do to address the question, and to design a plan.

1. With students arranged in their pre-designed groups, display the shipping boxes and a jewelry box (or images of these) to students.
2. After introducing the task, provide students time to grapple with the problem. Instruct them that they will be making several decisions about how to address the task. It is important that each member of their group be in agreement about the rationale behind the decisions.
3. Be available by walking around the room (students tend to ask questions if the teacher is nearby), but do not interfere with group discussions, unless asked.
4. Refer to the list of questions given previously in this document.
5. Some teams may need some gentle nudging if they seem to not have any idea of how to begin. Other teams may ask questions. Clarify without "telling", and answer with another question.



6. Look for opportunities to clarify vocabulary.
7. Lead students to recognize that they will need to do the following:
  - Find out the measurements of the shipping boxes.
  - Find out the measurements of the jewelry box.
  - Find the costs for mailing a shipping box of each size. This information is not found in the student handout. Two options for providing this information, each supporting a Mathematical Habit of Mind, are as follows:
    - Provide the information only when students determine the need. This option supports MHM1: Make sense of problems and persevere in solving them.
    - Ask students to find out this information themselves. This option supports both MHM1 and the modeling demand (MHM4: Model with mathematics).
  - Decide on an orientation for the jewelry boxes that would fit inside each shipping box.
  - Calculate how many jewelry boxes will fit in the shipping box with that particular orientation.
  - Test out different orientations and how they affect the number of jewelry boxes that will fit and the cost.

At this point, it is important to reconvene as a class. Ask each group to share a couple of specific steps they have taken toward answering the question and a couple of steps they plan on taking to move forward. Highlight any ideas students might have about making the problem-solving process more efficient and systematic. If not already mentioned by students, remind each group that each person is responsible for completing the calculations for one shipping box.

## **Day 2**

- Explain to students, in a whole-class format, that each member of their group will be working with a different shipping box.
- Read and discuss the two questions that students will address. Students will need to find the number of jewelry boxes that can fit into their assigned box. Students will need to determine the total cost of shipping all for the jewelry boxes in their assigned box.
- Make sure that students understand that they need to document their work for each question. Students will use this work as they explain their reasoning to others.
- Encourage students in the same group to collaborate by sharing their process and answering questions posed by others in their group.

### **Day 2 Teacher Notes:**

During Day 2, students model the problem, calculate the number of jewelry boxes that fit inside each shipping box, and determine the associated costs. Students experiment with different orientations for the jewelry boxes to optimize space and minimize cost.

1. As students implement their plan to minimize cost, continue to be available by walking around the room.
2. As students independently work to determine how many jewelry boxes will fit into their assigned shipping box and to calculate the total cost of shipping all of the necklaces using that type of box, encourage them to share their reasoning with the members of their group.
3. Circulate throughout the room, remembering that students tend to ask questions if the teacher is nearby. Do not interject into group discussions, unless asked. It will also be important to listen for trends that are emerging from the discussions so that you can refer to them during the subsequent

whole group discussion. Some teams may need some gentle nudging; other teams may ask questions. Continue to clarify without “telling”, and answer with another question. Possible prompts might include:

- What is the shape of your shipping boxes? How can you find the number of boxes that will fit inside? Is there a mathematical term for filling a shape? How can you find the volume of this shape?
- Is there a way to determine how many lengths of a jewelry box would fit into the length of the shipping box?
- How can you determine how many widths of the jewelry box would fit into the width of the shipping box?
- How can you determine how many heights of the jewelry box would fit inside of the shipping box?
- For students who are struggling, prompt them to draw jewelry boxes onto the shipping box. How many layers of jewelry boxes would you be able to fit in the shipping box?

Consider recording the number of jewelry boxes that the members of each group fit into each of the three shipping boxes. Students may find the differences in their determinations for the same shipping box interesting. As an extension to this task, consider providing an opportunity for each of the students who worked with a specific shipping box to form a new group to engage in a conversation. Pair students who worked on the same shipping box but whose findings differ to explain their strategies. As students compare and contrast their work, they are provided an opportunity to “construct viable arguments and critique the reasoning of others.” (MHM3) (An opportunity for this extension is provided in the Conclusion to this task.)

