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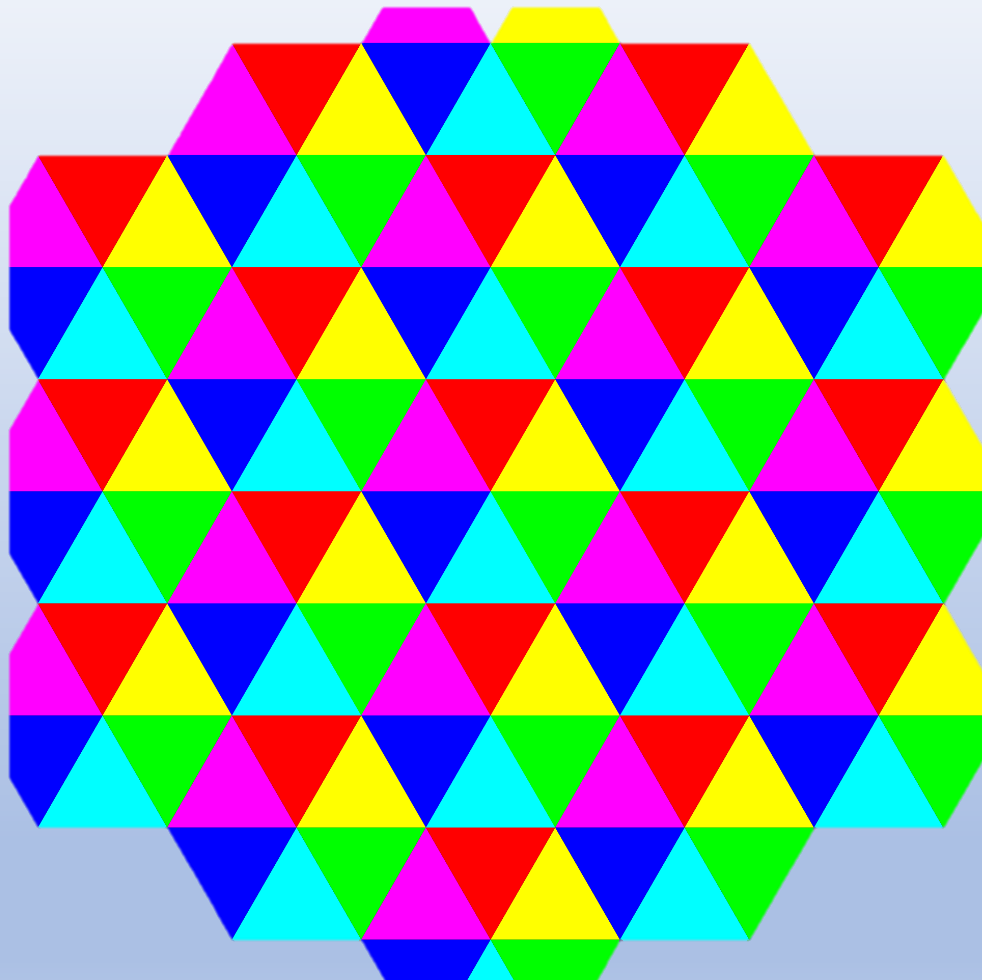
DEPARTMENT OF EDUCATION



MATHEMATICS

GRADE 8

Tessellate This

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Task Title: *Tessellate This*
Grade or Content Area: 8^{th} Grade
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Original Task Creator: Illustrative Mathematics
Quarter: 1

Rationale for Lesson and Associated Tasks

A tessellation is a fascinating combination of geometry and art. It is an arrangement of shapes that covers a surface without overlapping and without gaps. Tessellations can be analyzed mathematically. In this lesson, students investigate the characteristics and measure of the angles found in tessellations.

Tessellate This is a creative mathematical experience. Through experimentation and abstract reasoning, students discover repeating patterns. Students examine and create tessellations or patterns of shapes that fill the entire plane. Students also examine and create complex designs that exhibit rotational symmetry (that is, the design is congruent to itself by several rotations).

Lesson and Associated Tasks Overview

For Grade 8 students, the lesson and associated tasks serve as culminating activities to the study of rigid transformations and congruence. *Tessellate This* is a direct extension of prior work with triangles and other polygons. Students deduce the angle measures of polygons, determine what polygons can be tessellated, and apply knowledge of rigid motion transformation to create tessellations that maintain image congruence. Additionally, students use mathematical language to describe the transformations within a tessellation.
Rotate and Tessellate – (Unit 1 Lesson 17) (click here)
Tessellations of the Plane – (Unit 9 Lesson 1) (click here)
Regular Tessellations – (Unit 9 Lesson 2) (click here)
Tessellating Polygons – (Unit 9 Lesson 3) (click here)
Additional Resource: *Tessellations of Regular Polygons* - Source: TES, October 15, 2015
This lesson and associated tasks are scheduled to be completed over five class periods per the suggested sequence. The following is an overview of the lessons.

Day 1

1. Introduce Learning Target for the lesson
2. Distribute blank paper, geometry toolkits (see Materials), pattern blocks, and *Deducing Angle Measures of Polygons* (Lesson 17.1) handouts
3. Review the Triangle Sum Theorem by showing YouTube video, *Triangle Sum Theorem* (2:03) https://www.youtube.com/watch?v=xQfQ8RwZKNA
4. Find how many equilateral triangles can fit around a single vertex to determine the angle measure.
5. Use the equilateral triangle angle measure to deduce the angle measures of given regular polygons
6. Students share and justify their solutions
7. Student journal: What happens to the angle measure of polygons as the number of sides *decrease?* Why does it happen?

