WEST VIRGINIA
DEPARTMENT OF EDUCATION

MATHEMATICS

GRADE 9

Create Your Own Trail Mix

Modeling with Systems of Inequalities in Two Variables

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Task Title: *Create Your Own Trail Mix:* Systems of Inequalities in Two Variables
Grade or Content Area: 9th Grade
Toolkit Author:
Original Task Creator: Illustrative Mathematics
Quarter:

Rationale for Lesson and Associated Tasks

In *Create Your Own Trail Mix*, students generate a mathematical model of a system of inequalities in two variables for a familiar product – trail mix. Students choose quantities of two ingredients; determine how to represent the ingredients and quantities in a mathematical model using inequalities and a graph. Students use the model they create to make choices about their trail mix recipe.

Before students can successfully create a mathematical model for their trail mix, they must first build an understanding of inequalities, graphing inequalities, and determining a solution set based on the graph. Therefore, prior to the culminating task, *Create Your Own Trail Mix*, students complete tasks to develop understanding for graphing and solving problems using systems of linear inequalities.

Inequalities are arguably used more often in "real life" than equalities. Businesses use inequalities to control inventory, plan production lines, produce pricing models, and for shipping/warehousing goods and materials. Hurricanes are classified using inequalities that involve wind speed and storm surge. Inequalities are used too as a limiting factor: you must be taller than 42" to ride a ride at a park, shorter than 6'2" to be a pilot, or weigh at least 100 pounds to donate blood. Therefore, it is important for students to understand and interpret inequalities.

Linear inequalities can be particularly challenging for students to understand, solve, and construct. Students often treat inequalities as equations and expect that only one value will make the inequality true. This “single number answer” tendency is widespread and persists even after instruction (Vaiyavutjamai and Clements 2006). Additionally, when students solve inequalities, they often focus on manipulating symbols and do not think holistically about the meaning and the reasonableness of solutions. Writing inequalities to represent contextualized scenarios is also challenging; such errors as reversing variables and incorrectly representing relationships between quantities are common (MacGregor and Stacey 1993).

This lesson and associated tasks build understanding of inequalities while providing purpose for the constraint. By mathematizing situations in their daily lives, students move beyond solving textbook inequalities to using mathematics as a tool to represent and reason about real-world situations.
Sources:

- eNotes – (click here)
- *Creating Inequalities from Real-World Experiences*, Mathematics Teaching in Middle School, NCTM, November 2016, Vol. 22, Issue 4

Lesson and Associated Tasks Overview

Several lessons from Illustrative Mathematics have been blended to provide students the opportunity to explore Modeling with Systems of Inequalities in Two Variables. In Days 1 and 2, students write systems of inequalities in two variables using technology to graph the solutions; find solution sets by reasoning and graphing; and take a closer look at whether points on the boundary lines of the system's solution region are included in the solutions. In Days 3 and 4, the students interpret and analyze models that represent the constraints and conditions in a situation; and create their own models after specifying quantities of interest, identifying relevant information, and setting the constraints.

The following resources from Illustrative Mathematics provide the backbone for the task guide. Review all components (Preparation, Teacher Lesson Guide, Student Lesson, and Practice) of the tasks thoroughly. Both digital and print formats are available for most tasks.

Illustrative Mathematics: Algebra I – Unit 2 Linear Equalities, Inequalities, and Systems – Lesson 24
Solutions to Systems of Linear Inequalities in Two Variables

Preparation: (click here)
Teacher Guide: (click here)
Student Lesson: (click here)
Practice: (click here)
Cool Down Task 24.5 *Oh Good, Another Riddle*
Desmos Teacher Directions

Illustrative Mathematics: Algebra I – Unit 2 Linear Equalities, Inequalities, and Systems – Lesson 25
Solving Problems with Systems of Linear Inequalities in Two Variables

Preparation: (click here)
Teacher Guide: (click here)
Student Lesson: (click here)
Practice Problems: (click here)
Task 25.3 Info Gap Cards Blackline Master
Cool Down Task 25.4 *Widgets and Zurls*

Illustrative Mathematics: Algebra I – Unit 2 Linear Equalities, Inequalities, and Systems – Lesson 26
Modeling with *Systems of Linear Inequalities in Two Variables*

Preparation: (click here)
Teacher Guide: (click here)
Student Lesson: (click here)
Practice Problems: (click here)
A Trail Mix Recipe (Student Handout)
Nutrition Data (Student Handout)

This lesson and associated tasks are scheduled to be **completed over four class periods** per the suggested sequence. Depending on your students' previous work, the lesson can be entered at day one, two, or three. The following is an overview of the lessons.

Day 1

1. Introduce Learning Targets for the lesson:
   - Describe the graphs that represent solutions to a system of inequalities.
   - When given descriptions and graphs that represent two different constraints, find values that satisfy each constraint individually and values that satisfy both constraints at once.
2. Review graphing inequalities:
   Introduce, play, and discuss YouTube video *Linear Inequalities Graphing Explained!* (10:33)
3. Engage students in Desmos activity Point Collector Lines

4. Engage students in writing systems of inequalities representing situations and find the solutions by graphing using task 24.3 *Remember These Situations*
5. Engage students in task 24.4 Scavenger Hunt
6. Closure activity: Respond to task 24.5 *Oh Good, Another Riddle* in Student Journal

Day 2

1. Introduce Learning Target for Day 2 tasks:
   Determine if a point on the boundary of a graph of the solutions to a system of inequalities is or is not a solution.
2. Engage students in warm-up task 25.1 *Which One Doesn't Belong: Graphs of Solutions*
3. Explore with students' what points on the boundary lines are solutions to a system using task 25.2 *Focusing on the Details*
4. Engage students in 25.3 *Info Gap: Terms of a Team*
5. Closure activity: Respond to task 25.4 *Widgets and Zurls* in Student Journal

Day 3

1. Review learning target for the day: interpret inequalities and graphs in a mathematical model
2. Determine if an ordered pair is a solution for a given inequality using the warmup in task 26.1
3. Determine if an ordered pair is a solution for a system of inequalities using the *x* and *y*-axis as boundaries for the system
4. Graph system of inequalities using the Desmos App
5. Create a possible trail mix recipe based on the inequality graphs used in the mathematical model

Day 4

1. Review learning target for the day: Choose variables, specify constraints, and write inequalities to create a mathematical model
2. Determine constraints for a trail mix by choosing ingredients, quantities and nutrition amounts from the chart used for task 26.2
3. Graph inequalities for constraints. Determine answer region.
4. Determine 2 possible combinations of ingredients for small group trail mix recipe.
5. Gallery Walk: students display mathematical model for trail mix consisting of inequalities and graph.

