WEST VIRGINIA
DEPARTMENT OF EDUCATION

MATHEMATICS

GRADE 4

Plastic Building Blocks

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Task Title: Plastic Building Blocks
Grade or Content Area: 4th
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Quarter: 2nd

Rationale for Lesson and Associated Tasks

Two young boys, Dennis and Cody, are planning to build a castle out of plastic building blocks. This task applies mathematics to a child's setting and allows students to apply the math associated with adding mixed numbers with like denominators. It provides an opportunity for the student to illustrate a variety of approaches in addressing the challenge. There are generally two approaches commonly used: calculating precisely how many buckets of blocks the boys have and comparing the provided numbers to benchmark numbers.

Associated tasks expand beyond these two approaches and allow for a variety of ways to find the mathematical solution, such as adding the whole numbers together, then the fractions together, and combining the two. The task challenges the students to think of other solution pathways, such as finding equivalent fractions for the mixed numbers and then add the fractions. The task provides a platform for the students to transition among strategies in finding a solution.

In completing this task, students demonstrate their skills in adding mixed numbers with like denominators, making sense of a problem and solving it, and describing and modeling the strategy used. Students create an argument for their reasoning and critique the reasoning of others' strategies.

Lesson and Associated Tasks Overview

Task: Dennis and Cody are building a castle out of plastic building blocks. They will need $2\frac{1}{2}$ buckets of blocks for the castle they have in mind. Dennis used to have two full buckets of blocks but lost some and now has $1\frac{3}{4}$ buckets. Cody used to have two full buckets of blocks too, but now has $1\frac{1}{2}$ buckets. If Dennis and Cody combine their buckets of blocks, will they have enough to build their castle?

Task: (click here)

This lesson and associated tasks may be best addressed by student pairs. Students use mixed numbers with like denominators to model situations. They use mathematical strategies to solve a real-world task. The task can serve to connect students' prior knowledge regarding fractions represented in models, equations, or mental math solutions.

Part 1: Introduction of the Task

- Introduction of the task challenge (whole class).
- Distribute materials to complete the task.
- Check for Understanding.

Part 2: Task Activity

- Students begin to work on the task and searching for different methods to solve the challenge.
- Students prepare to present their solution to the task.

Part 3: Class Discussion and Analysis

- Presentation of several different student solutions.
- Whole class discussion and analysis of student solutions that highlight different approaches.
- Compare the mathematical ideas and strategies used in addressing the challenge.
- Student pairs create a fraction challenge to present to challenge another student pair.

Part 4: Application of Knowledge

Student's Task: Write your own real-world problem that will require another student to add mixed numbers with like denominators.

- Student pairs create and solve a problem similar to the task.
- Student pairs solve each other's problems.
- In a whole-class discussion, students share and compare their solution pathways for two or three of the student-created problems.
- In a whole-class discussion, students note the similarities and differences in the problems.

West Virginia College- and Career-Readiness State Standard

M 4.14

Add and subtract mixed numbers with like denominators by replacing mixed numbers with an equivalent fraction and/or by using properties of operations and the relationship between addition and subtraction.

Mathematical Habits of Mind (MHM)

1. Make sense of problems and persevere in solving them.
2. Reason abstractly and quantitatively.
3. Construct viable arguments and critique the reasoning of others.
4. Model with mathematics.
6. Attend to precision.

Mathematics Teaching Practices to Support Student Growth

- Implement tasks that promote reasoning and problem solving
- Facilitate meaningful mathematical discourse.
- Support productive struggle in learning mathematics.
- Pose purposeful questions
- Build procedural fluency from conceptual understanding.
- Elicit and use evidence of student thinking.

Essential Understandings

- An understanding of decomposing fractions, estimating with benchmark fractions, modeling with mixed numbers, and converting mixed numbers to equivalent fractions can serve as an effective tool in addressing real-world contexts.