Day 2

1. Review the Learning Target and the activity from Day 1
2. Introduce Learning Target for lesson
3. Distribute tracing paper, Lesson 1 handouts, and Blackline master 8.9.1.3 *Describing a Tessellation*
4. Engage students in *Notice and Wonder: Polygon Patterns* (1.1)
5. Introduce vocabulary through *Tessellations* (1.2)
6. Engage students in *Describing a Tessellation* (1.3)
7. Student journal: What was *challenging* about describing or identifying a tessellation?

Day 3

1. Review the Learning Target and the activity from Day 2
2. Introduce Learning Targets for the lesson
3. Distribute tracing paper, protractors, and Lesson 2 handouts
4. Explore what shapes tessellate: *Regular Tessellations* (2.1)
5. Investigate *Equilateral Triangle Tessellation* (2.2)
6. Experiment with other polygons: *Regular tessellations of Other Polygons* (2.3)
7. Student Journal: Regular tessellations exist in stores, homes, and schools. Describe two regular tessellations you have seen.

Day 4

1. Review the Learning Targets and the activity from Day 3
2. Introduce Learning Target for lesson
3. Distribute tracing paper and Lesson 3 handouts
4. Experiment with non-equilateral triangles: *Triangle Tessellations* (3 .1)
5. Investigate rigid motions and quadrilaterals to tessellate the plane: *Quadrilateral Tessellations* (3.2)
6. Explore pentagons that tessellate the plane: *Pentagonal Tessellations* (3.3)
7. Student Journal: Why do some pentagons tessellate the plane while others cannot?

Day 5

1. Review the Learning Target and the activity from Day 4
2. Introduce Learning Targets for the lesson
3. Use the PowerPoint *Tessellations* to discuss types of tessellations
4. Distribute geometry toolkits, pattern blocks, square or isometric graph paper and Lesson 17.2 and 17.3 handouts
5. Create a tessellation: *Tessellate This* (17.2) or *Rotate That* (17.3)
6. Gallery Walk to find and describe the transformations in three different student’s tessellations

West Virginia College- and Career-Readiness State Standards

M.8.16

Verify experimentally the properties of rotations, reflections, and translations:

a. Lines are taken to lines, and line segments to line segments of the same length.
b. Angles are taken to angles of the same measure.
c. Parallel lines are taken to parallel lines.

M.8.17
Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

M.8.18
Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

M.8.19
Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

Mathematical Habits of Mind (MHM)
MHM2. Reason abstractly and quantitatively.
MHM4. Model with mathematics**.**

Mathematics Teaching Practices to Support Student Growth
MHM2. Implement tasks that promote reasoning and problem solving.
MHM3. Use and connect mathematical representations
MHM4. Facilitate meaningful mathematical discourse.
MHM5. Pose purposeful questions.
MHM7. Support productive struggle in learning mathematics**.**

Essential Understandings
- Properties of angle sums can be used to reason about how shapes/figures will fit together.
- Rigid transformations can be used to make interesting repeating patterns of figures.
- Rigid transformations can be used to create tessellations and designs with rotational symmetry.

Set–up Phase
1. Become an Expert Regarding All Lessons and Associated Tasks Content
It is imperative that the teacher becomes very familiar with all *Tessellate This* materials (e.g. Teacher Guide, Teacher Presentation Materials, Student Task Statements, PowerPoint, YouTube video, digital app, and student handouts). Each lesson has several associated tasks and some tasks may be delivered via printed or digital resources.

The implementation of the tasks may vary class to class depending on the time frame available for the lessons, and the levels of student engagement and understanding. Suggestions for *Students with Disabilities* and *English Language Learners* are provided within the Units.

2. Establish Small Groups
Tessellate This tasks and activities engage students in both individual and group thinking. Working individually, in pairs and in small groups, students explore and share their reasoning about polygon angle measurement and why some regular polygons can tessellate and others cannot.

Small group instruction has significant impact on student achievement (Hattie, 2009) and allows teachers to work more closely with each student. This type of instruction provides the opportunity to monitor students' learning and identify gaps in the development of students' math skills.

Teachers will need to instruct students on how to work in small groups. The first step in the process is to establish ground rules and norms for interaction. Students must have a part in making the rules and these guidelines must be enforced by both teachers and students. Ground rules should encourage positive collaborative behaviors among all students. Guidelines/ground rules need to be posted in the classroom so students can readily refer to them. If students or teachers believe that additional rules are needed, they can be added later.

Teachers should assign groups intentionally based on skills and/or backgrounds. Skill levels, leadership skills, and personalities must all be considered when creating small groups. Small group collaboration works best when students have been provided previous opportunities to work together on a regular basis. Observations of leadership skills; personalities; ability to take criticism; ability to question; and the ability to think deeply about a task or problem will prove to be extremely helpful when creating small groups for this unit. This strategy minimizes the chance that high ability students will flock together leaving others out, allows teachers to create more diverse groups, and creates opportunities for students to work with peers with whom they otherwise might not have interacted.

Small group instruction gives teachers an opportunity to assess more closely what each student can do and build strategic plans around those assessments. Students who struggle to ask questions and participate in a whole group setting may thrive in a small group where they feel more comfortable and less overwhelmed. Furthermore, small group instruction tends to proceed at a fast pace, which typically helps students maintain focus and enjoy the learning experience.

Source: *Small Group Instruction: How to Make it Effective*, CORE: Excellence in Education Blog, September 27, 2018.

3. Develop Open-Ended Questions

Teachers of *Tessellate This* should use open-ended questions to support and scaffold the lessons and associated tasks for their students. These questions should purposefully direct students towards provided information, previously learned content, and similarities and differences in their work versus other group members. Within the context of open-ended mathematical tasks, teachers should select questions with four purposes in mind:

A. *Starter questions*

Starter questions focus the students' thinking in a general direction and give a starting point. Starter questions might include:

- How could you sort these.......?
- How many ways can you find to.......?
- What happens when we.........?
- What can be made from.........?
- How many different .... can be found?