West Virginia College- and Career-Readiness State Standards (Integrated / Traditional)

M.1HS.7 / M.A1HS.7

Represent constraints by equations or **inequalities**, **and by systems of** equations and/or **inequalities**, and interpret solutions as viable or non-viable options in a modeling context. (e.g., Represent inequalities describing nutritional and cost constraints on combinations of different foods.) Instructional Note: Limit to linear equations and inequalities.

M.1HS.11 / M.A1HS.17

Graph the solutions to a linear inequality in two variables as a half-plane (excluding the boundary in the case of a strict inequality) and **graph the solution set to a system of linear inequalities in two variables** as the intersection of the corresponding half-planes.

M.1HS.2 / M.A1HS.2

Define appropriate quantities for the purpose of descriptive modeling. Instructional Note: Working with quantities and the relationships between them provides grounding for work with expressions, equations and functions.

Mathematical Habits of Mind (MHM)

The tasks relate to all Standards for Mathematical Practice, but have an emphasis on:

MHM. 1. Make sense of problems and persevere in solving them.
MHM. 2. Reason abstractly and quantitatively.
MHM. 4. Model with mathematics.
MHM. 6. Attend to precision.

Mathematics Teaching Practices to Support Student Growth

The tasks relate to all Mathematics Teaching Practices, but have an emphasis on:

MTP.2 Implement tasks that promote reasoning and problem solving.
MTP.3 Use and connect mathematical representations.
MTP.4 Facilitate meaningful mathematical discourse.
MTP.5 Pose purposeful questions.

Essential Understandings

- The characteristics of linear inequalities and their representations are useful in solving real-world problems.
- The solution to a system of inequalities is the point(s) that the inequalities have in common.
- A mathematical model presented as a system of inequalities and the corresponding graph can be used to interpret a mathematical task.
- A mathematical model can be generated by identifying and using constraints for a real-world problem to create a system of inequalities and the corresponding graph to model the solution.

Set–up Phase

1. Become an Expert Regarding All Lessons and Associated Tasks Content

It is imperative that the teacher becomes very familiar with all *Create Your Own Trail Mix* curriculum and instructional materials: Teacher Preparation, Teacher Lesson Guides, Student Lessons, YouTube video, Desmos *Point Collector Lines* online activity, and student handouts. *Create Your Own Trail Mix* has several associated tasks and some tasks may be delivered via printed and/or digital resources.

The implementation of the tasks may vary class to class depending on the time frame available for the lesson and the levels of student engagement and understanding. Suggestions for *Students with Disabilities* and *English Language Learners* are provided within the lesson.

2. Establish Small Groups

Create Your Own Trail Mix tasks and activities engage students in both individual and group thinking. Working individually, in pairs and in small groups, students will explore and share their reasoning about graphing real-world scenarios.

Small group instruction has significant impact on student achievement (Hattie, 2009) and allows teachers to work more closely with each student. This type of instruction provides the opportunity to monitor students' learning and identify gaps in the development of students' math skills.

Teachers will need to instruct students on how to work in small groups. The first step in the process is to establish ground rules and norms for interaction. Students must have a part in making the rules and these guidelines must be enforced by both teachers and students. Ground rules should encourage positive collaborative behaviors among all students. Guidelines/ground rules need to be posted in the classroom so students can readily refer to them. If students or teachers believe that additional rules are needed, they can be added later.

Teachers should assign groups intentionally based on skills and/or backgrounds. Skill levels, leadership skills, and personalities must all be considered when creating small groups. Small group collaboration works best when students have been provided previous opportunities to work together on a regular basis. Small group instruction gives teachers an opportunity to assess more closely what each student can do and build strategic plans around those assessments. Students who struggle to ask questions and participate in a whole group setting may thrive in a small group where they feel more comfortable and less overwhelmed. Furthermore, small group instruction tends to proceed at a fast pace, which typically helps students maintain focus and enjoy the learning experience.
Source: *Small Group Instruction: How to Make it Effective*, CORE: Excellence in Education Blog, September 27, 2018.

3. Develop Open-Ended Questions

Teachers of *Creating Your Own Trail Mix: Modeling with Systems of Inequalities in Two Variables* should use open-ended questions to support and scaffold the lesson and associated tasks for their students. These questions should purposefully direct students towards provided information, previously learned content, and similarities and differences in their work versus other group members. Within the context of open-ended mathematical tasks, teachers should select questions with four purposes in mind:

A. *Starter questions*

Starter questions focus the students' thinking in a general direction and give a starting point. Starter questions might include:

- What details are important? (Task 24.3)
- What do you know about the graph of a system of inequalities? (Tasks 24.3 and 25.2))
- What are the criteria for checking if points are a solution? (Task 25.2)
- Can you tell me one of the rules? (Task 25.3)
- Why do you need to know the rules? (Task 25.3)
- What specific information do you need? (Task 25.3)
- Why do you need the information? (Task 25.3)
- What happens if inequalities $x > 0$ and $y > 0$ are included? (Task 26.2)

B. *Questions to stimulate mathematical thinking*

These questions focus on particular strategies aiding the formation of a strong conceptual network. The questions can serve as a prompt when students become 'stuck'. Questions might include:

- What do the variables represent? (Task 24.3)
- How are the graphs the same? (Task 24.4 Lesson Synthesis)
- How are the graphs different? (Task 24.4 Lesson Synthesis)
- How do you know what ingredients each person used? (Task 26.2)

C. *Assessment questions*

Assessment questions ask students to explain what they are doing or how they arrived at a solution. These open-ended questions are best asked after students have made progress with the problem.