Set-up Phase

1. **Become an Expert Regarding All Lesson and Associated Task Content**

   It is essential that the teacher complete the activity prior to classroom implementation to understand how the activity works and clearly appreciate the mathematics embedded in the format. Careful reading of the task will allow the teacher to identify questions students may have or areas in which they may struggle. It will provide a better understand the variety of opportunities to

embed in the activity to deepen student understanding of working with mixed numbers. Working through the task provides the teacher with a better appreciation of the activity and insight into the different ways students may approach the task and develop their solution.
The teacher should determine what documents students will create and hand in as they complete the task. The teacher should develop procedures and/or requirements for group work. As students engage in classroom discourse, they may be provided with prompts such as:

- Would you explain your thinking more about this?
- What are we trying to determine?
- What information do you need to complete the task?

2. Establish Small Groups

Where to start? This activity is best addressed by pairs of students. The teacher needs to take into consideration student skill levels, leadership skills, and personalities when creating small groups. Small group collaboration works best when students have been provided previous opportunities to work together on a regular basis.
Through weeks of teacher observation and making note of leadership skills, personalities, the ability to take criticism, to question, and think deeply about a task or problem will prove to be extremely helpful when creating small groups for this lesson and associated tasks. When creating the groups, all these factors help to eliminate the potential situation in which one student takes the lead and makes the decisions for the group. When you have this situation, one student is gaining all the benefits of the task, while others do not. If a student is not engaged in conversation, this lesson and associated tasks will not be beneficial in helping all students to have meaningful discussions about the mathematics involved nor in analyzing the relationships inherent to the tasks. Students will be working both individually and together when placed in small groups to complete this task.
If students have not previously developed standards for group work, provide students with an opportunity to collaborate to develop guidelines to help them stay focused during tasks and group work assignments. The teacher should post these standards in the classroom and reference prior to group task activities.

3. Develop Open-Ended Questions

The teacher should create a list of open-ended questions designed to support and scaffold the lesson and the associated tasks for the students. These questions should purposefully direct students towards provided information, previously learned content, and similarities and differences in their work among the group members. Some questions might include:

- What are we trying to determine?
- When using the information provided, what important details do you need to work on the task?
- How would you solve this problem?
- What information is needed to solve this task?
- What did you do that was similar? Different?
- Is there a strategy that is more efficient? Why or why not?

4. Gather Materials

- Copies of task statement (click here)
- Presentation device (required for teacher to show student work)
- White Paper (plain) or notebook paper
- Pencil
- Whiteboard (to present the task)

- White poster paper or sticky chart paper (large) for students in groups to post their created problem
- Math Toolkit (Optional: Items such as tile manipulatives, fraction bars, etc.)

Preorganization of the supplies will eliminate delays, disruptions, and confusion as students prepare to work. The teacher might have the items in a plastic tub or any container in the center of the group for easy access. Some students may need concrete items to address this task.

- Copies of task statement (click here)
- Presentation device (required for teacher to show the task or student work)
- Chart Paper (plain)
- White poster paper or sticky chart paper (large) for students in groups to post their results
- Pencil
- Whiteboard (to present the task)
- Math Toolkit (Optional: Items such as tile manipulatives, fraction bars, etc.)
  The materials in the math toolbox should include a variety to use while students work.

5. Anticipated Common Student Misconceptions

- Students may mistakenly believe that the fractions' numerators and denominators can be treated as separate whole *numbers*. (e.g., 2/4 + 5/4 = 7/8 or 3/5 – 1/2 = 2/3)
- Students may fail to find a common denominator when adding or subtracting fractions with unlike denominators.
- Students may believe that only whole numbers need to be manipulated in computations with fractions greater than one. When adding or subtracting mixed numbers, students may ignore the fractional parts and work only with the whole numbers. (e.g., $5\frac{3}{5} - 2\frac{1}{7} = 3$).