B. *Questions to stimulate mathematical thinking*

These questions focus on particular strategies aiding the formation of a strong conceptual network. The questions can serve as a prompt when students become 'stuck'. Questions to stimulate mathematical thinking might include:

- What is the same?
- What is different?
- Can you group these .... in some way?
- Can you see a pattern?
- How can this pattern help you find an answer?
- What do you think comes next? Why?
- Is there a way to record what you have found that might help us see more patterns?
- What would happen if....?

C. Assessment questions

Assessment questions ask students to explain what they are doing or how they arrived at a solution. They allow the teacher to see how students are thinking, what students understand, and at what level students are operating mathematically. These open-ended questions are best asked after time has been given to make progress with the problem, to record some findings, and perhaps to have achieved at least one solution. Assessment questions might include:

- What have you discovered?
- How did you find that out?
- Why do you think that?
- What made you decide to do it that way?

D. *Final discussion questions*

These questions draw together the efforts of the class and prompt the sharing and comparison of strategies and solutions. This is a vital phase in the mathematical thinking processes. It provides further opportunity for reflection and realization of mathematical ideas and relationships. It encourages students to evaluate their work. Final discussion questions might include:

- Who has the same answer/ pattern/ grouping as this?
- Who has a different solution?
- Are every one's results the same?
- Why/why not?
- Have we found all the possibilities?
- How do we know?
- Have you thought of another way this could be done?
- Do you think we have found the best solution?

*Additional questions are provided in the Teacher Guide.

Source: *Using Questioning to Stimulate Mathematical Thinking*, NRICH, February 2011.

4. Gather Materials

- Computer and presentation device (if the teacher uses the lesson digital format)
- Internet access (test app prior to implementation)
- Blank paper
- Graph paper – square or isometric
- Geometry toolkits (Colored pencils, scissors, and an index card) 1 per pair of students
- Pattern blocks
- Tracing paper (Patty Paper)
- Protractors (if not using a digital app)
- Video (Online) *Triangle Sum Theorem*
- PowerPoint *Tessellations*
- Illustrative Mathematics Handouts: *Deducing Angle Measures of Polygons* (Unit 1, Lesson 17.1); Blackline master *8.1.17.1 Deducing Angle Measures*; *Tessellations of the Plane* (Unit 9, Lesson 1);

Regular Tessellations (Unit 9, Lesson 2); *Tessellating Polygons* (Unit 9, Lesson 3); and *Tessellating Polygons* (Unit 1, Lesson 17.2 and 17.3)

5. Anticipated Common Student Misconceptions

Day 1

- When deducing angles measures, it is important students know that angles "all the way around" a vertex equals 360°. It is also important to know that angles that make a line when adjacent equal 180°.

Day 2

- Tessellations do not need to be edge to edge. That is, pieces do not need to fit together with edges of the same lengths matching exactly.
- Tessellations do not need to have symmetric, repeating patterns, though sometimes the shape forces it as with regular hexagons.

Day 3

- If students working with the pentagon and octagon add other shapes to make a more complicated tessellation, remind them that regular tessellations use copies of a single shape.
- Students may know that an equilateral triangle has 60-degree angles but may not be able to explain why. Consider prompting these students for the sum of the three angles in an equilateral triangle.
- Students may not see a pattern of hexagons within the triangle tessellation. Consider asking students what shape they get when they put 6 equilateral triangles together at a single vertex.

Day 4

- Students may struggle to put together copies of their triangle in a way that can be continued to tessellate the plane. Ask these students to put together two copies of the triangle. IF they have made a parallelogram, ask what kind of quadrilateral they have made. If they have not made a parallelogram, ask them if there is a different way to combine two copies of the triangle.
- Students may need to be reminded of the definition of a trapezoid: one pair of sides is contained in parallel lines.
- If the figures are not traced accurately, it may be difficult to determine if the pattern, using 180° rotations, can be continued. Ask these students what they know about the sum of the angles in a triangle and in a quadrilateral.
- Students may struggle tracing the rotated hexagon. Ask them what happens to the segments making angle A when the hexagon is rotated about A by 120° (or 240°).
- Students may wonder why the hexagon that they make by putting three pentagons together is a regular hexagon. Invite students to calculate the angles of the hexagon.

Day 5

- Watch out for students who choose shapes that almost-but-don't-quite fit together. Reiterate that the pattern must keep going forever – often small gaps or overlaps become more obvious when you try to continue the pattern.
- Students may think that reflection symmetry is the only kind of symmetry. Because of this, they may create a design that has reflection symmetry but not rotational symmetry

Explore Phase

In the grade eight Geometry domain, a major shift in the traditional curriculum occurs with the introduction of basic transformational geometry. In particular, the notion of congruence is defined differently than it has been in the past. Previously, two shapes were understood to be congruent if they had the "same size and same shape."

This informal notion is exchanged for a more precise one: that a two-dimensional figure is congruent to another if the second figure can be obtained from the first by a sequence of rotations, reflections, and translations. Therefore, students l need ample opportunities (e.g. "bell ringers") to explore these three geometric transformations and their properties prior to the introduction of the *Tessellate This* unit:

- Locate, given the coordinates of, and graph plane figures which are the results of **translations** or **reflections** in all quadrants of the coordinate plane.
- Locate, given the coordinates of, and graph plane figures which are the results of **rotations** (multiples of 90 degrees) with respect to a given point

Prior Instruction/Knowledge

Sixth-grade students build on their work with area from previous grade levels by ***reasoning about relationships among shapes*** to determine area, surface area, and volume. Students in grade six continue to understand area as the number of squares needed to cover a plane figure. They find areas of right triangles, other triangles, and special quadrilaterals by decomposing these shapes, rearranging or removing pieces, and relating the shapes to rectangles. As students compose and decompose shapes to determine areas, they learn that area is conserved when composing or decomposing shapes. For example, students decompose trapezoids into triangles and/or rectangles and use this reasoning to find formulas for the area of a trapezoid.