- How do you know if a system has no solution? (Task 24.4)
- Can you tell the number of solutions from the set of graphs? If so, how? (Task 24.4 Lesson Synthesis)

- Why does only Graph A represent a system of inequalities? (Task 25.1)
- How can the graph help you choose a recipe for your trail mix? (Task 26.3)

D. *Final discussion questions*

These questions draw together the efforts of the class and prompt sharing and comparison of strategies and solutions. This is a vital phase in the mathematical thinking processes. It provides further opportunity for reflection and realization of mathematical ideas and relationships. It encourages students to evaluate their work. Final discussion questions might include:

- Is everyone's graph the same? Why or why not? (Tasks 24.4 and 26.3)
- Can all rules be written as an inequality? Why or why not? (Task 25.3)
- What constraints did everyone use? (Task 26.3)
- How do the graphs of everyone's trail mix compare? (Task 26.3)

*Additional questions are provided in the Teacher Guides.

Source: *Using Questioning to Stimulate Mathematical Thinking*, NRICH, February 2011.

4. Gather Materials

- Computer and presentation device
- Internet access
- Graphing calculators or other graphing technology
- Online video: YouTube video *Linear Inequalities Graphing Explained!* (10:33) (click here)
- Online app: Desmos *Point Collector Lines* (click here)
- Illustrative Mathematics Student handouts:
  (24.3 *Remember These Situations;* 24.4 *Scavenger Hunt;* 24.5 *Oh Good, Another Riddle;* 25.1 *Which One Doesn't Belong: Graphs of Solutions;* 25.2 *Focusing on the Details;* 25. 3 *Info Gap: Terms of a Team;* 25.4 *Widgets and Zurls)*
- Illustrative Mathematics Handout Teacher Presentation Materials for task 26.2 used to create a chart for Trail Mix ingredients. Copy of this chart provided for each small group.
- Copy of A Trail Mix Recipe Handout provided for each small group
- Copy of Nutrition Data Handout provided for each small group
- Colored pencils
- Chart paper and markers
- Graph paper
- Student Journals

5. Anticipated Common Student Misconceptions

Day 1: Task 24.3 *Remember These Situations?*

Students may need reminding what a solution to the system would be in the given contexts.

Task 24.4 Scavenger Hunt

Some students may wonder if a point in either of the shaded regions on the graph could be where an item is hidden. Ask students to pick a point on the graph and consider whether it satisfies the first inequality, and then whether it satisfies the second inequality. Remind the students that a solution to a system needs to satisfy both.

Day 2: Task 25.3 Info Gap: Terms of a Team

Students might wonder if all clues on the data card constitute a rule. The student holding the data card may not know how to respond if asked, "What is the first rule?" or "What is one of the rules?" The students who are asking for information may not know if what is given counts as a rule. Clarify that a rule should be general enough to include multiple possibilities – not just one specific case.

Day 3: Task 26.2 Custom Trail Mix

When students analyze the graph for the system of inequalities, they may not understand if an ordered pair found on a boundary line is part of the solution. Additionally, students do not understand the difference between horizontal and vertical regions as related to the inequality to be graphed. Remind students that solutions must be part of the common shaded area for all inequality graphs.

Day 4: Task 26.3 Create Your Own Trail Mix

When students generate inequalities based on the determined restraints, they may confuse when to use >, ≥, <, or ≤. If a restraint is at least an amount, students mistakenly create a less than inequality, <, but the inequality needs ≥ symbol. If a restraint is at most for an amount, it needs the ≤ sign. Additionally, students must determine when the inequality includes the boundary as opposed to when it does not. For example, if a student wants to eat less than 2,000 calories each day, the boundary is not included, but if the students want to eat at least 35 grams of fiber each day, the boundary is included. Remind students to create restraints, that is inequalities, for the variables. Discuss with students if the variable can represent positive values, negative values, or both negative and positive values. Students must also determine if zero is included in the restraint

Explore Phase

One of the most important goals of instruction in mathematics is to illuminate connections between different mathematical concepts. In middle school, students build an understanding of how variables, expressions, equations, and **inequalities** are used to **represent quantities and relationships**. Students **make connections among different kinds of representations—algebraic, verbal, tabular, and graphical**.

In this lesson, students **further develop their capacity to create, manipulate, interpret, and connect mathematical representations and to use them for modeling.** Prior to teaching *Create Your Own Trail Mix,* students will need ample opportunities (e.g. "bell ringers") to explore and/or revisit these three standards:

- Solve systems of linear equations (exactly and approximately with graph), focusing on pairs of linear equations in two variables.
- Solve problems with inequalities in two variables.
- Graph linear inequalities in two variables.

Prior Instruction/Knowledge

In grade 8, students developed the skills and understandings needed to solve linear equations with one variable, and systems of linear equations that may also have one solution, an infinite number of solutions, or no solutions. The students discovered these cases as they graphed systems of linear equations and solved them algebraically. By making connections between algebraic and graphical solutions and the context of the system of linear equations, students made sense of their solutions. Students in grade eight also solved real-world problems leading to two linear equations in two variables.

Grade 9 students explore inequalities in two variables by **relying on their understanding of equations**. They see that inequalities are a handy way to express constraints that involve an upper or lower limit and can be satisfied by a range of values rather than a single value.

Please review the following:
Educators' Guide for Mathematics: Grade 8 (pages 17-18, pdf pages 19-20) (click here)
Educators' Guide for Mathematics: Math I (pages 15, 18 and 23, pdf pages 17, 20, 22) (click here)
Educators' Guide for Mathematics: Algebra I (Pages 20, 26 and 35, pdf pages 22, 28, 37) (click here)

Prerequisite Skills

- Graphically solve systems of linear equations
- Determine whether a linear equation has one solution, infinitely many solutions, or no solution
- Determine algebraically or graphically the solutions of a linear inequality in two variables

Impending Skills

- Solve systems of equations or inequalities algebraically and graphically that include nonlinear relationships.
- Use linear programming (systems of three or more inequalities) to solve problems.

