WEST VIRGINIA
DEPARTMENT OF EDUCATION

MATHEMATICS

GRADE 4

The Locker Game

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Task Title: The Locker Game
Grade or Content Area: 4th Grade Mathematics
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Quarter: 1

Rationale for Lesson and Associated Tasks

Mr. Wolf 4th grade class of 20 students is playing a game in the hallway. This hallway is lined with 20 lockers in a row. In this game, the students will go through rounds of opening and closing locker doors. By the end of the game, all 20 students in the class have passed through the hallway one time each.

The Locker Game is a mathematical investigation task that allows students to deepen their understanding of factors and multiples of whole numbers. As a classic mathematical puzzle, this task provides opportunities for students to look for and make use of structure in the relationship with factors and multiples. It provides opportunities for students to look for and express regularity in repeated reasoning.

During this task, students will:

- Explore numbers with an odd number of factors, an even number of factors, or a larger number of factors
- Establish which lockers will be left open and which will be left closed
- Establish which lockers are visited the most frequently and why
- Establish which lockers are visited exactly twice and why

Lesson and Associated Tasks Overview

The Locker Game* (click here)
**Review all components thoroughly.*

The Task:
The 20 students in Mr. Wolf's 4th-grade class are playing a game in a hallway that is lined with 20 lockers in a row. The first student starts with the first locker and goes down the hallway and opens all the lockers. The second student starts with the second locker and goes down the hallway and shuts every other locker. The third student stops at every third locker and opens the locker if it is closed or closes the locker if it is open. The fourth student stops at every fourth locker and opens the locker if it is closed or closes the locker if it is open. This process continues until all 20 students in the class have passed through the hallway.

A. Which lockers are still open at the end of the game? Explain your reasoning.
B. Which lockers were touched by only two students? Explain your reasoning
C. Which lockers were touched by only three students? Explain your reasoning.
D. Which lockers were touched the most?

For Grade 4 students, to help them understand the context of this task, simulate what happens to the locker doors in the first few rounds. This can be done either using actual lockers, the task handout, the PowerPoint simulation (link in materials), or the interactive website simulation (link in materials). During this simulation, do not suggest a way to approach the solutions. The simulation serves as clarification as to what the students are to do when determining if a locker door is opened or closed at the end of each round.

Students will work with factors of 20. They will see that there is a relationship between the numbers, discover patterns, and realize numbers that are factors and multiples. The task can serve to connect students' prior knowledge regarding factors, factor pairs, prime, composite, and square numbers. This task is scheduled to be completed over a single class period:

Lesson Implementation Outline

- Introduce "*The Locker Game"* to students (whole class)
- Provide students with student view of "*The Locker Game"*
- Simulate first few rounds (optional)
- Students move to work groups
- Teacher circulates
- Small group share, discuss, and analyze
- Whole group share, discuss, and analyze

West Virginia College- and Career-Readiness State Standard

M.4.4

Find all factor pairs for a whole number in the range 1–100, recognize that a whole number is a multiple of each of its factors. Determine whether a given whole number in the range 1–100 is a multiple of a given one-digit number. Determine whether a given whole number in the range 1–100 is prime or composite.

Mathematical Habits of Mind (MHM)

MHM2. - Reason abstractly and quantitatively.
MHM3. - Construct viable arguments and critique the reasoning of others.
MHM7 - Look for and make use of structure.
MHM8 - Look for and express regularity in repeated reasoning.

Mathematics Teaching Practices to Support Student Growth

- Implement tasks that promote reasoning and problem solving.
- Use and connect mathematical representations.
- Facilitate meaningful mathematical discourse.
- Build procedural fluency from conceptual understanding.
- Support productive struggle in learning mathematics.

Essential Understandings

- An understanding of multiples and factors can be used as an effective tool in addressing problem solving strategies.
- An understanding of patterns and relationships between numbers can be used as an effective tool in reasoning and problem-solving strategies.

Set-up Phase

1. Become an Expert Regarding All Lesson and Associated Task Content

It is essential that the teacher complete the task prior to classroom implementation to understand how the task works and clearly appreciate the mathematics embedded in the task. Completing the task provides the opportunity to identify specific questions and challenges that may arise as students work

through the task. It also allows the teacher to better understand the variety of opportunities embedded in the task to deepen student understanding of factors and multiples.

Students may benefit from additional tools designed to organize their thinking (such as images of a row of lockers, and/or multiplication chart). Students may also benefit from a classroom conversation about a method to keep track of and record the status of the lockers (i.e., open or closed). This information will be valuable as students work to determine the locker's final status and the number of times a locker was touched.