During grade seven, Standard M.7.11 lays the foundation for students to understand ***dilations as geometric transformations.*** This leads to a definition of the concept of similar shapes in eighth grade: shapes that can be obtained from one another through dilation. Standard M.7.11 is given significant attention in grade seven.

During grade seven, the ***notion of similarity has not yet been addressed***. Attempts to define similar shapes as those that have the "same shape but not necessarily the same size" should be avoided. Similarity is defined precisely in grade eight, and imprecise notions of similarity may detract from student understanding of this important concept. ***Shapes drawn to scale are indeed similar to each other, but could safely be referred to as "scale drawings of each other" at this grade level.***

Please review the following:

Educators Guide for Mathematics: Grade 6 (pages 44-47, pdf 46-49) Click here
Educators Guide for Mathematics: Grade 7 (pages 32-34, pdf 34-36) Click here

Prerequisite Skills

- Locate, given the coordinates of, and graph plane figures which are the results of translations or reflections in all quadrants of the coordinate plane.
- Locate, given the coordinates of, and graph plane figures which are the results of rotations (multiples of 90 degrees) with respect to a given point

Supporting Skills

- Predict results of tessellating, subdividing, and changing shapes by paper folding or dissecting and rearranging pieces of plane figures and solids.

- Verify how properties and relationships of geometric figures are maintained or how they change through transformations.

Impending Skills
- Transform (translate, reflect, rotate, dilate) polygons in the coordinate plane; describe the transformation in simple algebraic terms.

Source: *The Quantile Framework for Mathematics* 2020 MetaMetrics Inc.

Implementation Phase

Day 1
1. Introduce Learning Target for the lesson
2. Distribute blank paper, geometry toolkits (see Materials), pattern blocks, and *Deducing Angle Measures of Polygons* (Lesson 17.1) handouts
3. Review the Triangle Sum Theorem by showing YouTube video, *Triangle Sum Theorem* (2:03) https://www.youtube.com/watch?v=xQfQ8RwZKNA
4. Find how many equilateral triangles can fit around a single vertex to determine the angle measure
5. Use the equilateral triangle angle measure to deduce the angle measures of given polygons
6. Students share and justify their solutions
7. Student's journal: What happens to the angle measure of polygons as the number of sides *decrease?* Why does it happen?

Day 1 Teacher Notes:
To introduce the first task, briefly explain the Learning Target: Use properties of angle sums to reason about how figures will fit together. Distribute the supplies, blackline master, and student handouts.

1. Play the video, *Triangle Sum Theorem*, for all students to watch as a whole-class activity.
2. Whole-group discussion: Ask the students: How did the narrator prove that the sum of the measures of all interior angles of a triangle is 180°? Would the type of triangle (acute, right, or obtuse) affect the sum of the angle measures? How does the knowledge about the Triangle Sum Theorem help to find the sum of the measures of all interior angles of a quadrilateral?
3. If using pattern blocks, ask student groups to remove the equilateral triangles from their pattern blocks.
4. If using the blackline master *8.1.17.1* Deducing Angle Measures, ask student groups to find the equilateral triangle figure. Demonstrate how to use tracing paper to position and trace copies of the triangle around a single vertex.
5. Working in groups of 2-4, ask students to respond to the first question on the student handout: **H*ow many copies of an equilateral triangle can fit together around a single vertex? What is the measure of each angle in the equilateral triangle?*** Be sure students understand that the triangles' edges must have no gaps or overlaps. If students are using the blackline master, they need to trace the triangles on tracing paper.
6. Circulate the room listening to students as you allow students time to find the answers.
7. Based on your observations, select 2 groups to model and share their findings with the class.
8. Working in pairs, ask students to discover the measure of each interior angle in the other polygons on the blackline master 8.1.17.1. ***Important**: **Do not provide protractors.*** Instead, suggest students arrange multiple copies of the same polygon to fit around a single vertex to determine the angle measure as they did with the equilateral triangle. Students may use the equilateral triangle pattern

blocks or a tracing of an equilateral triangle to lay on top of the given polygon to aid in calculating the angle measures. Students might trace and cut in half known angle measures (e.g. trace and cut in half a 60° angle of an equilateral triangle to get a 30° angle; half the 30° angle to get a 15° angle) to help calculate the measure of an interior angle of a polygon on the blackline master. Ask students to record each angle measure for each polygon on the blackline master. The master is used later in another lesson's task.

9. Circulate the room listening to students as you allow time to calculate the angles.
10. Based on your observations, select pairs of students to share and justify to the class their angle measures for each given polygon.
11. Provide the following prompts for a student journal entry: What happens to the angle measure of polygons as the number of sides *decrease?* Why does it happen?

Day 2

1. Review the Learning Target and the activity from Day 1
2. Introduce Learning Target for lesson
3. Distribute tracing paper and Lesson 1 handouts. Do not distribute the Blackline master 8.9.1.3 *Describing a Tessellation* until needed for activity 1.3.
4. Engage students in *Notice and Wonder: Polygon Patterns* (1.1)
5. Introduce vocabulary through *Tessellations* (1.2)
6. Engage students in *Describing a Tessellation* (1.3)
7. Student journal: What was *challenging* about describing or identifying a tessellation?

Day 2 Teacher Notes:

Organize students into small groups. Review the Day 1 Learning Target by having students share their journal entries with members of their group. To introduce the tasks for Day 2, briefly explain the Learning Target: Understand congruence and similarity using physical models, transparencies, or geometric software. Distribute the supplies and student handouts.