Source: *The Quantile Framework for Mathematics* 2020 MetaMetrics Inc.

Implementation Phase

Day 1

1. Introduce Learning Targets for the lesson:
   - Describe the graphs that represent solutions to a system of inequalities.
   - When given descriptions and graphs that represent two different constraints, find values that satisfy each constraint individually and values that satisfy both constraints at once.
2. Distribute Lesson 24 Solutions to Systems of Linear Inequalities in Two Variables task handouts:
   - 24.3 Remember These Situations
   - 24.4 Scavenger Hunt
   - 24.5 Oh Good, Another Riddle
3. Review graphing inequalities:
   Introduce, play, and discuss YouTube video *Linear Inequalities Graphing Explained!* (10:33)
4. Engage students in Desmos activity Point Collector Lines – Screens 1-8: (click here)
   by graphing using task 24.3 *Remember These Situations*
5. Engage students in task 24.4 Scavenger Hunt
6. Closure activity: Respond to task 24.5 *Oh Good, Another Riddle* in Student Journal

Day 1 Teacher Notes:
To introduce the day's tasks, briefly explain the Learning Targets:

- Describe the graphs that represent solutions to a system of inequalities
- When given descriptions and graphs that represent two different constraints, find values that satisfy each constraint individually and values that satisfy both constraints at once.

Distribute colored pencils, graphing calculators or other graphing technologies, and the student handouts:

- 24.3 *Remember These Situations*
- 24.4 *Scavenger Hunt*
- 24.5 *Oh Good, Another Riddle*

1. Arrange students in groups of 2-4 members.
2. Review systems of equations and their solutions by asking students (whole class):
   Q. What does the solution to a **linear equation in two variables** represent?
   A. The solution to a linear equation in two variables is any pair of numbers that makes the equation true.
   Q. What does the solution to a **system of two equations in two variables** represent?
   A. A solution to a system of two equations in two variables is a pair of numbers that make both equations correct.
   Q. What do you remember about **how to solve a linear equation in two variables**?
   Record all student responses and display for all students to see. Be sure to record student language used (e.g. constraints, variables, *x*-axis, *y*-axis, solution point, etc.)
3. Play the YouTube video, *Linear Inequalities Graphing Explained!* (10:33)
4. Discuss in whole-class setting what the students learned from the video. Record the responses for all students to see.
   - Q. Was there new information in the video? If so, what was new to you?
   - Q. How is graphing linear inequalities the same as graphing linear equations?
   - Q. How is graphing linear inequalities different from graphing linear equations?
5. Review graphing inequalities using the online Desmos activity, *Point Collector Lines*.
   The activity may be facilitated whole class by using presentation devices (computer and data projector, Mimio, or other whiteboard) or students may log onto the site individually or in small groups using iPads or other internet-ready technologies. You may want to create a Class Code on Desmos if individual students or small groups will be logging onto the site. **Note: Prior to teaching the lesson, read the Desmos Teacher Directions, and use the Student Preview on Desmos to personally, experience the activity.** Open the Desmos *Point Collector Lines* site and use the student preview to inform students how the activity operates. Model how students respond by completing the Warmup activities #1 – #4 with the whole class using the instructor's presentation devices.
6. After making sure students understand how the Desmos activity works, tell students to complete the Challenge activities # 1 - # 4 on screens 5 through 8. Open the Teacher Moves and Sample Responses tabs at the bottom of the screen for additional information as you lead the class through the Challenge activities.
7. If students are working individually or in small groups to complete Challenges #1 - # 4, circulate the room to provide individual help and questions when appropriate. As students work, emphasize there is no need to find the optimal solution in one move. Tell the student to simply try something – anything. Then consider how to improve the result.
   For students who need help, consider asking them to describe the ideal boundary line, whether to include the boundary line (e.g., $<$ vs. $\leq$), and which direction to shade. Help students translate their description into an inequality.

9. Assign one of the three situations from task 24.3 Remember These Situations? to each group of students. Ask students to read the information provided about the situation and to answer the first question independently. Allow 5-6 minutes for student work. Circulate the room to provide individual help and questions when appropriate.
10. At the end of the work time, ask students to share their responses to question #1 with their group members. If different responses are given, the group should come to a consensus for the answer.

11. After sharing their system of inequalities to represent the constraints of the situation assigned, ask students to continue with the task by completing with their group members questions 2 and 3. Inform students that all group members must record the solutions on the student task handout.
12. Conduct whole-class discussion of the task. Have at least two groups share their solutions to each of the situations. During the discussion, emphasize the meaning of a point in the region where two graphs of linear inequalities overlap. Make sure students understand that all points in that region represent values that simultaneously meet both constraints in the situation. Ask students:
    - Q. Why does it make sense to think of the two inequalities in each situation as a system and find the solutions to the system, instead of only to individual inequalities?
    - A. If both constraints in the situation must be met, then we need to find values that satisfy both inequalities.
13. Ask students to **put away the graphing devices** as they will not be used for the next activity.
14. Introduce the situation involved in task **24.4 Scavenger Hunt**
15. The locations of the items can be narrowed down by solving the systems. A coordinate plane can be used to describe the solutions. **Assign each group of students one of the clues to solve and graph**.
16. Provide 7-8 minutes for the group work. Circulate the room to provide individual help and questions when appropriate.
17. Invite students to share their graphs and strategies for finding the solution regions. Draw attention to the group that found the system which had no solutions.
18. To help students make the connection between systems of equations and systems of inequalities, display the two graphs given under **Lesson Synthesis**. Ask students in a whole-class setting:
    - Q. How are the two sets of graphs alike?
    - A. The graphs have the same two lines. The graphs can tell the solutions to individual equations or inequalities, as well as the solutions to systems.
    - Q. How are they different? The first set of graphs show two regions that overlap, bounded by dotted lines. The second set shows two intersecting lines and the lines are solid. One set represents the solutions to a system of linear inequalities.
    - Q. How can we tell the number of solutions from each set of graphs?
    - A. The graph representing a system of equations shows one point of intersection, so there is only one solution. The graphs representing a system of inequalities shows one region of overlap, but there are many points in that region. This means that there are many solutions.
19. As closure activity and informal assessment, ask students to complete task **24.5 *Oh Good, Another Riddle*** and respond to Questions #1 - #3 in their Student Journals.