### Day 3

- Students in each group share and compare their results. Using their documented work (see Day 2), each student in the group explains his work.
- As each student shares, the other members of the group verify his reasoning and computation. Each student, when necessary, revises his work.
- If the idea of using a variety of boxes has not arisen in classroom discussion, present this possibility to the class. Ask students to explore using more than one size of box to see if they can minimize the shipping costs. The following questions can be used to guide conversation and students’ work:
  - Does the arrangement of the jewelry boxes inside of the shipping box affect the cost of shipping with each shipping box?
  - Could you or your group think of ways to reduce the amount of wasted space when shipping exactly 270 jewelry boxes?
  - Does the solution have to be only one size of the shipping boxes?
  - Did some of the shipping boxes have more wasted space than others? Why might this happen?
  - Could you use diagrams or picture representations to show and compare the unused spaces in different combinations of shipping boxes?
  - Is there an arrangement that would increase the number of jewelry boxes that would fit into a shipping box? What would this arrangement be?
  - After each group completes its collaboration and decides upon a combination of boxes to use to ship the jewelry boxes most economically, provide the group with a piece of chart

paper. On the chart paper, each group documents its findings, work, and reasoning for the most economical arrangement.

### Day 3 Teacher Notes

During Day 3, students present, reflect, and discuss. They explain their strategies and reasoning to each other. Students evaluate and make decisions about how to best fit all the jewelry boxes into a combination of jewelry boxes in order to determine the most economical arrangement. As a class, students reflect on how the orientation of the jewelry boxes and the size of the shipping boxes affect the shipping cost.

1. As the members of each group are presenting their conclusions and sharing their reasoning and their work, continue to circulate throughout the room, monitoring the conversations and group work.
2. Look for different methods or approaches to highlight among group conclusions and solutions.
3. Take care to note any misconceptions or inaccurate responses need to be clarified and corrected either by the teacher or by other students.

### Conclusion

- Monitor each group's readiness to report out their findings.
- Prepare an extension or alternate activity for student groups finishing quickly.
- After all groups have completed their poster, allow students to take part in an initial "Gallery Walk" to see the variety of solutions. (Each group's solution is written on large chart paper.) A video explanation of this technique can be found at <https://youtu.be/vablnDnJZ80>
- Initiate a group by group report from their findings.
- Have students "Gallery Walk" with sticky notes to post comments or ask questions about different groups' posted work. Students rotate around the room, studying each group's poster and leaving sticky notes with any questions or comments they might have. The comments will be in 'I Wonder...I Notice' format.
- In a class discussion, compare the variety of student results. Address the comments and questions posted during the "Gallery Walk".

### Conclusion Teacher Notes:

Because the student groups may complete their work at different times, this would be an opportune time to introduce the Extension previously mentioned. On Day 2 of the task, one student from each of the different groups worked with the same shipping box. Because these students may have arrived at a different number of jewelry boxes that could fit in the shipping box, they may find it interesting to compare their different approaches to the problem.

1. As student groups complete their poster, pair students who worked on the same shipping box to compare their findings from Day 2.
2. These conversations provide value to variety of student work and independent thinking. The conversations also provide an opportunity to "construct viable arguments and critique the reasoning of others." (MHM3).
3. As each group presents their findings, it is important to honor the group's solution and effort. If necessary, prompt the group to explain its decision-making, reasoning, and computation. This will provide an opportunity to have students explain their understanding of volume and their method(s) of finding volume.

4. Prompt students to explain their work with fractions, and how they decided which operation (multiplication or division) to use as they addressed the task. Effective group reporting “can make the difference between students’ feeling that they are just going through their paces and the sense that they are engaged in a powerful exchange of ideas.”

### **Share, Discuss, and Analyze Phase**

#### **Essential Understanding #1:**

*The ability to compute with fractions and interpret the results is an effective tool in addressing real-world contexts.*

**Share** - As students recognize the idea of filling a larger box with a number of smaller boxes is related to their understanding of volume, students’ prior experience with volume should prompt the use of the formulas  $V = l \times w \times h$  and  $V = B \times h$ . Students also determine the number of shipping boxes needed to hold all of the jewelry boxes.

**Discuss** – Using models and questions such as those noted below, the teacher can lead students to understand the relationship between filling the shipping box with jewelry boxes and finding volume. The teacher may also need to lead students to connect the concept of splitting the shipping box into jewelry-box sized sections with division.

- How can we determine how many jewelry boxes would fit inside a shipping box?
- What is the shape of the jewelry box?
- What is the shape of a shipping box?

As the students discuss finding volume, listen to their conversations. Do they recognize the measurements that are needed? Are they accurately multiplying the mixed numbers to determine volume? Are they accurately dividing the mixed numbers to determine the number of jewelry boxes that will fit into each shipping box?