1. Begin with the 1.1 *Notice and Wonder: Polygon Patterns* activity. Divide students into pairs and ask them to look at a set of 6 polygon patterns for 1 minute. (The patterns can be reproduced for students or displayed digitally). Ask students to individually record what they notice about the patterns and what they wonder about the patterns.
2. Have students share with a partner *what they noticed* about the polygon patterns and *what they wonder* about the polygon patterns. After sharing in pairs, have a whole-class discussion. Record student responses along with their reasoning. After each response, ask the class if they agree or disagree and to offer alternative ways of thinking about the patterns.
3. Extend the discussion by asking:
   - Q. How do you know what happens as the polygons continue to grow off the page?
   - A. The pattern continues – even off the page.
   - Q. Are the colors of the shapes important?
   - A. The colors are important for identifying the patterns and making the picture more attractive, but the colors could be changed, and the pattern would remain the same.
4. Begin activity 1.2 *Tessellations* by defining a tessellation of the plane by polygons (A tessellation covers the plane with copies of the shape with no gaps and no overlaps). Show the students the 6 polygon patterns used in the 1.1 *Notice and Wonder* activities and ask if each is a tessellation. (YES)

5. If using the print format of the activity, demonstrate for students how to use the tracing paper to create a tessellation. If using the Illustrative Mathematics digital app, demonstrate for students how to use the app to create a tessellation.
6. Arrange students in pairs. Ask the students to create a tessellation by tracing or digitally producing copies of one of the shapes on the student handout (1.2 *Tessellations*).
7. Circulate around the room as students work. Make sure the students in each pair use the same shape.
8. When the tessellations are completed, ask students to complete question 2 of the handout. After a few minutes, have 2 pairs of students present their tessellations to the class – explaining how the tessellations are the same and how they are different. (Use data projector or another digital device to show student work).
9. Ask students to follow the directions for question 3 to complete a third tessellation of the shape used for question 2.
10. Circulate the room as students work and assist any students who have questions.
11. Ask the students to share if they were successful at creating a third tessellation. If yes, project the tessellations with a data projector or another digital device and have a group discussion about the third tessellation. If no, ask students why they think a third tessellation could not be produced.
12. Working in pairs for activity 1.3 task #1, students describe tessellation patterns by using rigid motion words: translation, reflection, and rotation. Student A describes one of 4 polygon patterns as Student B tries to identify which picture Student A was describing. They switch roles and repeat the activity.
13. Engage students in a whole-group discussion about which word(s) were most helpful in identifying the polygon patterns and why they were helpful. Record the word(s) so all students can see them for the activity 1.3 task # 2 of student handout.
14. For activity 1.3 task #2, distribute the Blackline mater 8.9.1.3 Card A to Student A of each pair of students. Ask Student A to describe the tessellation on Card A as Student B tries to produce (draw) the tessellation. Student A must be careful to not let Student B see Card A until the tessellation is complete.
15. Ask Student A and Student B to make a list of any differences between Card A and the tessellation produced by Student B. (# 3 of student handout)
16. Repeat the activity (#4 of student handout) by giving Card B to Student B. Ask Student B to describe the tessellation on Card B as Student A tries to produce (draw) the tessellation. Student B must be careful to not let Student A see Card B until the tessellation is complete.
17. Ask Student A and Student B to make a list of any differences between Card B and the tessellation produced by Student A. (#5 of student handout)
18. Provide the following prompt for a student journal entry: What was *challenging* about describing or identifying a tessellation?

Day 3

1. Review the Learning Target and the activity from Day 2
2. Introduce Learning Targets for the lesson
3. Distribute tracing paper, protractors, and Lesson 2 handouts
4. Explore what shapes tessellate: *Regular Tessellations* (2.1)
5. Investigate *Equilateral Triangle Tessellation* (2.2)
6. Experiment with other polygons: *Regular tessellations of Other Polygons* (2.3)
7. Student Journal: Regular tessellations exist in stores, homes, and schools. Describe two regular tessellations you have seen.

Day 3 Teacher Notes:
Organize the students into small groups. Review the Day 2 Learning Target by having students share their journal entries with members of their small group. To introduce the tasks for Day 3, briefly explain the Learning Targets: Understand congruence and similarity using physical models and Use informal arguments to establish facts about the angle sum and exterior angles of triangles. Distribute the supplies and student handouts.

1. Launch the 2.1 *Regular Tessellations* activity. Introduce the concept of regular tessellations: *Only one type and size of polygon is used; and if the polygons meet, they either share a single vertex or a single side.* Using the Teacher Presentation Materials, project the pictures of 6 tessellations. Engage students in a discussion of which of the 6 tessellations are regular (only the one with squares) and which are not. Ask students to identify why the tessellation is regular or irregular (e.g., several different polygons are used; edges of polygons do not match up completely).
2. Using a data projector or another digital device, display the following data table. Ask students to copy the table on their student handout under Question 1.

SHAPE	TESSELLATE?	NOTES
Octagon		
Hexagon		
Pentagon		
Square		
Triangle		

3. Ask students to look at the shapes on the student handout. Their task is to determine which shapes can make regular tessellations of the plane and which cannot. Note: If using the digital format, review how to use the app to tessellate the plane with the shape. If using the print format, remind students how to use the tracing paper to create a tessellation. Ask students to complete the table as they experiment to see which shapes tessellate.
4. Circulate the room as students work in their small groups and provide assistance as needed.
5. Referring to the completed table, ask if anyone can explain why certain shapes make a regular tessellation of the plane. (Question 1 of student handout).
6. Referring to the completed table, ask if anyone can explain why certain shapes could not make a regular tessellation of the plane. (Question 2 of student handout).
7. Begin activity 2.2 *Equilateral Triangle Tessellation* by asking students what is the measure of each angle of an equilateral triangle? Ask them how they know? Refer back to Day 1 (Lesson 17.1).
8. Ask students to complete the remaining questions on the *2.2 Equilateral Triangle Tessellation* student handout.
9. Circulate the room as students work in their small groups.
10. Lead a whole-class discussion by asking:
    - Q. How did you find the angle measures in an equilateral triangle?
    - A. The sum of the angles is 180 degrees and they are all congruent, so each is 60 degrees.
    - Q. Why is there no space between six triangles meeting at a vertex?
    - A. The angles total 360 degrees, which is a full circle.
    - Q. How does your tessellation with triangles relate to hexagons?
    - A. You can group the triangles meeting at certain vertices into hexagons, which tessellate the plane.
    - Q. Are there other tessellations of the plane with triangles?