Day 2

1. Review the Learning Targets and tasks from Day 1
2. Introduce Learning Target for Day 2 tasks:
   Determine if a point on the boundary of a graph of the solutions to a system of inequalities is or is not a solution.
3. Engage students in warm-up task 25.1 Which One Doesn't Belong: Graphs of Solutions.
4. Explore with students what points on the boundary lines are solutions to a system using task 25.2 *Focusing on the Details*
5. Engage students in 25.3 *Info Gap: Terms of a Team*
6. Closure activity: Respond to task 25.4 *Widgets and Zurls* in Student Journal

Day 2 Teacher Notes:

Organize students into groups of 2-4 members. Review the Day 1 Learning Targets and tasks by having students share their journal entries with their group members.

To introduce the tasks for Day 2, briefly explain the Learning Target:

- Determine if a point on the boundary of a graph of the solutions to a system of inequalities is or is not a solution.

Distribute colored pencils and the student handouts: 25.1 *Which One Doesn't Belong: Graphs of Solutions;* 25.2 *Focusing on the Details;* 25.3 *Info Gap: Terms of a Team;* 25.4 *Widgets and Zurls*

1. Display the graphs of the warm-up task **25.1 *Which One Doesn't Belong: Graphs of Solutions*** for all students to view.
2. Ask students to independently study the graphs for 2 minutes looking for graph(s) that do not belong. Tell students they must be able to explain why the selected graphs(s) do not belong.
3. At the end of 2 minutes, ask students to share their thinking with their small group. As a group, the students must come to consensus about which graph(s) do not belong and why.
4. Engage students in a whole-class discussion by asking each group to share with the class one reason why a particular graph does not belong? Record and display the responses for all students to see. Ask students to explain any terminology they use such as infinitely many solutions or boundary line. Since there is no single correct answer to the question, carefully listen to students' explanations, and ensure the reasons given are valid. Press students on unsubstantiated claims.
5. Inform students they are about to **take a closer look at whether points on the boundary lines are solutions** to a system of inequalities.
6. Display or project the system of inequalities and graph of task **25.2 *Focusing on the Details*** for all students to see. Ask students to think independently about which region represents the solutions to each inequality given in the task. Give students 3 minutes of quiet time and tell students they must be able to explain their selections.
7. At the end of 3 minutes, have students share with their thinking about the inequalities with the members of their group.
8. Follow the group sharing with a whole class discussion. Look for multiple entry points to the task:
   - Most students are likely to identify the inequality representing each graph by considering the equation of the boundary line (solid line vs dotted line)
   - Other students may test some of the coordinate pairs in each region to see if the pairs make the inequality true.

   Make sure these strategies for connecting the algebraic and graphical representations are discussed.
9. Ask students to focus on the given solutions to the system. Ask them to decide whether each point is a solution and to be prepared to explain how they know
10. When all students have an answer for each point, conduct class discussion. Focus the discussion on the points on the boundary lines and how the students determined if the points are or are not solutions to the system
11. Rearrange students into groups of 2 and ask students to put away any graphing devices.
12. Explain the structure of the task **25.3 *Info Gap: Terms of a Team***. Tell students they will either be a data card player or a problem card player. Discuss the list of activities for each player found on the student handout 25.3 Info Gap: Terms of a Team.
13. Once students understand what to do based on the card received, distribute either a Problem Card 1 or a Data Card 1 to each team member. Remind students they are not to show the card received

to their partner.

14. Direct students to read the problem card or data card received and to begin the activity.
15. As students finish the first problem, check their work.
16. Give students a new set of cards to repeat the activity. If the student received the Problem Card in problem 1, he/she receives the Data Card for problem 2. If the student received the Data Card in problem 1, he/she receives the Problem Card for problem 2.
17. After all students have completed the two problems, invite students who solved the problems by graphing a system of inequalities to share their graphs and thinking. Project the students' graphs and engage the students in a whole-class discussion.
    - Q. Which membership rule does each shaded region on the graphs represent?
    - Q. Can all the clues be written as an inequality? If so, what are they?
    - Q. Should we graph them all? Why or why not?
    - Q. How do the problem 1 graphs help us answer questions about whether 6 adults and 8 children are acceptable?
    - Q. How do they tell us about the maximum allowable number of adults on a team?
    - Q. How does the second set of graphs help us find the minimum number of team members and the composition of the team?
18. Project the solution graphs for Task 25.3: Info Gap: Terms of a Team for all students to see. Reinforce the importance of attending carefully to the boundary lines of the regions by asking:
    - Q. In the graphs of problem 1, are points on the two lines solutions to the system? Give students two points to consider such as (14, 2) and (8, 4).
    - A. Yes. Points on a solid line are included. Substituting 14 and 2 or 8 and 4, makes the inequality true.
    - Q. In this situation, are points on the horizontal axis solutions to the system?
    - A. Yes. Points on the horizontal axis represent no adults. The rules say that a team with only children is allowed.
    - Q. In the graphs of problem 2 are points on the boundaries of the overlapping triangular region solutions to the system?
    - A. Yes.
19. Direct students to read task 25.4 *Widgets and Zurls* and respond to questions 1-3 in their journals.