Possible options the teacher may develop to help students with organizing and keeping track of touches to the locker are:

- The rows and columns on grid paper may provide an option to organize the student's record
- For students who struggle to record their findings in an organized manner, after a whole-group discussion in which students verify their results, the teacher may want to provide these students with a pre-created organized record of the task.

2. Establish Small Groups

The Locker Game promotes both individual and small group thinking. Students will explore their thoughts about patterns while gaining familiarity with factors and multiples relationships and contribute to meaningful conversations about factor pairs, prime numbers, and composite numbers while working in small groups.

Skill levels, leadership skills, and personalities are all considered when creating small (e.g. three students) groups. Small group collaboration works best when students have been provided previous opportunities to work together on a regular basis. When creating small groups for this lesson and associated tasks be mindful of student's leadership skills, personalities, their ability to take criticism, to question, and think deeply about a task or problem. These factors help to eliminate the potential situation in which one student takes the lead and makes the decisions for the group. In this scenario, one student is gaining all the benefits of the task, while others do not. If a student is not engaged in conversation, this lesson and associated tasks will not be beneficial in helping all students to have meaningful discussions about the mathematics involved nor in analyzing the relationships inherent to the tasks. Students will be working both individually and together when placed in small groups (e.g. three students) to complete this lesson and associated tasks.

3. Develop Open-Ended Questions

Teachers should create a list of open-ended questions designed to support and scaffold the lesson and associated tasks for their students. These questions should purposefully direct students towards provided information, previously learned content, and similarities and differences in their work versus group members.

Making Sense of the Problem:

- What problem are you trying to solve?
- What is it asking you to do?
- What question or questions are you attempting to answer?
- What are the important key words or numbers in the problem?
- What is your first step in getting started on this problem?)

- How could you use pictures, a number line, or manipulatives to help you solve this problem?
- Is this answer reasonable? Does it make sense? Why or why not?
- What do you already know that will help you?
- What information do you need to collect in order to answer the questions?

Patterns and Properties:

- What patterns or relationships do you see between these numbers?
- Do you see a pattern? If so, describe the pattern you see.
- What decisions can you make from the pattern that you noticed?
- What rule or math term that could help you?

Support or Contradict Their Ideas or the Ideas of Others:

- How did you show that your answer was correct?
- How can you explain it in a different way?
- Does _____'s answer make sense to you? Why or why not? What question can you ask that makes your answer more easily understood?
- Do you agree or disagree with the way _____ solved it? Why or why not?
- Tell me about your thinking.
- How did you know this was a reasonable answer?
- Tell me about your calculation. How do you know what to do?

Reason Abstractly, Quantitatively, or Concretely:

- How could you use a different operation or strategy to solve this problem?
- What do these numbers represent?
- What is the relationship between _____ and _____?
- How can you make a model to show your answer?
- Show me what you mean so I can see it too.
- Which representation is the most helpful to you and why? (chart, graph, diagram, table, equation, etc.)

Restate or Explain Their Strategies:

- How did you solve the problem?
- What strategy did you use?
- What math words did you use or learn?
- What were the steps you used?
- What have you discovered?
- How are these similar?
- How are these different?
- How did you know what operation to use?

Predict, Invent, or Problem Solve:

- What would happen if _____?
- How else might you solve the problem?
- What other choices do you have?
- What if _____?
- I really like how you did _____. Explain to me what to do next.
- Is there another way to do that?
- What do you predict the answer will be?

Reflect on Their Work and Make Connections:

- What problem/strategy were you investigating today?
- What questions arose as you worked?

- What were you thinking when you made decisions or selected the strategies to solve the problem?
- How does knowing _____ help you to answer the questions?
- What other math strategies can you connect to this?
- How is this like something you have done before?

List Generated From: "Guiding Questions for Math Tasks." *Create-Abilities*, (click here)

4. Gather Materials

- Task – The Locker Game student view (click here)
- Graph paper (optional)
- Paper with 20 lockers numbered 1-20 - (optional)
- Multiplication Chart - (optional)
- Pre-created record of the results of the task - (optional)
- Locker Game Simulation (click here) (optional)
- Locker Simulation PPT – click here or Webview - (optional) (Remember to hit the "Start PowerPoint" button at the top of the screen to view the entire PPT)
- Manipulatives with two sides to represent the open lockers and closed lockers (e.g., playing cards, two-color chips, etc.) – (optional)