A. Yes. You can make infinite rows of triangles that can be placed on top of one another – and displaced relative to one another.

11. Launch activity 2.3 *Regular Tessellation for Other Polygons* by asking students "Are there some other regular polygons in addition to equilateral triangles, squares, and hexagons that can be used to create a regular tessellation of the plane? Record their responses.
12. Tell students they are going to investigate polygons with 7, 8, 9, 10, and 11 sides to see if they do or do not tessellate and why. If using a print format, tell students to use tracing paper and protractors to explore their conjectures. If using the Illustrative Mathematics digital format, tell students to use the app to explore their conjectures.
13. Ask students to record their observations under question 1 of the 2.3 *Regular Tessellation for Other Polygons* student handout.
14. Circulate the room as students work. Provide students with assistance when necessary.
15. Upon completion of the investigation, ask the students to share their findings in small groups and to note any differences.
16. Lead a whole-class discussion by asking questions 2 – 4 of student handout:

    Q #2. How does the measure of each angle in a square compare to the measure of each angle in an equilateral triangle? How does the measure of each angle in a regular 8-sided polygon compare to the measure of each angle in a regular 7-sided polygon?

    A #2. The angles in a square are 90 degrees; greater than the 60-degree angles in a triangle. The angles in a regular 8-sided polygon have greater measure than the angles in a regular 7-sided polygon.

    Q #3. What happens to the angles in a regular polygon as you add more sides?

    A #3. The angles increase in measure. Imagine opening up a polygon with 6 sides to add a 7^{th} equal side. In order to fit the new side, all of the other sides must be spread out or opened up, increasing the measure of the angles.

    Q #4. Which polygons can be used to make regular tessellations of the plane?

    A #4. The equilateral triangle, square and regular hexagon are the only regular polygons that can tessellate the plane. As more sides are added, the angles get greater, 120 degrees is the biggest divisor of 360 that can be the measure of an interior angle of a regular polygon.
17. Provide the following prompt for a student journal entry: *Regular tessellations exist in stores, homes, and schools. Describe **two** regular tessellations you have seen and where you saw them.*

Day 4

1. Review the Learning Targets and the activity from Day 3
2. Introduce Learning Target for lesson
3. Distribute tracing paper and Lesson 3 handouts
4. Experiment with non-equilateral triangles: *Triangle Tessellations* (3 .1)
5. Investigate rigid motions and quadrilaterals to tessellate the plane: *Quadrilateral Tessellations* (3.2)
6. Explore pentagons that do tessellate the plane: *Pentagonal Tessellations* (3.3)
7. Student Journal: Why do some pentagons tessellate the plane while others cannot?

Day 4 Teacher Notes:

Review the Day 3 Learning Target by having students share their journal entries with members of their small group. To introduce the tasks for Day 4, briefly explain the Learning Target: Understand congruence and similarity using physical models, transparencies, or geometric software. Distribute the supplies and student handouts.

1. Begin the lesson by assigning students to small groups to experiment with the tessellation of a non-equilateral triangle.
2. Assign each group of students one of the triangles on the student handout *3.1 Triangle Tessellations***.** Ask each student to create a tessellation of the assigned triangle. If using the print format of the activity, remind students how to use the tracing paper to create a tessellation. If using the Illustrative Mathematics digital app, remind students how to use the app to create a tessellation.
3. Circulate the room as students work. Look for students who use two copies of the triangle to form a parallelogram in order to create their tessellation.
4. Have students share their tessellations in their small group. Ask students if the tessellations were alike or if they were different and how.
5. In a whole-class discussion, ask students if anyone used two copies of the triangle to form a parallelogram in order to create the tessellation. If yes, ask the student(s) to share their tessellation with the class. Use a data projector or a digital device to display the tessellation(s). If no, demonstrate for the students how to use two non-equilateral triangles to form a parallelogram, and show how to use the parallelogram to create a tessellation of the plane.
6. Begin activity 3.2 *Quadrilateral Tessellations* by reviewing how any triangle can tessellate the plane and that some triangles can do so in many ways.
7. Ask students if **some** quadrilaterals tessellate the plane and if so, which ones. (Yes: squares, rectangles, rhombuses, and parallelograms).
8. THINK, PAIR AND SHARE: Ask students if **any** quadrilateral can be used to tessellate the plane. Give students a few moments to think before sharing with a partner. Poll the class for the number of yes and the number of no responses. Record the numbers in a place visible to all students. The poll is revisited later in the lesson.
9. Remind students of the definition of a trapezoid. Tell students they will now investigate the ability of a trapezoid to tessellate the plane. Direct students to complete questions 1, 2, and 3 of the student handout.
10. Circulate the room as students work. Assist students when necessary.
11. Invite students to share their tessellations in their small group.
12. In a whole-group discussion, ask students:
    - Q. Were you able to tessellate the plane with copies of the trapezoid?
    - A. Yes, two of the trapezoids can fit together to make a parallelogram and the plane can be tessellated with copies of the parallelogram.
    - Q. What did you notice about the trapezoid and 180-degree rotations?
    - A. The trapezoids fit together with no gaps and no overlaps and leave space for four more quadrilaterals.
    - Q. How do you know that there are no overlaps?
    - A. The sum of the angles in a quadrilateral is 360°. At each vertex in the tessellation, copies of the four angles of the quadrilateral come together.
13. Revisit the poll concerning if any quadrilateral can tessellate the plane. Ask if any students have changed their answers and ask the students to explain their reasoning.
14. Launch activity 3.3 *Pentagonal Tessellations* by asking:
    - Q. Can you tessellate the plane with a regular pentagon?
    - A. No
    - Q. Can you think of a type of pentagon that could be used to tessellate the plane?