Explore Phase

Prior to teaching of this the lesson and the associated activities the following content should be reviewed (e.g. Warm-up activities one week prior to the introduction of this lesson):

- Model and identify mixed numbers and their equivalent fractions.
- Write equivalent fractions with smaller or larger denominators.
- Use models to write equivalent fractions, including using composition or decomposition.

Prior Instruction/Knowledge:

In the Grade 4 Domain, Numbers and Operations with Fractions, students develop proficiency with fractions and mixed numbers. In Grade 3, students develop an understanding of fractions as built from unit fractions.

The concept of fractions is a critical instructional area that includes developing an understanding of fraction equivalence, addition and subtraction of fractions with like denominators, and multiplication of

fractions by whole numbers. In grade four, fractions include those with denominators 2, 3, 4, 5, 6, 8, 10, 12, and 100. Students should develop a solid understanding that a mixed number indicates the sum of a whole number and a fraction. They develop a method for converting mixed numbers to fractions that is connected to the meaning of fractions. Students use this understanding in adding and subtracting fractions and mixed numbers.

Please review the following:
Educators Guide for Mathematics: Grade 4 pages 21-31, pdf 23-33 pages) (click here)

Prerequisite Skills

- Use addition and subtraction facts to 20.
- Use multiplication facts through 144.
- Model and identify mixed numbers and their equivalent fractions.
- Write equivalent fractions with smaller or larger denominators.
- Add and subtract fractions and mixed numbers using models and pictures to explain the process and record the results in number and word problems.
- Use models to write equivalent fractions, including using composition or decomposition or showing relationships among halves, fourths, and eighths, and thirds and sixths.
- Understand that the fractional relationships that occur between zero and one also occur between every two consecutive numbers.
- Write equivalent fractions with smaller or larger denominators.
- Use models to represent a fraction as a product of a whole number and a unit fraction in number and word problems.

Supporting Skills

- Add 3 single-digit numbers in number and word problems.
- Describe the probability of a chance event using a fraction or ratio.
- Locate points on a number line**.**

Impending Skills

- Describe the effect of operations on size and order of numbers.
- Rewrite or simplify algebraic expressions including the use of the commutative, associative, and distributive properties, and inverses and identities in number and word problems.
- Determine the area of rectangles, squares, and composite figures using nonstandard units, grids, and standard units in number and word problems.
- Use the commutative, associative, and distributive properties, and inverses and identities to solve number and word problems with rational numbers.
- Add and subtract fractions and mixed numbers with unlike denominators in number and word problems
- Determine the complement of an event.
- Add and subtract fractions and mixed numbers with like denominators (without regrouping) in number and word problems.
- Convert fractions and terminating decimals to the thousandths place to equivalent forms without models; explain the equivalence.
- Multiply two fractions or a fraction and a whole number in number and word problems.
- Compare and order fractions using common numerators or denominators.
- Add and subtract fractions and mixed numbers using models and pictures to explain the process and record the results in number and word problems.
- Represent multiplication or division of mixed numbers with and without models and pictures.
- Represent division of fractions and mixed numbers with and without models and pictures in number and word problems; describe the inverse relationship between multiplication and division.

Source: *The Quantile Framework for Mathematics*
https://metametricsinc.com/educators/quantile-for-educators/

Implementation Phase

Part 1

- Introduction of the task challenge (whole class): If Dennis and Cody combine their buckets of blocks, will they have enough to build their castle?
- Distribute materials to complete the task.
- Check for Understanding.

Part 1 Teacher Notes:

Preset Activities- The students are in pairs. The materials are provided for the students.
Thinking starter for students: Today, we are going to apply what we know about mixed numbers with like denominators to find if two children have enough plastic blocks to build a castle.