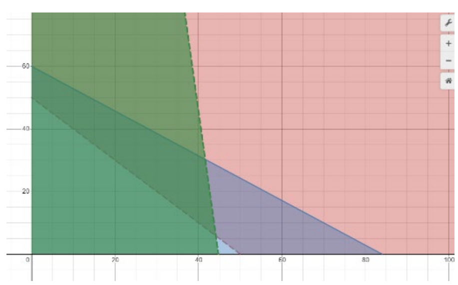
Day 3

1. Review learning target for the day: interpret inequalities and graphs in a mathematical model.
2. Using the Desmos App, complete warmup in Task 26.1.
3. Determine if an ordered pair is a solution for a system of inequalities using the *x* and *y*-axis as boundaries for the system.
4. Introduce Task 26.2, Interpreting a Mathematical Model for Jada and Tyler's trail mix
5. Graph the system of inequalities for Jada or Tyler's constraints using the Desmos App.
6. Determine ingredients for Jada and Tyler's trail mix using the mathematical model provided in Task 26.2.
7. Create a possible trail mix recipe based on the inequality graphs used in the mathematical model.
8. Discuss solutions found by students for both Jada and Tyler's recipes.

Day 3 Teacher Notes:

Provide each small group (2-4 members) with the trail mix ingredients chart and a computer or other device to run the online Desmos App.

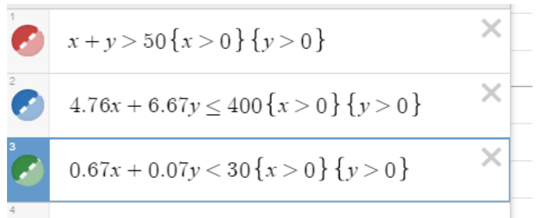
1. Review learning target for the day: interpret inequalities and graphs in a mathematical model.
2. Students determine if (5.43, 0) is a solution for a given inequality using warmup in Task 26.1
3. Project warmup Task 26.1 or write the problem on the whiteboard. (click here)
4. Whole group class discussion to determine student understanding and to correct student misconceptions. (5.43, 0) is a solution for $x > 0$, $x \geq 0$, and $y \geq 0$, but not $y > 0$.
5. Working in small groups, students describe the graph for the following inequalities: $x \geq 0$, $x > 0$, $y > 0$, and $y \geq 0$
    - Describe the difference between graphs $x > 0$ and $x \geq 0$
    - Describe the solution for the graph $y \geq 0$
    - Describe the solution for $x > 0$. Is the *y*-axis included in the solution?
6. Working in small groups, students sketch the solution for the following system of inequalities. $x > 0$ and $y > 0$. Students should notice the solution is the first quadrant and the boundaries are not included in the solution.
7. Students determine if the following ordered pairs are found in the answer region for the inequality graph $x > 0$ and $y > 0$: (3, 5), (-2, 0), (0, 6), (7, -2)
   As students work in small groups, circulate the room. Check with each group to determine student understanding and address misconceptions as a whole class:
    - Students do not understand when the boundary is part of the solution
    - Students do not understand the difference between a horizontal and vertical region as related to the inequality
    - Students do not correctly label quadrants when addressing answer regions
8. Introduce **Task 26.2** and provide students with a copy of the trail mix chart found in **Task 26.2**.
   A link for the word document Teacher Presentation Materials is found at the bottom of the teacher preparation page using the following link: (click here)
   The chart is on page 2 of the word document. The document may be edited as needed.
9. Project the inequalities for Jada and Tyler (or write inequalities on the whiteboard) found in Activity 26.2. Do not give students access to the provided graphs until later in the activity.



10. Working in small groups, student "notice and wonder" at least 5 things from the information found in the chart and inequalities for Jada and Tyler.
    Q: What do the variables in the inequalities represent in terms of units?
    A: The variables represent the ingredients chosen. The units are grams.
    Q: What do you notice regarding the inequality signs?
    A: Answers will vary. Discuss with students each inequality sign.
    Example: $x + y > 50$ *from Tyler's recipe*
    x grams of chocolate added to y grams of coconut must total more than 50 grams of the mixture
    Q: What do the coefficients of the variables represent in terms of units?
    A: The coefficients represent the nutritional amount per gram for each column

14. Working in small groups, students graph either Jada or Tyler's inequalities using Desmos (www.desmos.com/graphing). Assign each group which person to graph.
Desmos is a free app that can be downloaded by both teachers and students.
Open Desmos: www.desmos.com/graphing

The following screens are examples from Desmos using Tyler's inequalities.



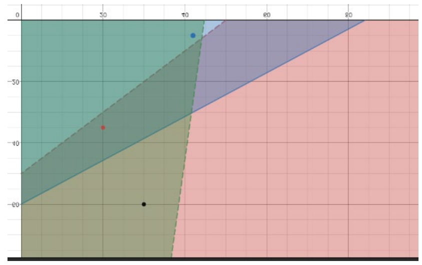
- Students do not need to rearrange the inequalities when using the Desmos App
  For this lesson, leaving the inequalities in standard form eliminates possible computation errors.
- The braces at the end of each inequality limit the graph to the first quadrant.
- To change the window, use . Adjust values until all inequalities are visible.

1. Discuss the following as a class before determining the ingredients used:
   Q. What is the difference between a dotted and solid line?
   A. A dotted line means the boundary is not included and a solid line means the boundary is included as part of the solution.
   Q: How does the shaded portion of the graph relate to the inequality sign?
   A: Answers may vary. Students may notice that shading for a greater than symbol is above the graph, while shading for a less than symbol is below the graph. However, remind students that this may not be the case if the dependent variable is negative in standard form.
   Q: What if the coefficient for two variables in two different equations was the same?
   A: The nutritional amount for both is the same.

16. Working in small groups, students determine which ingredients Jada or Tyler used for their trail mix.
Analyze decisions based on constraints and graph using the following questions.
Project from the front of the room, or create a handout with the following questions:
Q: Which two ingredients did Tyler or Jada choose?
Q: How did you determine what each variable represents?
Q: How did you determine what each constraint represents?
Q: Which graph represents each constraint?
Q: If two quantities used the same value, how did you determine which ingredient was represented by the inequality?
Students record their answers in their student journal.

17. As a whole group, students discuss which ingredients Tyler and Jada used for their trail mix.

18. Working in groups, students test possible combinations for their trail mix using the Desmos App.
Tyler: x = grams of chocolate pieces and y = grams of coconut
Below demonstrates how to graph ordered pairs in Desmos: 35) is in the answer region for the model. This represents a trail mix consisting of 20 grams of chocolate pieces and 35 grams of coconut pieces meeting all restraints for the model.