A. Example: A square base with a 45-45-90 triangle on top.

15. Group students in pairs. Ask the students to complete question 1 on the 3.3 *Pentagonal Tessellations* student handout. If using a print format, tell students to use tracing paper to explore their conjectures. If using the Illustrative Mathematics digital format, tell students to use the app to explore their conjectures. Invite the students to talk about their ideas with their partner before writing the ideas down. Provide the students with sentence frames to help explain their answer:
    - First, I _________ because ….
    - At the central vertex, I noticed that …..
    - I agree/disagree because…..
    - What other details are important?
16. Circulate the room as students work, and provide students with assistance when necessary.
17. In a whole-class discussion, ask students to share their findings for question 1.
18. Direct students to complete questions 2, 3, and 4 of their student handout.
19. In a whole-class discussion, ask students to share their findings for questions 2, 3, and 4.
20. Provide the following prompt for a student journal entry: Why do some pentagons tessellate the plane while others cannot?

Day 5

1. Review the Learning Target and the activity from Day 4
2. Introduce Learning Targets for the lesson
3. Use the PowerPoint *Tessellations* to discuss types of tessellations
4. Distribute geometry toolkits, pattern blocks, square or isometric graph paper and Lessons 17.2 and 17.3 handouts
5. Create a tessellation: *Tessellate This* (17.2) or *Rotate That* (17.3)
6. Gallery Walk to find and describe the transformations in three different student's tessellations

Day 5 Teacher Notes:

Review the Day 4 Learning Target by having students share their journal entries with members of their small group. To introduce the tasks for Day 5, briefly explain the Learning Targets: Repeatedly use rigid transformations to make interesting repeating patterns of a shape <u>and</u> Use properties of angle sums to reason about how figures will fit together. Distribute the supplies and student handouts.

1. Begin the lesson by projecting the Launch Image from the Teacher Presentation materials.
2. Engage students in a whole-class discussion. Share with students the definition of tessellation. "A tessellation of the plane is a regular repeating pattern of one or more shapes that covers the entire plane." Consider showing students several examples of tessellations. A true tessellation covers the entire plane. While this is impossible to show, we can identify a pattern that keeps going forever in all directions. This is important when we think about tessellations and symmetry. One definition of symmetry is, "You can pick it up and put it down a different way and it looks exactly the same." Look at the image projected. The **translation** that takes point **Q** to point **R** results in a figure that looks exactly the same as the one we started with. Does the **translation** that takes **S** to **Q** do the same? (Yes).
3. Engage students in a discussion of the types of tessellations using the PowerPoint *Tessellations.*
4. Arrange the students in small groups and provide them with pattern blocks or the blackline master 8.1.17.1 Deducing Angle Measures. Tell students to experiment with putting the shapes together by either tracing the pattern blocks of their choosing or by cutting out shapes from the blackline master

to trace. Tell the students they do not need to use all of the shapes – two or three shapes will be enough for the activity.

5. Circulate the room providing assistance when necessary. Students who are struggling may do better if you suggest they use two of the simpler shapes for their experiment.
6. In their small groups, ask students to share their findings.
7. Project the *Launch Image 3 Rotate That* (Teacher Presentation materials) so all students may see the figure.
8. Ask students what transformation they could perform on the figure so that it matches up with its original position. There are a number of rotations using A as the center that would work: 72° or any multiple of 72°. Make sure students understand that the 5 triangles in the pattern are congruent and that $5 \cdot 72° = 360°$. This is why multiples of 72° with center A match this figure up with itself.
9. Each student may choose to complete either Lesson 17.2 *Tessellate This* or Lesson 17.3 *Rotate That*. In lesson 17.2, the students create a tessellation. Students decide which shapes to use. The tessellation must be a repeating pattern that goes on forever to fill up the entire plane. In lesson 17.3, students make a design with rotational symmetry.
10. Encourage the students to be creative and to use color in their designs.
11. Circulate the room as the students work. Provide students with assistance when necessary.
12. Once the designs are completed, display all student work.
13. Ask students to engage in a Gallery Walk to find and describe the transformations (translation, reflection, and/or rotation) in three different student designs. Tell students to record their observations in their Student Journals.

Share, Discuss, and Analyze Phase

Essential Understanding #1:

Properties of angle sums can be used to reason about how shapes/figures will fit together.

Share - On Day 1, students explore how many copies of an equilateral triangle can fit together around a single vertex. The students must also determine the measure of each angle in the equilateral triangle. After the exploration, the students use their knowledge of the equilateral triangle to deduce the interior angle measures of given polygons. After each activity, students share and justify within their group and the whole class their findings.

Discuss - At the beginning of the lesson, students watch the video, *Triangle Sum Theorem*. At the end of the video, students are engaged in a discussion of how the narrator proved that the sum of the measures of all interior angles of a triangle is 180° and how the knowledge of the Triangle Sum Theorem helps to find the measures of interior angles of a quadrilateral.

Analyze - At the closure of the lesson on Day 1, the students are asked to respond to the prompt: *What happens to the angle measure of polygons as the number of sides decrease? Why does it happen?* In order to respond, students analyze the angle measures they deduced for the given polygons.