5. Anticipated Common Student Misconceptions

- Students may think all odd numbers are prime, when in fact, several are composite, for example, 9 and 15.
- Students may think 1 is a prime number, please remind students that a prime number must have 2 factors, 1 and itself. Since 1 has only one factor, it is neither prime nor composite.
- When listing multiples of numbers, students may omit the number itself. Students should be reminded that the smallest multiple is the number itself. Multiples of 3: 3, 6, 9, 12, 15…
- When listing the factors of numbers, students may omit 1. Students should be reminded that 1 is a factor of all numbers.
- Students may think larger numbers have more factors. (Counterexample: 16 has six factors: 1, 2, 3, 6, 8, and 16; 19 has two factors: 1 and 19)
- Student misconceptions may relate to vocabulary (factors, multiples, prime, and/or composite) and an appreciation of the differences between factors and multiples, prime and composite, etc. Prompting students' prior knowledge of this vocabulary prior to introducing the task would be beneficial for all students, and especially for struggling students. The *Which One Doesn't Belong?* website might provide an opportunity through which students can collaborate to share their understanding of the needed vocabulary.

Explore Phase

Prior to completing this task with your students, the following content should have been reviewed (e.g. as "bell ringers" one week prior to the introduction of this lesson):

- Number patterns
- Skip-counting and its connection to the multiplication tables (counting groups of specific numbers and knowing how many groups have been counted).
- Decomposition of known facts
- Vocabulary: even, odd, factors, factor pairs, multiples, prime, composite, square numbers

Prior Instruction/Knowledge:

During grade four, students' prior understanding of and skill with decomposition of multiplication will contribute to their study of multiples and factors. Students will extend the idea of decomposition of multiplication and learn to use the term multiple as well as prime and composite.

Please review the following:

Educators Guide for Mathematics: Grade 4 (pages 10-12, pdf pages 12-14) (click here)

Prerequisite Skills

- Know and use division facts related to multiplication facts through 144.
- Describe and demonstrate patterns in skip counting and multiplication; continue sequences beyond memorized or modeled numbers.
- Multiply a multi-digit whole number by a 1-digit whole number or a 2-digit multiple of 10.
- Understand that many whole numbers factor in different ways.
- Find factors, common factors, and the greatest common factor of numbers; explain.

Supporting Skills

- Find the value of an unknown in a number sentence.
- Find factors, common factors, and the greatest common factor of numbers; explain.
- Write numbers using prime factorization.
- Find multiples, common multiples, and the least common multiple of numbers; explain.
- Write numbers using prime factorization.
- Use the identity properties for addition and multiplication and the zero property for multiplication.
- Identify and use the rules for divisibility (2, 3, 4, 5, 6, 9, and 10).

Impending Skills

- Find factors, common factors, and the greatest common factor of numbers; explain.
- Add and subtract fractions and mixed numbers with unlike denominators in number and word problems.
- Write and solve rational equations; identify extraneous solutions, including checking the solution in the original equation.
- Identify prime and composite numbers less than 100.
- Find the greatest common factor of a polynomial.
- Use rules of exponents to simplify numeric and algebraic expressions.
- Write numbers using prime factorization.

Source: *The Quantile Framework for Mathematics; https://metametricsinc.com/educators/quantile-for-educators/; 2020 MetaMetrics Inc.*

Implementation Phase

Lesson Implementation Outline

- Introduce "*The Locker Game"* to students (whole class)
- Provide students with student view of "*The Locker Game"*
- Simulate first few rounds (optional but recommended)
- Students move to work groups
- Teacher circulates
- Small group share, discuss, and analyze
- Whole group share, discuss, and analyze

Teacher Notes:
The teacher should provide each student with a copy of the task. After reading the prompt to introduce the task, provide students with a few minutes of independent work time to formulate questions about the task. To check for understanding, have students restate the task in their own words.

It may be beneficial to initially work through the opening and closing of the lockers together using one of the following if actual lockers are not available:

- Locker Game Simulation (click here) (optional)
  This simulation is set for 1000 students and 1000 lockers. Change the setting from **Simulation** to **Explore**, to be able to demonstrate the task by manually clicking on the individual lockers. This will allow a limited number of lockers used in the demonstration. When selected, "Change every ☐ locker(s):" will slowly go through all 1000 lockers.
- Locker Simulation PPT – click here or Webview - (optional) (Remember to hit the "Start PowerPoint" button at the top of the screen to view the entire PPT)
- Task – The Locker Game student view (click here)
- Manipulatives with two sides to represent the open lockers and closed lockers (e.g., playing cards, two-color chips, etc.)