- Introduce the task to the students. Introduce the task challenge: If Dennis and Cody combine their buckets of blocks, will they have enough to build their castle?
- Provide the student task handout and have students working as a team.
- Check for understanding of the task.
- Optional: Provide individual task cards with guidelines, directions, and the task challenge. Example:
  If Dennis and Cody combine their buckets of blocks, will they have enough to build their castle? With your partner, use a math strategy to answer the essential question.
  1. *Model your strategy in your math notebook or paper provided and explain it to your partner.*
  2. *Look for a different strategy for how to solve the problem.*
  3. *Model and describe the new strategy to your partner.*
  4. *List and model any other strategies you can use to solve this problem.*

  Be ready to report or present to the class the different strategies you used.

Part 2: Task Time

- Students begin to work on the task and searching for different methods to solve the challenge.
- Students prepare to present their solution to the task.

Part 2 Teacher Notes:

- Organize student pairs to address the task challenge.
- As students begin to work on the task, the teacher should circulate through the class and observe student pairs finding ways to solve the task. The teacher should pose questions to the groups to prompt their thinking. The teacher should avoid identifying student solutions as correct or incorrect.

Part 3: Class Discussion and Analysis

- Presentation of several different student solutions.
- Whole class discussion and analysis of student solutions that highlight different approaches.
- Compare the mathematical ideas and strategies used in addressing the challenge.
- Student pairs create a fraction challenge to present to challenge another student pair.

Part 3 Teacher Notes: Class Discussion and Analysis

- Complete a check on each group's readiness to report out their findings.
- Guide the discussion about each groups' method of solving the task.
- Present the class the opportunity to write their own fraction situation and exchange with each other in the group to solve it.

Part 4: Application of Knowledge

Student's Task: Write your own real-world problem that will require another student to add mixed numbers with like denominators.

- Student pairs create and solve a problem similar to the task.
- Student pairs solve each other's problems.
- In a whole-class discussion, students share and compare their solution pathways for two or three of the student-created problems.
- In a whole-class discussion, students note the similarities and differences in the problems.

Part 4 Teacher Notes:

- Students create and solve a problem similar to the task to practice and demonstrate their skill and understanding. The teacher challenges student pairs to use their skill and understanding to create a challenge that involves mixed numbers for other student pairs to solve.

  As the students create their new problems, the teacher should monitor for any confusion or an inability to successfully create a fractional word problem. The teacher should also note students who might need reteaching. The teacher should identify problems and solution pathways that students will present to the class for discussion. It may be appropriate to use the Math Congress style of presentation. In this instructional strategy, the student groups or pairs post their solution on chart paper and decide what to share in their presentation to the class. While every student's solution is valuable, the teacher needs to effectively ensure that students can present the necessary information about the methods used. This strategy allows the teacher to have whole-class discussions on two or three, carefully chosen student solutions where connections can be made to every student's mathematical learning. To aid in this process, the teacher should address the following questions:
  - What ideas/strategies are represented in the word problems?
  - How does the sample word problems selected continue to develop students' understanding and develop connections between the task's learning goals and previous knowledge?
  - Which ideas can be generalized and how do I develop a strategy for students to come to these generalizations about solving problems?
  - With the word problems I want to be presented, in what order will I have students present, so that it is in a manner that scaffolds learning for students?

- Report out and discuss two or three of the created problems and solutions by the students. As the students present, the teacher poses questions such as:
  - What are the similarities and differences in the thinking necessary in these problems?
  - What strategy/strategies were used in solving these problems?
  - Will these strategy/strategies always work? Why or why not?
  - Have you tried any of the strategies presented? Will you try any of the strategies?
  - In what future situation would you use this strategy?

  The intent of the discussion is to extend student learning. Students who are not familiar with being asked to defend their thoughts may initially struggle. With practice, they will become more comfortable in communicating their thoughts and understanding of task's concepts and in sharing their understanding of solving word problems.

Share, Discuss, and Analyze Phase

Essential Understanding:

An understanding of decomposing fractions, estimating with benchmark fractions, modeling with mixed numbers, and converting mixed numbers to equivalent fractions can serve as an effective tool in addressing real-world contexts.