**Analyze** - It is important for the teacher to highlight the conversations that were heard and provide students an opportunity to explain their results. When rounding was necessary, did students appreciate the need to determine the number of “whole jewelry boxes” that fill each shipping box? In circulating through the student groups, were student misconceptions related to operations with fractions uncovered? Did students believe it necessary to find a common denominator in multiplying or dividing fractions? Did students find writing an equivalent improper fraction for the mixed numbers challenging? Did students remember that division is the inverse of multiplication and, as a result, understand that they should multiply by the reciprocal? Provide opportunities to address these misconceptions and possibly uncover additional misconceptions. As individuals share their work in multiplying or dividing mixed numbers, provide students an opportunity to compare their own computation to these models.

#### **Essential Understanding #2:**

*The ability to find the volume of a shape with whole-number measurements can be extended to shapes with fractional measurements.*

**Share** - The lesson opens with students challenged to find the most economical method to ship a quantity of jewelry boxes. Students collaborate to design a plan to address the question. They model the problem and calculate the number of jewelry boxes that fit inside of a variety of shipping boxes. To find a successful solution, students experiment with different orientations of the jewelry boxes to optimize space and minimize cost.

**Discuss** - Student teams work together and should discuss the following as they outline a plan:

- To answer the question, what would we need to determine?
- What information was provided?

- How could we use this information?
- Is there any other information that would be helpful?
- How can we determine how many jewelry boxes would fit inside a shipping box?
- How can we determine how many shipping boxes are needed?
- What is the shape of the jewelry box?
- What is the shape of a shipping box?

As students work to develop a plan, they may not appreciate the many aspects of the problem that need to be considered. In asking the above questions, the teacher can lead students to understand the relationship between the important aspects of the problem.

**Analyze** - As the problem is presented to the students, the teacher should not highlight that the situation can be analyzed by finding the volume of the boxes. Through discussion, students should be led to recognize that they can create and use models and that they can fill a larger box with a number of smaller boxes. The teacher can lead students to appreciate that this relationship is related to their prior experience in finding volume. In making these connections, students come to realize that they can apply their knowledge and skills about volume to address the problem:

- They know that finding the number of boxes that will fill a shipping box is related to finding the volume of the shipping box.
- They can find the volume of a shipping box or a jewelry box (right rectangular prisms) using the formulas  $V = lwh$  and  $V = Bh$ .
- They can find the number of jewelry boxes that will fit into each of the different shipping boxes by dividing into sections, each the size of a jewelry box; i.e. we can divide the total volume of a shipping box by the volume of a jewelry box.
- They can determine the number of each of the different sized shipping boxes needed to hold 270 jewelry boxes.
- They can find the total cost of the needed number of different sized shipping boxes.
- They can compare the total cost for the different sized shipping boxes to determine which would be the most economical.

**Discuss** - After students develop a method to compare the total cost when they use just one specific size of shipping box, students should be led to consider using a combination of shipping boxes. In asking the questions such as those found below, the teacher can lead students to explore the option of using a combination of different sized shipping boxes to minimize costs.

- What ways could an individual or a group think of ways to reduce the amount of wasted space when shipping exactly 270 jewelry boxes?
- Did some of the shipping boxes have more wasted space than others?
- Does the solution have to be only one size of the shipping boxes?
- Could diagrams or picture representations be used to show and compare the unused spaces in different combinations of shipping boxes?

**Analyze** - Through discussion, students can be led to develop an organized method of analyzing the cost of different combinations of shipping boxes:

- They can completely fill as many of the large postal boxes with jewelry boxes as possible and then determine the number of jewelry boxes that remain.
- They can split these remaining jewelry boxes into combinations of the mid-size and small shipping boxes.
- They can find the total cost of these different combinations of shipping boxes for the total 270 jewelry boxes.
- They can compare the total costs of these arrangements to find the most economical method.

- They can complete the above reasoning and process with the mid-size postal boxes and compare the total costs of these arrangements to find the most economical method.
- They can analyze total costs for the different combinations of shipping boxes to find the most economical method.

### **Task In Action**

The video clips below provide a demonstration of the task being implemented in a classroom as it aligns with the Effective Mathematics Teaching Practice indicated. These clips should be used by the teacher to model the implementation of the task in his or her classroom.

- Establish Mathematics Goals to Focus Learning:
  - [Video Clip #1](#)
  - [Video Clip #2](#)
- Implement Tasks That Promote Reasoning and Problem Solving:
  - [Video Clip #3](#)
- Use and Connect Mathematical Representations:
  - [Video Clip #4](#)
- Facilitate Meaningful Mathematical Discourse:
  - [Video Clip #5](#)
- Pose Purposeful Questions:
  - [Video Clip #6](#)
- Build Procedural Fluency from Conceptual Understanding:
  - [Video Clip #7](#)
- Support Productive Struggle in Learning Mathematics:
  - [Video Clip #8](#)
- Elicit and Use Evidence of Student Thinking:
  - [Video Clip #9](#)