The goal of the simulation is to make sure the students understand the problem. The teacher is not to suggest a way to solve the problem. This should be left up to the individual groups to decide. One problem-solving method that works well for one group may not work for all groups.

During the simulation, the students are utilizing an organizational method to make sure they understand the process necessary to find and record the data. If the students struggle with developing an organizational method, as a class or with the struggling group, develop one or two methods from which students in the class/group may choose. If students present more than one method, discuss the merits of each and decide as a class which method will be used to record the status of the lockers.

Possible organizational method:

		LOCKERS																			
		1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
STUDENT	1	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O	O
	2		C		C		C		C		C		C		C		C		C		C
	3			C			O			C			O			C			O		
	4				O				O				C				O				O
	5					C					O					O					C
	6						C						O						C		
	7							C							O						
	8								C								C				
	9									O									O		
	10										C										O
	11											C									
	12												C								
	13													C							
	14														C						
	15															C					
	16																O				
	17																	C			
	18																		C		
	19																			C	
	20																				C
# of students		1	2	2	3	2	4	2	4	3	4	2	6	2	4	4	5	2	6	2	6
Outcome		O	C	C	O	C	C	C	C	O	C	C	C	C	C	C	O	C	C	C	C

2 touches - prime
3 touches - perfect squares
6 touches - 6 factors
Number of touches = number of factors

O = Open
C = Closed

The teacher should circulate to identify any misconceptions and pose prompting questions for struggling groups. Prompting questions are ones in which the teacher gives appropriate clues that help the students develop and improve their answers.

Examples of prompting questions for struggling groups include:

- How many students will touch Locker 1? Who touches it? *(One student will touch Locker 1. Student 1 touches Locker 1.)*
- How many students will touch Locker 2? Who? Locker 3? Who*? (Two students will touch Locker 2. Students 1 and 2 will touch Locker 2. Two students will touch Locker 3. Students 1 and 3 will touch Locker 3.)*
- Who will touch Locker 10? *(Students 1, 2, 5, and 10 will touch Locker 10.)*
- Will anyone touch every single locker? Who? Will anyone else? *(Yes, Student 1 will touch every locker, but no one else will.)*
- If I am the 17-person, which lockers will I change? *(You would change Locker 17.)*
- How is the student number related to the lockers that students will change? (The student number is a factor of the locker that will be changed. OR The locker is a multiple of the student number.*)*

As student groups complete the opening and shutting of lockers, allow them to compare their results with students in other groups(s) and if necessary, revise their findings. As a class, compare and verify student findings.

After students have verified and/or revised their findings, they are prepared to address the task questions. (For students who had difficulty organizing findings and/or need to make multiple revisions to their data, it may be beneficial to provide them with a previously created document that illustrates the results.)

Student teams will use their data sheets to answer the task questions:

- Which lockers are still open at the end of the game? Explain your reasoning.
- Which lockers were touched by only two students? Explain your reasoning.
- Which lockers were touched by only three students? Explain your reasoning.
- Which lockers were touched the most?

After student groups collaborate to answer Questions A-D, allow students to compare and share their results in a whole-class discussion. Write the answer to the first question for all to see – the list of lockers that are open at the end of the task. Ask students if they notice anything interesting about these locker numbers.

- If students do not notice that each of these locker numbers is a square, do not mention it now. Allow students the opportunity to at a few more sets of locker numbers to see if they begin to notice and look for patterns. The class can always return to this list of numbers once they begin to look for patterns.
- If students do notice, celebrate their insight!

Continue to look at the sets of locker numbers and prompt students to look for patterns.

If students finish early, extension questions/ideas, such as the following, may be asked:

- Without using visual representations, determine the number of students who would touch each of the lockers if we had 50 students and 50 lockers? 100 students and 100 lockers?

- Choose a random locker number, write the rule for determining who will touch that locker. Make sure your rule applies to all locker numbers *(The students whose numbers are factors of the locker numbers will touch the locker.)*
- A student touches two different lockers. What do you know about the student number and the locker number? *(The student number is a common factor of the two locker numbers, and the locker numbers are multiples of the student number.)*
- If I give you two locker numbers, write the steps for the strategy you would use to determine which students touched both?
- After the answers to Questions A-D have been answered and the patterns in their numbers determined, ask students to see if they can find a connection between the numbers in the different sets. Because this conversation may spark additional insights, provide teams an additional 10-15 minutes to refine and/or extend their insights into connections between the sets of numbers. Based on teacher observations of student findings and reasoning, select different students/groups to share their group's results.