19. Students determine a possible trail mix recipe and record ingredients and grams for the recipe.
20. Discuss as a whole class combination of the recipe for both Jada and Tyler.

Day 4

- Review learning target for the day: Choose variables, specify constraints, and write inequalities to create a mathematical model
- Introduce activity: Design your own trail mix recipe (Task 26.3).
- Determine constraints for a different trail mix by choosing ingredients, quantities, and nutrition amounts from the chart used for Task 26.2 and create an equality statement for each.
- Determine 2 possible combinations of ingredients for small group trail mix recipe.
- Gallery Walk: students display mathematical model for trail mix.
- Discussion of the mathematical model using activity synthesis in Task 26.3

Day 4 Teacher Notes:
Provide each small group with a handout, A Trail Mix Recipe, (attached document) to record information for this activity.
Provide each small group with the Nutrition Data chart (attached document).
Students continue using the chart with ingredients for trail mix from Day 1.
Provide students with graph paper, a computer or other device for Desmos, colored pencils, ruler, and chart paper.

1. Review learning target for the day: Choose variables, specify constraints, and write inequalities to create a mathematical model
2. Introduce activity: Design Your Own Trail Mix Recipe (**Task 26.3**)
   This activity allows students to determine their own recipe for trail mix using the same chart of ingredients from Day 1. Provide each small group with a chart displaying daily nutritional requirements for calories, protein, sugar, fat, and fiber. The ***Nutrition Data*** chart (attached document) can be used for this purpose.
3. In small groups students use the Trail Mix chart and the Nutrition Data chart to come to a group consensus for the following:
   - Which nutritional requirements are important to your group to create a good trail mix recipe (choose 2 – 3 nutritional requirements)? Explain your reasoning.
   - How can you determine the total number of grams for sugar, fat, protein, and fiber?
   - Which 2 ingredients did your group choose for the trail mix recipe?
   - How many grams of trail mix do you need to make for your small group to share (50 grams = $\frac{1}{4}$ cup)
4. Students in small groups determine constraints for their trail mix recipe based on the following:
   Provide students with A Trail Mix Recipe handout to record their decisions

- Nutritional requirements chosen as important for your small group
- Ingredients chosen for the trail mix
- Quantity of trail mix
- Total grams for nutritional needs
- Identify variables for trail mix
- Restrictions on the chosen variables

5. Working in small groups, students discuss each restraint and determine an inequality to represent the constraint for the mathematical model. Students need inequalities for the variable for each ingredient, total quantity of trail mix, and total quantity for nutritional requirements.
   - Students discuss the ingredients to use for the recipe and label the ingredients with a variable.
     Example: Ingredients for trail mix recipe: Peanuts and Almonds
     x = peanuts in grams   y = shredded almonds in grams
   - **Restraints for variables**: Students discuss possible values for variables.
     Example: Inequalities for restraint: $x > 0$ and $y > 0$
   - Students discuss quantity totals for trail mix: students determine the quantity needed.
     Example: Each person in the small group wanted at least 50 grams (1/4 cup) of trail mix.
     There are 3 people in the group so at least 150 grams of trail mix is needed
   - Students decided to focus on calories and fat found in the trail mix mixture. Students discussed the total calories and fat that should be in the mixture using the provided charts.
     Example: Students decided to allow 44 grams of fat for the entire recipe.
     They determine the coefficients for the variables from the chart:
     $$0.46x + 0.67y \leq 44$$
6. Students use the inequalities to create a graph representing each inequality.
   As students create inequalities, circulate the room to determine student understanding. The following are suggestions for questions to ask students as they create inequalities
   - Is every constraint represented by an inequality statement?
   - Does the inequality sign correctly reflect the restraint?
     Example: Create at most 100 grams of trail mix
     $x + y \leq 100$
7. Students graph the created system of inequalities using a different color for each restraint.
   Note: Since most of the inequalities contain decimals, it may be easier to graph using intercepts rather than using slope-intercept form
   Note: Notice any computation errors students make as they graph the inequalities.
8. After all inequalities are graphed, determine the region for the answer and label this region.
9. Using Desmos, students graph the system of inequalities.
   Graph each inequality in Desmos with restrictions for the variables:
   Review with students how to enter the inequality with braces for the restrictions on the variables.
   Students will need to edit the window to display all inequalities.
10. Students compare the graph generated with pencil and paper with the graph using Desmos.
11. Students determine at least 2 possible combinations for their trail mix. Check the combinations using the Desmos graph. Record combinations on the Trail Mix Recipe handout.
12. Create a mathematical model for your work including inequalities and graph on chart paper. Do not identify ingredients for your recipe or include mixtures from your small group for the recipe. Display chart together with the graph generated in Desmos for the gallery walk.
13. Gallery walk - students display their mathematical models for the gallery walk. Allow students 5 minutes to look at each small group's model.

14. Small-Group Assessment: Students determine trail mix ingredients and a possible combination of ingredients for the trail mix recipe for another small group.
15. As a whole class discussion, each group shares with the class ingredients chosen and a combination for their trail mix.
16. Class discussion of the mathematical model Task 26.3 using the synthesis found at the conclusion of the activity:

    Students record answers to the following questions in their student journals.

    - What constraints did every group use?
      The variables for the ingredients must be positive, so each group should have included $x > 0$ and $y > 0$ as inequalities for the model
    - How do the graphs of the various mixes compare?
      Responses will vary.
    - Did anyone need to revise their model to create a solution they could use?
      Responses will vary. If students could not locate a solution for the recipe, they would need to change the amount for the total.
    - How did you use the graph to choose a recipe for the trail mix?
      The ordered pairs for the recipe must be in the portion of the graph shaded for all inequalities.
    - What is a possible trail mix recipe you could use?
      Answers will vary.

Share, Discuss, and Analyze Phase

Essential Understanding #1*:*

The characteristics of linear inequalities and their representations are useful in solving real-world problems.