Share - On Day 3, students investigate if polygons with 7, 8, 9, 10, and 11 sides do or do not tessellate. (Task 2.3) Upon completion of the investigation, students share their findings in small groups.

Discuss - Students complete questions on the 2.2 Equilateral Triangle Tessellation handout and participate in a whole-class discussion:

Q. How did you find the angle measures in an equilateral triangle?

A. The sum of the angles is 180 degrees and they are all congruent, so each is 60 degrees.

Q. Why is there no space between six triangles meeting at a vertex?

A. The angles total 360 degrees, which is a full circle.

Q. How does your tessellation with triangles relate to hexagons?

A. You can group the triangles meeting at certain vertices into hexagons, which tessellate the plane.

Q. Are there other tessellations of the plane with triangles?

A. Yes. You can make infinite rows of triangles that can be placed on top of one another – and displaced relative to one another.

Analyze - Students analyze their data from the investigation of polygons with 7, 8, 9, 10, and 11 sides to answer questions 2 – 4 of student handout:

Q # 2. How does the measure of each angle in a square compare to the measure of each angle in an equilateral triangle? How does the measure of each angle in a regular 8-sided polygon compare to the measure of each angle in a regular 7-sided polygon?

A # 2. The angles in a square are 90 degrees, greater than the 60-degree angles in a triangle. The angles in a regular 8-sided polygon have greater measure than the angles in a regular 7-sided polygon.

Q # 3. What happens to the angles in a regular polygon as you add more sides?

A # 3. The angles increase in measure. Imagine opening up a polygon with 6 sides to add a 7^{th} equal side. In order to fit the new side, all of the other sides must be spread out or opened up, increasing the measure of the angles.

Q # 4. Which polygons can be used to make regular tessellations of the plane?

A # 4. Only the equilateral triangle, square and regular hexagon can be used to make regular tessellations of the plane. As more sides are added, the angles get greater; 120 degrees is the biggest divisor of 360 that can be the measure of an interior angle of a regular polygon.

Essential Understanding #2:

Rigid transformations can be used to make interesting repeating patterns of figures.

Share - On Day 2, students participate in activity 1.1 *Notice and Wonder: Polygon Patterns*. Working in pairs, the students look at a set of 6 polygon patterns for 1 minute and record what they notice about the patterns and what they wonder about the patterns. Students share with their partner *what they noticed* about the polygon patterns and *what they wonder* about the polygon patterns. After sharing in pairs, students have a whole-class discussion. Student responses along with their reasoning are recorded. After each response, the students share if they agree or disagree and offer alternative ways of thinking about the patterns.

Discuss - Students work in pairs to describe tessellation patterns by using rigid motion words (translation, reflection, and rotation). Activity 1.3 #1. Student A describes one of 4 polygon patterns as Student B tries to identify which picture Student A is describing. They switch roles and repeat the activity. Students engage in a whole-group discussion about which word(s) were most helpful in identifying the polygon patterns and why they were helpful. The teacher records the word(s) so all students can see them for the Activity 1.3 # 2.

Analyze: Working in pairs, Student A describes a tessellation as Student B tries to produce or draw the tessellation from the description. To analyze the drawing, the students create a list of differences between Card A and the tessellation produced by Student B. (Step 3 of student handout)

Essential Understanding #3:

Rigid transformations can be used to create tessellations and designs with rotational symmetry.

Share - On Day 4, students create a tessellation of an assigned triangle (Activity *3.1 Triangle Tessellation****s).*** Students share their tessellations in their small group. In their groups, they decide if the tessellations are alike or if they are different and how.

Discuss - Students investigate the ability of a trapezoid to tessellate the plane. Students complete questions 1, 2, and 3 of the student handouts. In whole-group, the students discuss:

Q. Were you able to tessellate the plane with copies of the trapezoid?

A. Yes, two of the trapezoids can fit together to make a parallelogram and the plane can be tessellated with copies of the parallelogram.

Q. What did you notice about the trapezoid and 180-degree rotations?

A. The trapezoids fit together with no gaps and no overlaps and leave space for four more quadrilaterals.

Q. How do you know that there are no overlaps?

A. The sum of the angles in a quadrilateral is 360°. At each vertex in the tessellation, copies of the four angles of the quadrilateral come together.

Analyze - In a Think, Pair and Share activity, students are asked if **any** quadrilateral can be used to tessellate the plane. After thinking a few minutes, they share with a partner their reasoning. The class is polled for their opinions. Later in the lesson, the students are asked if they wish to revise their response to the poll based on analysis of the investigation with tessellating a trapezoid.

Share - On Day 5, students experiment with putting shapes together by either tracing pattern blocks of their choosing or by cutting out shapes from a blackline master to trace. At the end of the activity, the students share their findings in small groups.

Discuss - Students discuss what transformations they could perform on the Image 3 *Rotate That* figure so that it matches up with its original position.

Analyze - Students engage in a Gallery Walk to analyze the tessellations created by their classmates. Students study the designs of 3 students and must describe the transformations (translations, reflections, and/or rotations) that were used in creating the tessellations.

Task In Action

The video clips below provide a demonstration of the task being implemented in a classroom as it aligns with the Effective Mathematics Teaching Practice indicated. These clips should be used by the teacher to model the implementation of the task in his or her classroom.

- Implement Tasks That Promote Reasoning and Problem Solving:
    - **Video Clip #1**

- Use and Connect Mathematical Representations:
    - **Video Clip #2**

- Facilitate Meaningful Mathematical Discourse:
    - **Video Clip #3**

- Pose Purposeful Questions:
    - **Video Clip #4**

- Support Productive Struggle in Learning Mathematics:
    - **Video Clip #5**