Based on teacher observations of student findings and reasoning, select different students/groups to share their group's results. If students are having difficulty seeing the relationships between the sets of numbers and factors and multiples, the teacher may want to set the stage for students to find one or more of the patterns included below.

Some of the patterns that student groups may notice include:

- Each of the locker numbers that was touched only two times are prime; the numbers in this set have only two factors.
- Each of the locker numbers touched exactly three times are squares; the numbers in this set have exactly three factors.
  As a formative assessment, if student teams identify this pattern or if the teacher presents this pattern, consider asking student teams to verify this conjecture by finding the three factors of each number in this set.
- Each of the locker numbers that remained open is a square number.
- Each of the locker numbers that was touched the most (6 times) is even; the numbers in this set have exactly six factors.
  As a formative assessment, if student teams identify this pattern or if the teacher presents this pattern, consider asking each student team to verify this conjecture by finding the six factors of a different number in this set.
- The only locker that was touched five times is also a square; this number has exactly five factors. Consider asking student teams to find these five factors.
- All lockers with numbers that are squares have been touched an odd number of times; each of these numbers has an odd number of factors.
- The locker that was touched only once is Locker #1; the number is neither composite nor prime; the number only has one factor.

Share, Discuss, and Analyze Phase

Essential Understanding #1: *An understanding of multiples and factors can be used as an effective tool in addressing problem solving strategies.*

Share - The investigation opens with students being presented the task where Mr. Wolf's students are in

the hallway opening and closing specific lockers. The goal of the investigation is to determine which lockers will remain open at the end of the game.

Discuss - During the discussion, it is important to remember to make the transition from the language of lockers and students to mathematics. Moving from lockers and students to factors and multiples is essential. In the language of lockers and students, the question would be "Which students touched both Locker 6 and Locker 15?" In mathematical language, the question would be, "What are the common factors of 6 and 15?" Once the students have some success in this transition, give them the opportunity to write other questions pairs showing the Locker to Mathematical that can be answered by analyzing their data.

Students might answer the questions without thinking about the underlying justifications for their answers. In the first question, for example, a student might say "1, 4, 9, 16 are all still open because I tried it out and those were the ones that were left open." If a student goes this route, the teacher can guide the conversation back to factors by asking the students if they notice anything special about the numbers of the lockers which are still open.

Student groups work together and discuss the following question with regards to The Locker Game:

- Q #1: Which lockers are still open at the end of the game? Explain your reasoning.
- Q #2: Which lockers were touched by only two students? Explain your reasoning
- Q #3: Which lockers were touched by only three students? Explain your reasoning.
- Q #4: Which lockers were touched the most

Analyze - Students review, analyze, and respond to The Locker Game questions:

- Q #1: What rule or math term could help you?
- Q #2: Is there a connection between a given locker number and the numbers of the students who touch it?
- Q #3: How can you determine how many students have touched a specific locker?

Essential Understanding #2: *An understanding of patterns and relationships between numbers can be used as an effective tool in reasoning and problem-solving strategies.*

Share- The investigation opens with students being presented the task where Mr. Wolf's students are in the hallway opening and closing specific lockers. The goal of the investigation is to determine which lockers will remain open at the end of the game.

Discuss- Student groups work together and discuss the following question with regards to The Locker Game:

- Q #1: Which lockers are still open at the end of the game? Explain your reasoning.
- Q #2: Which lockers were touched by only two students? Explain your reasoning
- Q #3: Which lockers were touched by only three students? Explain your reasoning.
- Q #4: Which lockers were touched the most?

Analyze- Students review, analyze, and respond to The Locker Game questions:

- Q #1: What patterns or relationships do you see between the student's number and the locker number that was touch?
- Q #2: Do you see a pattern?
- Q #3: Describe the pattern you found.
- Q #4: Were all the patterns useful or were some more effective in predicting which lockers were left open?
- Q #5: What decisions can you make from the pattern that you noticed?
- Q #6: How can you use the patterns you discovered to make predictions?

Task In Action

The video clips below provide a demonstration of the task being implemented in a classroom as it aligns with the Effective Mathematics Teaching Practice indicated. These clips should be used by the teacher to model the implementation of the task in his or her classroom.

- Establish Mathematics Goals to Focus Learning:
  - Video Clip #1
- Implement Tasks That Promote Reasoning and Problem Solving:
  - Video Clip #2
- Facilitate Meaningful Mathematical Discourse:
  - Video Clip #3
- Pose Purposeful Questions:
  - Video Clip #4
- Support Productive Struggle in Learning Mathematics:
  - Video Clip #5