Share – In **Task 24.3 *Remember These Situations,*** students write systems of inequalities representing real-life situations and find solutions by graphing. The students explore inequalities involving bank accounts, concert ticket purchases, and advertising packages. Students work together – sharing ideas and strategies - to solve and graph one of the systems of inequalities and come to a consensus for the answer.

Discuss – In **Task 24.3 Remember These Situations,** much discussion occurs in small groups. Whole class discussion of **Task 24.3 *Remember These Situations*** allows all students to experience all three of the real-life situations.

Q. Why does it make sense to think of the two inequalities in each situation as a system and find the solutions to the system, instead of only to individual inequalities?

A. If both constraints must be met, then we need to find values that satisfy both inequalities.

Analyze – In **Task 24.3 Remember These Situations, s**tudents must analyze the situation given, determine the constraints and write a system of inequalities.

Essential Understanding #2:

The solution to a system of inequalities is the point(s) that the inequalities have in common.

Share – In **Task 25.3 Info Gap: Terms of a Team*,*** students use their understanding of systems of linear inequalities to solve problems that involve satisfying multiple constraints simultaneously. The task requires students to make sense of a problem and persevere in solving it. To be successful in solving the problem, the paired students must share with each other.

Discuss – In **Task 25.3 Info Gap: Terms of a Team**, students need to determine what information is necessary to solve the problem, ask for the information, and then explain their requests. This process takes several rounds of discussion if the first requests do not yield the information needed. It also requires students to refine the mathematical language they use and ask increasingly more precise questions until they get the information needed.

Analyze – In **Task 25.3 Info Gap: Terms of a Team, s**tudents must analyze the situation given, determine the constraints, and write a system of inequalities. In the task synthesis, students display their graphs and then analyze the different solutions:

Q. Which membership rule does each shaded region on the graphs represent?

Q. Can all the clues be written as an inequality? If so, what are they?

Q. Should we graph them all? Why or why not?

Essential Understanding #3:

A mathematical model presented as a system of inequalities and the corresponding graph can be used to interpret a mathematical task.

Share – Working in small groups, students discuss the following questions using the chart and inequalities from Jada and Tyler.

Q: What do the variables in the inequalities represent in terms of units?

A: The variables represent the ingredients chosen. The units are grams.

Q: What do you notice regarding the inequality signs?

A: Answers will vary. Discuss with students each inequality sign.

Q: What do the coefficients of the variables represent in terms of units?

A: The coefficients represent the nutritional amount per gram for each column

Discuss – As students compare their graph with the graph in the activity (projected in the front of the room), answer the following questions using a whole-class discussion:

Q: What is the difference between a dotted and solid line?

A: A dotted line means the boundary is not included as part of the solution and a solid line means the boundary is included as part of the solution

Q: How does the shaded portion of the graph relate to the inequality sign?

A: Answers may vary. Students may notice that shading for a greater than symbol is above the graph, while shading for a less than symbol is below the graph. However, remind students that this may not be the case if the dependent variable is negative in standard form.

Q: What does it mean if the coefficient for two variables in two different equations was the same?

A: The nutritional amount for both ingredients is the same value.

Q: How do you determine the answer region for your model?

A: The common shaded area for all inequalities.

Analyze – Working in small groups, students determine which ingredients Jada and Tyler used for their trail mix. Students determine a possible trail mix recipe for Jada and Tyler based on the mathematical model. Analyze decision-based on constraints and graph using the following questions:

Q: Which two ingredients did they choose?

Q: How did you determine what each variable represents?

Q: How did you determine what each constraint represents?

Q: Which graph represents which constraint?

Q: If two quantities used the same value, how did you determine which ingredient was represented by the inequality?

Essential Question #4:
A mathematical model can be generated by identifying and using constraints for a real-world problem to create a system of inequalities and the corresponding graph to model the solution for the system of inequalities.

Share – In small groups, students use the Trail Mix chart and the Nutrition Data chart to form a group consensus for the following:

- Which nutritional requirements are important to your group to create a good trail mix recipe (choose 2 – 3 nutritional requirements)? Explain your reasoning.
- How can you determine the total number of grams for sugar, fat, protein, and fiber?
- Which 2 ingredients did your group choose for the trail mix recipe?
- How many grams of trail mix do you need to make for your small group to share?
  50 grams = $\frac{1}{4}$ cup

Discuss – Students create a set of constraints based on the mathematical task to create a trail mix recipe. Students discuss each restraint and create an inequality to represent the constraint. The final piece of the model is generated by graphing the inequalities on a coordinate system.
Students need inequalities for the variable for each ingredient, total quantity of trail mix, and total quantity for nutritional requirements.

- Students discuss the ingredients to use for the recipe and label the ingredients with a variable.
  Example: Ingredients for trail mix recipe: Peanuts and Coconut
  x = peanuts in grams   y = shredded coconut in grams
- **Restraints for variables**: Students determine the variables must represent positive values.
  Example: Inequalities for restraint: $x > 0$ and $y > 0$
- Students discuss quantity totals for trail mix: students determine the quantity of trail needed.
  Example: There are 3 people in the group so at least 150 grams of trail mix is needed
  $$x + y \geq 150$$
- Students decided to focus on calories and fat found in the trail mix mixture.
  Example: Students decided to allow 44 grams of fat for the entire recipe.
  They determine the coefficients for the variables from the chart.
  $$0.46x + 0.67y \leq 44$$

Analyze – Following the gallery walk, students analyze their mathematical model using the following questions first in their small groups, and then as a whole class.

Q. What constraints did every group use?
A. The variables for the ingredients must be positive, so each group should have included $x > 0$ and $y > 0$ as inequalities for the model
Q. How do the graphs of the various mixes compare?
A. Responses will vary.
Q. Did anyone need to revise their model to create a solution they could use?
A. Responses will vary. If students could not locate a solution for the recipe, they would need to change the amount for the total.
Q. How did you use the graph to choose a recipe for the trail mix?
A. The ordered pairs for the recipe must be in the portion of the graph shaded for all inequalities.
Q. What is a possible trail mix recipe you could use? A. Answers will vary