Share

The lesson opens with the students presented with the task challenge: If Dennis and Cody combine their buckets of blocks, will they have enough to build their castle?

In groups of two, the students work together and use their previous knowledge about adding mixed numbers to decide if the two boys have enough plastic blocks to build the castle.

Student pairs collaborate in developing their solution(s). Representative student pairs model their strategies for the class.

Discuss

Students work together and discuss different methods to solve the task challenge. As students develop a solution, discussion might include a variety of ways to reach an answer.

Students may discuss their understanding of decomposing the fractional part of the mixed number. The students' conversation would include decomposing the fractions in the mixed numbers $1\frac{3}{4}$ and $1\frac{1}{2}$ For example, in $1\frac{3}{4}$ the $\frac{3}{4}$ would be seen or understood as:

$$\frac{3}{4} = \frac{1}{4} + \frac{1}{4} + \frac{1}{4}$$

Some students might discuss comparing the mixed numbers in the word problem to a whole number. For example, comparing $1\frac{3}{4}$ to 1 or 2. This strategy involves using benchmark fractions to solve the problem.

Some students may notice that if Cody has $1\frac{1}{2}$ buckets and the boys need $2\frac{1}{2}$ buckets, they only need one more bucket to create the castle. Because Dennis has $1\frac{3}{4}$ buckets, or more than one bucket, students may deduce that the boys will be able to build the castle.

Other students may talk about converting the mixed numbers to improper fractions. Students would discuss multiply the whole number part by the fraction's denominator, add that to the numerator, and then, write the result on top of the denominator. In this solution pathway, the would discuss that to add the improper fractions, the fractions need to have the same denominator.

For the whole class discussion, the teacher selects a representative number of solutions and has the students present their strategy to the class. Students see that others have different strategies that may include drawing a picture or model, estimating, using properties of operations, converting mixed numbers to equivalent fractions, etc. From the discussion, students see there are many ways to solve this task.

Analyze

Students review, analyze, and determine a solution for task challenge. They create a word problem involving mixed numbers. In these tasks, students develop connections between their prior knowledge and the mathematical concepts embedded in the task. Students analyze their created word problem:

- Does the problem present a real-world application of adding mixed numbers?
- Does the problem present sufficient mathematical information to solve?
- Does the word problem have any extra or unnecessary numerical information?
- (Optional) Is the problem exciting and challenging to solve?

As a selection of different problems is presented to the whole class, the students analyze and solve using a variety of solution techniques that will be similar to those used in the first problem. Students determine if solutions are correct and the mathematical explanations clear and accurate. With the teacher acting as facilitator, the students compile a list of strategies to the problems.

Task In Action

The video clips below provide a demonstration of the task being implemented in a classroom as it aligns with the Effective Mathematics Teaching Practice indicated. These clips should be used by the teacher to model the implementation of the task in his or her classroom.

- Establish Mathematical Goals to Focus Learning
  - Video Clip 1
  - Video Clip 2
- Implement Tasks that Promote Reasoning and Problem Solving
  - Video Clip 1
  - Video Clip 2
  - Video Clip 3
- Use and Connect Mathematical Representations
  - Video Clip 1
  - Video Clip 2
  - Video Clip 3
- Facilitate Meaningful Mathematical Discourse
  - Video Clip 1
  - Video Clip 2
  - Video Clip 3
- Pose Purposeful Questions
  - Video Clip 1
  - Video Clip 2
  - Video Clip 3
- Build Procedural Fluency from Conceptual Understanding
  - Video Clip 1
  - Video Clip 2
- Support Productive Struggle in Learning Mathematics
  - Video Clip 1
  - Video Clip 2
  - Video Clip 3
- Elicit and Use Evidence of Student Learning
  - Video Clip 1
  - Video Clip 2